This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

EducT
118
75.441

Educ T 118.75.441

HARVARD COLLEGE
Feb. 6, 1940
LIBRARY

Harold F. Eastman

A
KEY
TO THE
NATIONAL ARITHMETIC,
EXHIBITING THE OPERATION OF
THE MORE DIFFICULT QUESTIONS
IN THAT WORK;

FOR THE USE OF TEACHERS ONLY.

By BENJAMIN GREENLEAF, A. M.,
AUTHOR OF THE "COMMON SCHOOL ARITHMETIC," "ALGEBRA," ETC.

NEW ELECTROTYPE EDITION.

BOSTON:
ROBERT S. DAVIS & CO.
NEW YORK: BAKER, PRATT, & CO., 142 & 144 GRAND STREET.
PHILADELPHIA: J. A. BANCROFT & COMPANY.
ST. LOUIS: GRAY, BAKER, & COMPANY.
CHICAGO: JANSEN, McCLURG, & CO.
1875.

Educ T 118.75.441

LIBRARY

GREENLEAF'S SERIES OF MATHEMATICS.

1. NEW PRIMARY ARITHMETIC; OR, MENTAL ARITHMETIC, on the Inductive Plan; with Easy Exercises for the Slate. Designed for Primary Schools. 84 pp.

2. NEW INTELLECTUAL ARITHMETIC, on the Inductive Plan; being an Advanced Intellectual Course, for Common Schools and Academies. 180 pp.

3. COMMON-SCHOOL ARITHMETIC; OR, INTRODUCTION TO THE NATIONAL ARITHMETIC. A Complete Treatise. Improved electrotype edition. 324 pp.

4. THE NATIONAL ARITHMETIC, being a complete Course of Higher Arithmetic, for advanced Scholars in Common Schools, High Schools, and Academies. New electrotype edition, with additions and improvements. 444 pp.

5. NEW ELEMENTARY ALGEBRA; in which the First Principles of Analysis are progressively developed and simplified. For Common Schools and Academies. 324 pp.

6. NEW HIGHER ALGEBRA; an advanced Analytical Course, for High Schools, Academies, and Colleges. 394 pp. [*Just Published.*]

7. ELEMENTS OF GEOMETRY, with Practical Applications to Mensuration. 320 pp.

8. ELEMENTS OF TRIGONOMETRY, with Practical Applications and Tables.

9. ELEMENTS OF GEOMETRY AND TRIGONOMETRY; or the last two named works in one volume. 490 pp.

10. TREATISE ON SURVEYING AND NAVIGATION; with Practical Applications and Tables. [*In Preparation.*]

☞ KEYS to the ARITHMETICS, ALGEBRAS, GEOMETRY AND TRIGONOMETRY. *For Teachers only.* 6 volumes.

Entered according to Act of Congress, in the year 1837, by
BENJAMIN GREENLEAF,
in the Clerk's Office of the District Court of the District of Massachusetts.

Entered according to Act of Congress, in the year 1857, by
BENJAMIN GREENLEAF,
in the Clerk's Office of the District Court of the District of Massachusetts

Entered according to Act of Congress, in the year 1865, for renewal, by
LUCRETIA K. GREENLEAF, widow, and BETSEY P. KENDALL, EMILY A. TEWKSBURY,
and LYDIA K. SYKES, children, of BENJAMIN GREEFLEAF,
in the Clerk's Office of the District Court of the District of Massachusetts.

RIVERSIDE, CAMBRIDGE:
PRINTED BY H. O. HOUGHTON AND COMPANY.

PREFACE.

THE object of the author, in this publication, is to aid the teacher in communicating instruction to his pupils, and in detecting any error which they may have made in the operation of the examples.

Every instructor who has a large number of scholars under his care is aware that it is a great tax on his time, especially when in school, to examine the operation of many arithmetical questions; whereas, by the aid of a Key, he may readily detect any mistake in the operation. Besides, amid the labors of the school-room, it is often very difficult for the most able arithmetician to recollect, at the moment, all the principles involved in the solution of difficult questions; but, by recurring to a Key, this difficulty will be obviated.

The author would recommend to teachers never to point out *directly* to the pupil the method of solving a problem, nor perform the labor for him; but suggest and explain such principles as will enable him to perform the question himself.

The answers to all the examples in the Arithmetic are inserted in the Key, for the convenience of those teachers who may prefer to use the edition of the Arithmetic which does not contain the answers.

B. GREENLEAF.

BRADFORD, MASS., *September*, 1857.

CONTENTS.

PAGE
Numeration, 5
Addition, 5
Subtraction, 6
Multiplication, 7
Division, 8
Cancelation, 9
Contractions in Division, 12
Problems, 14
United States Money, 16
Addition of United States Money, 17
Subtraction of " " 17
Multiplication of " " 17
Division " " " 17
Practical Questions by Analysis, 18
Bills, 18
Ledger Accounts, 20
Reduction of Compound Numbers, 20
Addition of Compound Numbers, 38
Subtraction of Compound Numbers, 39
Multiplication of " 40
Division " " 42
Principles and Applications, 43
Difference of Latitude, 45
Difference of Longitude, 46
Longitude and Time, 46
Examples by Analysis, 49
Factoring, 51
Divisors or Measures, 52
Common Divisors, 53
Greatest Common Divisor, 54
Least Common Multiple, 54
Common Fractions, 57
Common Denominator, 59
Addition of Common Fractions, 59
Subtraction of " 60
Multiplication of " 62
Division " " 66
Reduction of Complex Fractions, 70
Proposed Numerator or Denominator, 75
Common Numerator, 75
Greatest Common Divisor, 76
Least Common Multiple of Fractions, 77
Reduction of Denominate Fractions, 78
Addition of Denominate Fractions, 82
Subtraction of " " 83
Examples by Analysis, 87
Decimal Fractions, 90
Addition of Decimals, 91
Subtraction of Decimals, 92
Multiplication of Decimals, 93
Contractions in Multiplication of Decimals, 93
Division of Decimals, 94
Contractions in Division of Decimals, 94

PAGE
Reduction of Decimals, 94
Infinite or Circulating Decimals, 103
Transformation of Repetends, 104
Addition of Circulating Decimals, 104
Subtraction of Circulating Decimals, 106
Multiplication of Circulating Decimals, 107
Continued Fractions, 108
Ratio, 111
Analysis by Ratio, 114
Simple Proportion, 118
Compound Proportion, 122
Conjoined Proportion, 125
Percentage, 130
Interest, 135
Promissory Notes, 142
Compound Interest, 146
Discount and Present Worth, 151
Banking, 153
Stocks, 156
Brokerage and Commission, 157
Account of Sales, 158
Profit and Loss, 158
Partnership, 161
Bankruptcy, 166
Taxes, 167
General Average, 168
Equation of Payments, 169
Average of Accounts, 172
Account of Storage, 177
Insurance, 180
Life Insurance, 181
Custom-House Business, 182
Coins and Currencies, 183
Exchange, 184
Arbitration of Exchange, 186
Alligation, 188
Involution, 193
Evolution, 194
Extraction of the Square Root, 194
Extraction of the Cube Root, 194
Extraction of any Root, 195
Horner's Method of Extracting Roots, 196
Application of Powers and Roots, 199
Arithmetical Progression, 209
Geometrical Progression, 210
Annuities, 212
Permutations and Combinations, 213
Analysis by Position, 214
Scales of Notation, 220
Duodecimals, 223
Mensuration, 226
Solids, 232
Tonnage, 236
Miscellaneous Examples, 237

KEY

TO

GREENLEAF'S ARITHMETIC.

NUMERATION.

1. **(Art. 40, p. 22.)** 29
2. 407
3. 23,007
4. 5,000,027
5. 7,205,005
6. 2,207,604,009
7. 105,909,308,201
8. 9,000,000,008,000,000,046
9. 15,000,000,000,031,000,017
10. 507,000,000,000,203,000,057,000,018
11. 9,000,000,000,000,000,047,007,002,000,392
12. 15,000,000,000,000,000,000,000,000,010,127,026,320,426

Note. — The above is the French method.

ADDITION.

(Art. 47, p. 26.)

17.	30,530	24.	300,000	31.	3,837,156
18.	31,643	25.	264,088	32.	150
19.	26,798	26.	357,477	33.	26,199
20.	28,578	27.	276,605	34.	264
21.	34,383	28.	3,980,839	35.	4,801,393
22.	29,340	29.	4,183,478	36.	5,067,696
23.	283,649	30.	31,881,050	37.	5,640,426

38.	4,344,737	42.	62,075	46.	119
39.	4,935,497	43.	10,601	47.	$228
40.	1,937,678	44.	11,087	48.	$ 37,443
41.	118,106	45.	$ 82,871		

(Art. **48**, p. **28.**)

2.	296	4.	25,976	6.	936,318
3.	1,832	5.	643,322	7.	23,191,876

SUBTRACTION.

4.	(Art. **52**, p. **31.**)	5,676	24.	408,881,883,715
5.		5,119	25.	61,475,423
6.		4,409	26.	999,999
7.		2,589	27.	1
8.		48,447	28.	6,686,136
9.		46,698	29.	760,702,380
10.		17,672	30.	31,309,891
11.		53,859	31.	16,680,605
12.		411,001	32.	10,014,098,379
13.		426,944	33.	85
14.		6,202,102	34.	110
15.		799,081	35.	190
16.		1,439	36.	993,044
17.		92,690	37.	$11,810
18.		243,334	38.	
19.		617,441	39.	173
20.		900,981	40.	1,026
21.		98,999,080	41.	4,004
22.		788,889	42.	45 and 38
23.		9,393,239,896,461	43.	519,853,026

1.	(Art. **53**, p. **33.**)	138	5.	1,237,311
2.		525 dollars.	6.	2,500,000
3.		389 dollars.	7.	49,632 dollars.
4.		9,187 dollars.	8.	572,206 dollars.

MULTIPLICATION.

5.	(Art. **63**, p. **40.**) 3,156,492	26.	59,784
6.	6,172,835	27.	3,545,304
7.	1,979,796	28.	584,720,181,340
8.	30,316,704	29.	594,731,545
9.	16,294,896	30.	119,109,094,835
10.	22,204,188	31.	406,781,410,014
11.	45,005,091	32.	318,697,622,634
12.	77,377,566	33.	230,896,467,247
13.	293,468,329	34.	137,260,338,494
14.	161,539,842	35.	213,255,462,816
15.	274,135,320	36.	395,018,272
16.	17,247,986,832	37.	70,136,114,040
17.	$26,645	38.	475,065,601,536
18.	$5,529	39.	20,406,081,008,060,402
19.	$2,779	40.	915,527,086,788,307
20.	$21,053	41.	454,115,186,861,492
21.	13,505	42.	12,032,109,124,168,023
22.	24,386	43.	81,000,108,000,036
23.	$4,886	44.	52,370,625
24.	4,888	45.	114,972
25.	9,021	46.	29,657,416,470,704

(Art. **64**, p. **42.**)

2.	252,801	5.	2,639,559,272
3.	$11,025	6.	897,264
4.	2,784		

(Art. **64**, p. **43.**)

3.	18,190	8.	63,126,063,000
4.	410,600	9.	3,720
5.	70,000,000	10.	$888,000
6.	9,594,000,000	11.	$2,050,000
7.	700,000,000	12.	2,850,000,000

DIVISION.

(Art. 77, p. 49.)

	Quotients.	Rem.		Quotients.	Rem.
8.	25,569	2	30.	17,327	
9.	151,617	2	31.	69,255	1
10.	66,930	2	32.	71,451	
11.	12,090,447	2	33.	8,650	111
12.	20,747	8	35.	4,598	297
13.	39,936	4	36.	226,447	174
14.	260,171	28	37.	5,091	5091
15.	68,241	80	38.	7,060,504	4267
16.	111,946,492	1	39.	88,888	2341
17.	23,762,387	2	40.	800,008	4567
18.	20,166,474	1	41.	908,007,004	8765
19.	17,964,186	4	42.	2,069	
20.	33,081,425	3	43.	2,700 pounds.	
21.	13,698,246	4	44.	134	
22.	26,316,692	1	45.	987	
23.	169,739,167	3	46.	17	
24.	133,557,795	1	47.	$85\frac{148}{365}$	
25.	129,629,629	3	48.	$384\frac{8}{13}$ hours.	
26.	126,984,126	6	49.	\$12,402	
27.	17,166	0	50.	35	
28.	153,227	44	51.	A's 76; B's 68; C's 48.	
29.	275,175	0			

(Art. 78, p. 51.)

3.	321		7.	138	45
4.	308	1	8.	273	18
5.	38	38	9.	$121\frac{19}{45}$	
6.	507	40			

(Art. 79, p 52.)

3.	12,345,678	9	8.	37	411,111
4.	9,876,543	0	9.	89,765	432,156
5.	5	2,100	10.	164,000	
6.	11	91,853	11.	$\$21.42\frac{6}{7}$	
7.	3	137,851	12.	$494\frac{152}{192}$	

CANCELLATION.

(Art. **85**, p. **55**.)

3. $\dfrac{\overset{2}{\cancel{24}} \times 16}{\underset{1}{\cancel{12}}} = 32.$

4. $\dfrac{\overset{3}{\cancel{48}} \times 8}{\underset{1}{\cancel{16}}} = 24.$

5. $\dfrac{\overset{1}{\cancel{7}} \times \overset{5}{\cancel{10}} \times \overset{\cancel{2}}{\cancel{12}} \times 5}{\underset{\cancel{2}}{\cancel{14}} \times \underset{9}{\cancel{18}} \times \cancel{6}} = \frac{25}{9} = 2\frac{7}{9}.$

6. $\dfrac{15 \times \overset{1}{\cancel{7}} \times \overset{1}{\cancel{27}} \times \overset{\cancel{4}}{\cancel{40}}}{\underset{\cancel{2}}{\cancel{54}} \times \underset{\cancel{2}}{\cancel{14}} \times \underset{1}{\cancel{10}} \times 2} = \frac{15}{2} = 7\frac{1}{2}.$

7. $\dfrac{\overset{1}{\cancel{13}} \times \overset{5}{\cancel{15}} \times \overset{\cancel{2}}{\cancel{20}} \times 5}{\underset{2}{\cancel{26}} \times \cancel{10} \times \cancel{2} \times \cancel{3}} = \frac{25}{2} = 12\frac{1}{2}.$

8. $\dfrac{\overset{4}{\cancel{28}} \times \overset{1}{\cancel{27}} \times \overset{\cancel{3}}{\cancel{21}} \times 15 \times \overset{\cancel{2}}{\cancel{18}}}{\underset{1}{\cancel{7}} \times \underset{\cancel{2}}{\cancel{54}} \times \underset{1}{\cancel{7}} \times \cancel{3} \times \cancel{9}} = 60.$

9. $\dfrac{\overset{2}{\cancel{56}} \times 11}{\cancel{28}} = 22.$

10. $\dfrac{\overset{2}{\cancel{14}} \times \overset{8}{\cancel{24}} \times \overset{1}{\cancel{9}}}{\underset{\cancel{7}}{\cancel{63}} \times \underset{1}{\cancel{3}}} = 16$ cents.

11. $\dfrac{5 \times \overset{\cancel{3}}{\cancel{12}} \times 7}{\cancel{3} \times \cancel{4}} = 35.$

12. $\dfrac{\cancel{8} \times \overset{2}{\cancel{60}} \times \overset{\overset{5}{\cancel{15}}}{\cancel{75}}}{\underset{\cancel{3}}{\cancel{90}} \times \underset{\cancel{5}}{\cancel{40}}} = 10.$

(Art. **87**, p. **57**.)

(2.)	(3.)	(4.)
63013 × 17	79245 × 19	32067812 × 16
441091	713205	192406872
1071221	1505655	513084992

(Art. **88**, p. **57**.)

(2.)	(3.)	(4.)
6651 × 108	111223 × 104	2042 × 1009
53208	444892	18378
718308	11567192	2060378

(Art. 89, p. 58.)

(2.)
13317 × 51
66585
679167

(3.)
71389 × 21
142778
1499169

(4.)
12062 × 91
108558
1097642

(Art. 90, p. 58.)

(2.)
8360 × 7001
58520
58528360

(3.)
10613 × 801
84904
8501013

(4.)
91603 × 2001
183206
183297603

(Art. 91, p. 59.)

(2.)
915
$22\frac{2}{5}$
1830
1830
20130 = product by 22.
366 = product by $\frac{2}{5}$.
20496 = product by $22\frac{2}{5}$.

(3.)
$1224\frac{1}{7}$
18
9792
1224
22032
$2\frac{4}{7} = \frac{1}{7}$ of 18.
$22034\frac{4}{7}$

(4.)
180
$69\frac{1}{6}$
1620
1080
30 = $\frac{1}{6}$ of 180.
12450

(Art. 92, p. 59.)

(2.)
8)6805600
850700

(3.)
4)179240
44810

(4.)
6)19237800
3206300

(5.)
8)12345678000
1543209750

(6.)
3)3130
$1043\frac{1}{3}$

(7.)
3)53400
17800

(8.)
4)771000
192750

(9.)
6)9168000
1528000

(10.)
8)1993000
249125

(11.)
3)28044000
9348000

(ART. 93, p. 61.)

(3.) 61370913
96488
490967304 = the product by 8.
2945803824 = the last product × 6.
5891607648 = the last product × 2.
5921556653544

(4.) 8649347864 multiplicand.
1325769612 multiplier.
103792174368 = the product by 12.
830337394944 = the foregoing product × 8 for 96.
4982024369664 = the last product × 6 for 576.
1141713918048 = the first product × 11 for 132.
11467042561708308768 product, Ans.

(ART. 94, p. 61.)

(2.)
77777770000
7777777
77769992223

(3.)
41623100000
416231
41622683769

(4.)
987654000000
987654
3)987653012346
329217670782

(5.)
87654300000
876543
3)87653423457
29217807819
2
58435615638

(6.)
9999990000
999999
9998990001

(7.)
325678950000
32567895
3)325646382105
108548794035

(8.)

$$\begin{array}{r} 6666600000 \\ 66666 \\ \hline 3)6666533334 \\ \hline 2222177778 \\ 2 \\ \hline 4444355556 \end{array}$$

(9.)

$$\begin{array}{r} 91234567800 \\ 912345678 \\ \hline 90322222122 \end{array}$$

(10.)

$$\begin{array}{r} 12345670000 \\ 1234567 \\ \hline 12344435433 \end{array}$$

(11.)

$$\begin{array}{r} 98123452000000 \\ 98123452 \\ \hline 98123353876548 \end{array}$$

CONTRACTIONS IN DIVISION.

(Art. **95**, p. **62**.)

(2.)

$$\begin{array}{r} 89630 \\ 3 \\ \hline 26889|0 \end{array}$$

(3.)

$$\begin{array}{r} 123450 \\ 6 \\ \hline 7407|00 \end{array}$$

(4.)

$$\begin{array}{r} 18621 \\ 8 \\ \hline 1489|68 \end{array}$$

(5.)

$$\begin{array}{r} 317121 \\ 4 \\ \hline 126848|4 \end{array}$$

(6.)

$$\begin{array}{r} 876735 \\ 3 \\ \hline 26302|05 \end{array}$$

(7.)

$$\begin{array}{r} 123456 \\ 8 \\ \hline 987|648 \end{array}$$

(8.)

$$\begin{array}{r} 61678500 \\ 4 \\ \hline 246714|000 \end{array}$$

(9.)

$$\begin{array}{r} 9500 \\ 6 \\ \hline 57|000 \end{array}$$

(10.)

$$\begin{array}{r} 12000 \\ 3 \\ \hline 36|000 \end{array}$$

(11.)

$$\begin{array}{r} 150000 \\ 8 \\ \hline 1200|000 \end{array}$$

(12.)

$$\begin{array}{r} 333\frac{1}{3} \\ 3 \\ \hline 10|00 \end{array}$$

(13.)

$$\begin{array}{r} 120 \\ 4 \\ \hline 48|0 \end{array}$$

(14.)

$$\begin{array}{r} 616350 \\ 4 \\ \hline 24654|00 \end{array}$$

(Art. **96**, p. **64.**)

(4.)

$$\begin{array}{r|l} 12332 & 655 \\ 12 & 987 \\ 1 & \cancel{9}\cancel{9}\cancel{9} \\ \hline \end{array}$$

12345 Ans.

(5.)

$$\begin{array}{r|l} 98755 & 1235 \\ 9 & 9990 \\ \hline \end{array}$$

$98765\frac{\cancel{9}\cancel{9}\cancel{9}\cancel{9}}{\cancel{9}\cancel{9}\cancel{9}\cancel{9}}$. Ans.

(5.)

$$\begin{array}{r|l} 98755 & 1235 \\ 9 & 9990 \\ 1 & \cancel{9}\cancel{9}\cancel{9}\cancel{9} \\ \hline \end{array}$$

98765 Ans.

(6.)

$$\begin{array}{r|l} 9123456779 & 876543211 \\ 9 & 999999990 \\ 1 & \cancel{9}\cancel{9}\cancel{9}\cancel{9}\cancel{9}\cancel{9}\cancel{9}\cancel{9}\cancel{9} \\ \hline \end{array}$$

9123456789 Ans.

(Art. **97**, p. **64.**)

(2.)

$$\begin{array}{l} 44\overline{)39006}(886\frac{22}{44} \\ \quad\ \underline{380} \\ \qquad \underline{286} \\ \qquad\ \ 22 \end{array}$$

(3.)

$$\begin{array}{l} 34\overline{)1088}(32 \\ \qquad \underline{68} \\ \qquad\ \ 0 \end{array}$$

(4.)

$$\begin{array}{l} 191\overline{)5157}(27 \\ \qquad \underline{1337} \\ \qquad\quad\ 0 \end{array}$$

(5.)

$$\begin{array}{l} 7750\overline{)1133123}(146\frac{1623}{7750}. \\ \qquad\quad \underline{35812} \\ \qquad\qquad \underline{48123} \\ \qquad\qquad\ \ 1623 \end{array}$$

(Art. **98**, p. **65.**)

(2.)

$$\begin{array}{l} 9\frac{1}{4})13120 \\ 4 \qquad\quad\ 4 \\ \hline 37)52480(1418\frac{14}{37} \\ \quad\ \underline{37} \\ \quad\ 154 \\ \quad\ \underline{148} \\ \qquad\ 68 \\ \qquad\ \underline{37} \\ \qquad\ 310 \\ \qquad\ \underline{296} \\ \qquad\quad 14 \end{array}$$

(3.)

$$\begin{array}{l} 36)\ \ 76672\frac{6}{7} \\ \ \ 7 \qquad\quad 7 \\ \hline 252)536710(2129\frac{202}{252}. \\ \qquad \underline{504} \\ \qquad\ \ 327 \\ \qquad\ \ \underline{252} \\ \qquad\quad 751 \\ \qquad\quad \underline{504} \\ \qquad\quad 2470 \\ \qquad\quad \underline{2268} \\ \qquad\qquad 202 \end{array}$$

(4.)
$205)2090\frac{1}{3}$
3 3
$615)6271(10\frac{121}{615}$
615
121

(5.)
$16\frac{1}{2})10626$
2 2
33) 21252(644
198
145
132
132
132

(6.)
$69\frac{1}{6})12450$
6 6
415) 74700(180
415
3320
3320
0

(7.)
$17\frac{3}{4})5591\frac{1}{4}$
4 4
71)22365(315
213
106
71
355
355

(8.)
$10\frac{1}{2})2667$
2 2
21) 5334(254
42
113
105
84
84

(9.)
$272\frac{1}{4})136125$
4 4
1089) 544500(500
5445
00

(10.)
$2\frac{3}{7})119$
7 7
17) 833(49
68
153
153

(11.)
$31\frac{1}{4})12968\frac{3}{4}$
4 4
125) 51875(415
500
187
125
625
625

PROBLEMS.

1. (Art. **99**, p. **67**.) **$2763 + $4650 + $ 8950 = $16363.**
2. $929 — $279 = $650.
3. 6476 — 242 = 6234 feet.

4. 1519 + 328 = 1847.
5. 1963 — 199 = 1764; 1764 ÷ 2 = 882, miles B travelled; 882 + 199 = 1081, miles A travelled.
6. $250 + 410 = $660 : $4698 — 660 = $4038 : $4038 ÷ 3 = $1346, George received; $1346 + $250 = $1596, James received; $1346 + $410 = $1756, Edwin received.
7. $8463 ÷ 217 = $39.
8. 19 × 3 = 57; 684 ÷ 57 = 12 weeks.
9. 3808 ÷ 224 = 17 men.
10. $575 × 99 = $56925.
11. 96 × 22 = 2112; 63360 ÷ 2112 = 30.
12. 1101 × 13 = 14313.

MISCELLANEOUS EXAMPLES.

(PAGE 68.)

1. 200 + 305 + 230 + 282 + 171 = 1188, Ans.
2. $175 + $87 + $31 = $293; $38 + $12 = $50; $293 — $50 = $243, Ans.
3. 97 × 5 = $485; 97 — 17 = 80; 80 × 8 = $640; $640 — $485 = $155, gain, Ans.
4. 3787 × 1728 = 6543936 cubic inches, Ans.
5. 175686lb. ÷ 987 = 178lb., Ans.
6. 120 + 80 + 160 = 360 acres, Ans.
7. $8395 ÷ 365 = $23, Ans.
8. 12 × 6 = 72; 12 × 12 × 6 = 864; 864 — 72 = 792, Ans.
9. $7 × 8 = $56; $8 × 3 = $24; $56 + $24 = $80, Ans.
10. $31 + $45 = $76; 3952 ÷ 76 = 52, Ans.
11. 13 × 4 = 52, Ans.
12. $250,000 ÷ 500 = $500, Ans.
13. 127 + 212 = 339; 500 — 339 = 161; $47 × 127 = $5969; $96 × 212 = $20352; $37 × 161 = $5957; $5969 + $20352 + $5957 = $32,278; $32,278 — $17,876 = $14,402, Ans.

14. $17 - 7 = 10$; $18 - 10 = 8$ miles, Ans.
15. $15 \times 5 = 75$ days, Ans
16. $2 + 8 = 10$; $10 \times 4 = 40$; $40 + 32 = 72$; $72 \div 2 = 36$; $36 \times 10 = 360$; $360 \div 24 = 15$, Ans.
17. $16 + 4 = 20$; $20 \times 2 = 40$; $126 + 40 = 166$; $48 \div 2 = 24$; $34 \times 6 = 204$; $17 - 5 = 12$; $204 \div 12 = 17$; $17 + 24 = 41$; $166 - 41 = 125$, Ans.
18. $683 - 16 = 667$; $667 \div 23 = 29$ pupils, Ans.
19. $12 + 40 = 52$; $40 \times 5 = 200$; $12 \times 6 = 72$; $52 + 200 + 72 + 7 = 331$ miles, Ans.
20. $9891 - 1211 = 8680$; $8680 \div 2 = 4340$, A received; $9891 - 4340 = 5551$, B received, Ans.
21. $15 \times 16 = 240$; $4080 \div 240 = 17$, third number, Ans.
22. $\$4 \times 17 = \$68$; $\$2 \times 32 = \$64$; $\$68 + \$64 = \$132$; $132 \div 6 = 22$ tons, Ans.
23. $100 + 200 + 300 = 600$; $5608 - 600 = 5008$; $5008 \div 4 = 1252$, first year; $1252 + 100 = 1352$, second year; $1352 + 100 = 1452$, third year; $1452 + 100 = 1552$, fourth year, Ans.
24. $\$45 - \$35 = \$10$; $\$10 \times 12 = \$120$; $\$1100 - \$620 = \$480$; $480 \div 120 = 4$ years, Ans.
25. $19782 + 31 = 19813$, Ans.
26. $1885 \div 65 = 29$; $\$4473 + \$812 = \$5285$; $\$65 + \$6 = \$71$; $4473 \div 71 = 63$ shares; $63 - 29 = 34$ shares remaining; $\$5285 - \$1885 = \$3400$; $\$3400 \div 34 = \$100$, Ans.

UNITED STATES MONEY.

(Art. **107**, p. **73.**)

2. 76500 cents.
3. $72\frac{6}{10}$ cents.
4. $ 3.29
5. $ 12.345
6. 123560 mills.
7. 2220 cents.

ADDITION.

	(ART. **108**, p. **73.**)	6.	$1717.062
3.	$328.722	7.	$154.355
4.	$1805.847	8.	$7.105
5.	$3320.675		

SUBTRACTION.

	(ART. **109**, p. **74.**)	7.	$13.875
3.	$278.486	8.	$909.75
4.	$69.066	9.	$3.43
5.	$154.069	10.	$2.36
6.	$2058.08		

MULTIPLICATION OF UNITED STATES MONEY.

	(ART. **110**, p. **75.**)	10.	$44.748
2.	$16.38	11.	$109.25
3.	$58.59	12.	$790.92
4.	$591.25	13.	$65.10
5.	$3.358	14.	$142.02
6.	$249.28	15.	$48.222
7.	$1120.32	16.	$48910.95
8.	$358.28	17.	$67.16
9.	$102.69	18.	$70.664

DIVISION OF UNITED STATES MONEY.

	(ART. **111**, p. **77.**)	13.	$3.75
4.	$1.625	14.	$2.28
5.	425	15.	47
6.	$3.89	16.	17
7.	$1.75	17.	691
8.	329	18.	$4.68
9.	$0.75	19.	$0.18
10.	$4.00	20.	$132.55
11.	144	21.	$1.12
12.	$0.06		

PRACTICAL QUESTIONS BY ANALYSIS.

(Art. **116**, p. **79.**)

2.	$118.82
3.	$7.24½
4.	$310.20
5.	$1126.93½
6.	$4668595.00
8.	$120.66⅔
9.	$105.00
10.	$72.68¾
12.	$3800.00
13.	$337.50
14.	$502.25
15.	$22.50
16.	$6996.00
18.	$190.00
19.	$3255.75
20.	$103.35
21.	$52.95
22.	$129.525
24.	$876.375
25.	$472.50
26.	$7.98¾
27.	$60.17½
28.	$12467.25
30.	$28.20
31.	$42.75
32.	$9.035¼
33.	$771.65
34.	$283.12½
35.	$35.526⅞
37.	$0.13
38.	$4.16⅔
39.	$72.25
40.	$29.70
41.	$2.50
42.	$0.22
43.	$0.20
45.	152
46.	84
47.	362
48.	27
49.	50
50.	216

BILLS.

(Art. **122**, p. **83.**)

(1.) James Dow.

$0.45 × 17 = $7.65
.37 × 19 = 7.03
.46 × 16 = 7.36
.87 × 13 = 11.31
.63 × 9 = 5.67
.56 × 25 = 14.00
.31 × 17 = 5.27
.16 × 19 = 3.04
$61.33

(2.) Samuel Smith.

$0.98 × 13 = $12.74
.15 × 16 = 2.40
.13 × 36 = 4.68
.9 × 47 = 4.23
.19 × 12 = 2.28
.17 × 7 = 1.19
.61 × 13 = 7.93
$35.45

(3.) **Wilson, Niles & Co.**

$6.00 × 2 = $12.00
5.00 × 3 = 15.00
1.80 × 5 = 9.00
.25 × 17 = 4.25
.60 × 13 = 7.80
.50 × 19 = 9.50
.60 × 3 = 1.80
1.12½ × 7 = 7.87½
.22 × 15 = 3.30
15.00 × 5 = 75.00
4.50 × 3 = 13.50
1.50 × 5 = 7.50
4.50 × 1 = 4.50

$171.02½

(4.) **Albert Crawford Dr.**

$5.25 × 17 = $89.25
1.62 × 29 = 46.98
.17 × 60 = 10.20
.27 × 49 = 13.23
3.19 × 18 = 57.42
2.75 × 27 = 74.25
.61 × 75 = 45.75
.75 × 36 = 27.00
.18 × 49 = 8.82

$372.90

Cr.

Cash, $83.00
$30.00 × 3 = 90.00
4.00 × 7 = 28.00
2.00 × 4 = 8.00
1.75 × 5 = 8.75
2.25 × 7 = 15.75
Cash, = 60.00
Draft, = 45.00

$338.50

Remains due, $34.40

(5.) **Benjamin Treat.**

$25.50 × 37 = $943.50
16.17 × 41 = 662.97
97.75 × 40 = 3910.00
169.37 × 13 = 2201.81

$7718.28

(6.) **J. C. Porter.**

$8.25 × 17 = $140.25
.50 × 50 = 25.00
.08½ × 140 = 11.90
.63 × 120 = 75.60

$252.75

(7.) JOHN CUMMINGS.

$6.25 × 97 =	$606.25	Am't brought up	$12811.36
5.95 × 167 =	993 65	1.61 × 39 =	62.79
6.07 × 87 =	528.09	.17 × 197 =	33.49
5.75 × 196 =	1127.00	.69 × 86 =	59.34
7.25 × 275 =	1993.75	1.17 × 78 =	91.26
1.16 × 69 =	80.04	.85 × 187 =	158.95
.67 × 136 =	91.12	11.61 × 91 =	1056.51
.76 × 68 =	51.68	17.15 × 83 =	1423.45
1.37 × 169 =	231.53	3.16 × 47 =	148.52
9.67 × 76 =	734.92	18.15 × 35 =	635.25
69.70 × 89 =	6203.30	9.47 × 47 =	445.09
3.47 × 49 =	170.03	6.83 × 57 =	389.31
Am't carried up,	$12811.36		$17315.32

LEDGER ACCOUNTS.

(ART. **123**, p. **87**.)

1. $ 461.97
2. $ 3165.60
3. $4130.23
4. $21995.54

REDUCTION OF COMPOUND NUMBERS.

ENGLISH MONEY.

(ART. **128**, p. **90**.)

(3.)

127£. 15s. 8d.
20
2555s.
12
30668d.
4
122672far.

(4.)

4)122672far.
12)30668d.
20)2555s. 8d.
127£. 15s. 8d.

(5.)

28£. 19s. 11d. 3far.
20
579s.
12
6959d.
4
27839far.

(6.)

4)27839far.
12)6959d. 3far.
20)579s. 11d.
28£. 19s. 11d. 3far.

(7.)

378£.
20
7560s.
12
90720d.

(8.)

12)90720d.
20)7560s.
378£.

AVOIRDUPOIS WEIGHT.

(Art. **129**, p. **91.**)

(1.) 165T. 13cwt. 3qr. 19lb. 14oz.
20
3313cwt.
4
13255qr.
25
66284
26511
331394lb.
16
1988368
331395
5302318oz.

(2.) 16)5302318oz.
25)331394lb. 14oz.
4)13255qr. 19lb.
20)3313cwt. 3qr.
165T. 13cwt. 3qr. 19lb. 14oz.

(3.) 3T. 16cwt. 2qr. 18lb:
20
76cwt.
4
306qr.
25
1538
613
7668lb.
16
46008
7668
122688oz.

(4.) 16)122688oz.
25)7668lb.
4)306qr. 18lb.
20)76cwt. 2qr.
3T. 16cwt. 2qr. 18lb.

(5.) 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.
20
57cwt.
4
231qr.
25
1161
463
5791lb.
16
34751
5792
92671oz.
16
556029
92672
1482749dr., Ans.

(6.) 16)1482749dr.
16)92671oz. 13dr.
25)5791lb. 15oz.
4)231qr. 16lb.
20)57cwt. 3qr.
Ans. 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.

(7.) 7T. 17cwt.
20
157cwt.
4
628qr.
25
3140
1256
15700lb.
.07
$1099.00, Ans.

(8.) 19cwt. 3qr. 20lb.
4
79qr.
25
395
160
1995lb.
.09
$179.55, Ans.

TROY WEIGHT.

(Art. **130**, p. **92.**)

(1.) 28lb. 11oz. 12pwt. 15gr.
12
347oz.
20
6952pwt.
24
27813
13905
166863gr., Ans.

(2.) 24)166863gr.
20)6952pwt. 15gr.
12)347oz. 12pwt.
Ans. 28lb. 11oz. 12pwt. 15gr.

(3.) 3lb. 10oz.
12
46oz.
20
920pwt.
24
22080gr., Ans.

(4.) 24)22080gr.
20)920pwt.
12)46oz.
3lb. 10oz., Ans.

(5.) 73lb. 11oz.
12
887oz.
20
17740pwt.
$.062
35480
106440
$1099.88, Ans.

(6.) .062)$1099.88
20)17740pwt.
12)887oz.
Ans. 73lb. 11oz.

(7.) 57lb. 7oz.
12
691oz.
20.593$\frac{1}{5}$
2073
6219
3455
1382
138$\frac{1}{5}$
$ 14229.901$\frac{1}{5}$, Ans.

(8.) 19lb. 6oz. 16pwt.
12
234oz.
20
4696pwt.
.93
$4367.28, Ans.

(9.) 13lb. 9oz.
12
165
1.3857
1155
825
1320
495
165
$228.6405, Ans.

APOTHECARIES' WEIGHT.

(Art. 131, p. 93.)

(1.) 23℔ 9℥ 0ʒ 2℈ 13gr.
12
285℥
8
2280ʒ
3
6842℈
20
136853gr., Ans.

(2.) 20)136853gr.
3)6842℈
8)2280ʒ
12)285℥
Ans. 23℔ 9℥ 0ʒ 2℈ 13gr.

(3.) 23℔
12
276℥
8
2208ʒ
3
6624℈, Ans.

(4.) 3)6624℈.
8)2208ʒ
12)276℥
23℔, Ans.

(5.) 47℔ 0℥ 0ʒ 1℈ 19gr.
12
564℥
8
4512ʒ
3
13537℈
20
270759gr., Ans.

(6.) 20)270759gr.
3)13537℈ 19gr.
8)4512ʒ 1℈
12)564ʒ
Ans. 47℔ 0℥ 0ʒ
[1℈ 19gr.

(7.) 1℔
12
12℥
8
96ʒ
3
288℈
20
5)5760gr.
1152
12½
$144.00
1.80
$142.20, Ans.

AVOIRDUPOIS, TROY, AND APOTHECARIES' WEIGHT COMPARED.

(Art. 132, p. 94.)

(1.) 13lb. 6oz.
7000 437½gr
91000 6
2625 2625gr.
5760)93625(16lb.
576
3602
3456
1465
12
5760)17580(3oz.
1728
300
20
6000 (Carried forward to p. 26.)

(2.) 16lb. 3oz. 1pwt. 1gr.
12
195oz.
20
3901pwt.
24
7000)93625(13lb.
7000
23625
21000
2625
16
7000)42000(6oz.
42000

(Brought forward.)
5760)6000(1pwt.
5760
240
24
5760)5760(1gr.
5760

(3.)
3lb. 8oz. 10pwt.
12
44oz.
20
890pwt.
24
20)21360gr.
3)1068
356dr., Ans.

(4.)
356dr.
3
1068
20
24)21360
20)890
12)44oz. 10pwt.
Ans. 3lb. 8oz. 10pwt.

(5.)
2lb. 8oz.
16
40oz.
2.40
$96.00

2lb. 8oz.
7000
14000
3500
20)17500gr.
875℈
.20
$175
96
$ 79 gain, Ans.

437½gr.
8
3500gr.

(6.)
12lb.
16
192oz.
37½
$ 72.00
70
Ans. $ 2.00 loss.

12lb.
7000
24)84000gr.
20)3500pwt.
175oz.
.40
$ 70.00

LINEAR OR LONG MEASURE.

(Art. **133**, p. **96**.)

(1.) 96deg. 56m. 7fur. 32rd. 12ft. 6in.
$69\frac{1}{6}$
870
581
16
6696m.
8
53575fur.
40
2143032rd.
$16\frac{1}{2}$
12858194
2143033
1071516
35360040ft.
12
Ans. 424320486in.

(2.) 12)424320486in.
$16\frac{1}{2}$)35360040ft. 6in.
40)2143032rd. 12ft.
8)53575fur. 32rd.
$69\frac{1}{6}$)6696m. 7fur.
Ans. 96deg. 56m. 7fur. 32rd. [12ft. 6in., Ans.

(3.) 79m.
8
632fur.
40
25280rd.
$16\frac{1}{2}$
417120ft., Ans.

(4.) $16\frac{1}{2}$)417120ft.
40)25280rd.
8)632fur.
Ans. 79m.

(5.) 396
40
15840rd.
$16\frac{1}{2}$
95040
15840
7920
261360ft.
12
3136320in., Ans.

(6.) 12)3136320in.
$16\frac{1}{2}$)261360ft.
2 2
33)522720
40)15840rd.
396fur., Ans.

(7.) 30ms.
8
240fur.
40
9600rd.
16½
57600
9600
4800
158400ft.
12
1900800in., Ans.

(8.) 12)1900800in.
16½)158400ft.
2 2
33)316800
40)9600rd.
8)240fur.
30ms., Ans.

CLOTH MEASURE.

(Art. **134,** p. **96.**)

(1.)
17yd. 3qr. 2na.
4
71qr.
4
286na., Ans.

(2.)
4)286na.
4)71qr. 2na.
17yd. 3qr. 2na., Ans.

(3.)
365yd. 1qr. 3na.
4
1461qr.
4
5847na., Ans.

(4.)
4)5847na.
4)1461qr. 3na.
365yd. 1qr. 3na., Ans.

(5.)
71E.E. 4qr.
5
359qr.
4
1436na., Ans.

(6.)
4)1436na.
5)359qr.
Ans. 71E.E. 4qr.

(7.)

47yd. 3qr.
4
191qr.
1.25
955
382
191
$238.75, Ans.

(8.)

31⅕E.E.
5
4)156qr.
39
3
$117, Ans.

SURFACE OR SQUARE MEASURE.

(ART. **135**, p. **97**.)

(1.) 57A. 3R. 27p. 21yd. 8ft. 57in.
4
231R
40
9267p.
30¼
278031
2316¾
280347¾yds.
9
2523137¾ft.
144
10092555
10092553
2523127
108
363331893in., Ans.

(2.)

144)363331893in.
9)2523138ft. 21in.
30¼)280348yd. 6ft.
4 4
121)1121392
40)9267p. 85 ÷ 4 = 21yd. 2ft. 36in.
4)231R. 27p.
57A. 3R. 27p. 21yd. 2ft. 36in.
6 21
Ans. 57A. 3R. 27p. 21yd. 8ft. 57in.

(3.) 25A.
4
100R.
40
4000p.
272¼
8000
28000
8000
1000
1089000ft., Ans.

(4.) 272¼)1089000ft.
4 4
1089)4356000
40)4000p.
4)100R.
25A., Ans.

(5.) 365S. M.
640
14600
2190
233600A.
160
14016000
2336
Ans. 37376000sq. rd.

(6.) 160)37376000 square rods.
640)233600 acres.
365 square miles.

(7.) 144)12345678
9)85733ft. 126 in.
30¼)9525yd. 8ft.
40)314p. 26½yd.
4)7R. 34p.
1A. 3R.
1A. 3R. 34p. 26½yd. 8ft. 126in.
½yd. = 4ft. 72in.
Ans. 1A. 3R. 34p. 27yd. 4ft. 54in.

(8.) 39A. 2R. 16p.
4
158R.
40
6336p.
3.75
31680
44352
19008
$23760.00

6336 poles.
272¼
12672
44352
12672
1584
1724976 square feet.
0.25
8224880
3449952
431244.00
23760.00
$407,484.00, Ans.

SURVEYOR'S MEASURE.

(Art. **136**, p. **98.**)

(1.)
46m. 3fur. 5ch. 25l.
8
371fur.
10
3715ch.
100
371525l., Ans.

(2.)
100)371525l.
10)3715ch. 25l.
8)371fur. 5ch.
46m. 3fur.
Ans. 46m. 3fur. 5ch. 25l.

(3.)
97m.
8
776fur.
10
7760ch.
100
776000l., Ans.

(5.)
1377sq. ch.
16
8262
1377
40)22032
4)550R. 32p.
137A. 2R.
137A. 2R. 32p., Ans.

(6.)
$ 80 ÷ 160 = $ 0.50.
625)2,126,250 square links.
3402sq. rd.
0.50
$ 1701.00, Ans.

CUBIC OR SOLID MEASURE.

(Art. **137**, p. **100.**)

(1.)
29 cords.
128
232
58
29
3712ft.
1728
29696
7424
25984
3712
Ans. 6414336in.

(2.)
1728)6414336 cubic inches.
128)3712 cubic feet.
29 cords, Ans.

(3.)
19 tons.
40
760ft.
1728
6080
1520
5320
760
1313280 cubic inches, Ans.

(4.)
1728)1313280 cubic inches.
40)760 feet.
19 tons, Ans.

(5.) 128 × 128 = 16384 cubic feet. Ans.
(6.) 16384 ÷ 128 = 128 cords, Ans.
(7.) 4 × 40 × 7 = 1120 cubic feet, Ans.
(8.) 8650ft. ÷ 128 = 67 cords, 74 cubic feet, Ans.
(9.) 17 × 11 × 9 = 1683 cubic feet, Ans.

LIQUID OR WINE MEASURE.

(Art. **138**, p. **101.**)

(1.) 57T. 3hhd. 50gal. 3qt.
4
231hhd.
63
14603gal.
4
58415qt.
2
116830pt.

(2.)
2)116830 pints.
4)58415qt.
63)14603gal. 3qt.
4)231hhd. 50gal.
57T. 3hhd.
57T. 3hhd. 50gal. 3qt., Ans.

(3.) 96hhd. 47gal. 2qt.
63
295
580
6095gal.
4
24382qt.
2
48764pt.
4
195056 gills, Ans.

(4.)
4)195056 gills.
2)48764pt.
4)24382qt.
63)6095gal. 2qt.
96hhd. 47gal.
Ans. 96hhd. 47gal. 2qt.

(5.)
40hhd.
63
120
240
2520gal.
4
10080qt.
2
20160pt.
0.37½
141120
60480
10080
$7560.00, Ans.

(6.) 2
63
126
0.40
$50.40

126
4
504
0.12
$60.48
50.40
$10.08, Ans.

BEER MEASURE.

(ART. 139, p. 102.)

(1.) 46 × 54 = 2484gal.; 2484gal. + 49gal. = 2533gal.; 2533 × 4 = 10132qt.; 10132 × 2 = 20264pt., Ans.

(2.) 20264 ÷ 2 = 10132qt.; 10132 ÷ 4 = 2533gal.; 2533 ÷ 54 = 46hhd. 49gal., Ans.

(3.) 368 × 54 = 19872gal.; 19872 × 4 = 79488qt.; 79488 × 2 = 158976pt., Ans.

(4.) 158976 ÷ 2 = 79488qt.; 79488 ÷ 4 = 19872gal.; 19872 ÷ 54 = 368hhd., Ans.

(5.) 76 × 54 = 4104gal.; 4104 × 0.29 = $ 1190.16, Ans.

(6.) 47 × 54 = 2538gal.; 2538gal. + 36gal. = 2574gal.; 2574 × 4 = 10296qt.; 10296 × 0.05 = $ 514.80, Ans.

DRY MEASURE.

(ART. 140, p. 103.)

(1.) 35 × 4 = 140pk.; 140pk. + 3pk. = 143pk.; 143 × 8 = 1144qt.; 1144 × 2 = 2288pt., Ans.

(2.) 2288 ÷ 2 = 1144qt.; 1144 ÷ 8 = 143pk.; 143 ÷ 4 = 35bu. 3pk., Ans.

(3.) 676 × 36 = 24336bu.; 24336 × 4 = 97344pk., Ans.

(4.) 97344 ÷ 4 = 24336bu.; 24336 ÷ 36 = 676ch., Ans.

(5.) 50 × 0.60 = $ 30.00; 50 × 5 = 250pk.; 250 × 0.15 = $ 37.50; $ 37.50 — $ 30.00 = $ 7.50 gain, Ans.

(6.) 96 × 0.42 = $ 40.32; 2304 ÷ 32 = 72bu.; 72 × 0.45 = $ 32.40; $ 40.32 — $ 32.40 = $ 7.92 loss, Ans.

DRY, LIQUID, AND BEER MEASURE COMPARED.

(ART. **141**, p. **104**.)

(1.) 4hhd. 15gal. beer = 231gal.; 231 × 282 = 65142; 65142 ÷ 231 = 282gal. wine = 4hhd. 30gal., Ans.

(2.) 4hhd. 30gal. = 282 wine gal.; 282 × 231 = 65142; 65142 ÷ 282 = 231 beer gal. = 4hhd. 15gal., Ans.

(3.) 2820 × 4 = 11280qt.; 11280 × 0.04 = $ 451.20; 2820 × 282 = 795240; 795240 ÷ 231 = $3442\frac{46}{77}$gal. wine measure; $3442\frac{46}{77}$ × 4 = $13770\frac{30}{77}$qt.; $13770\frac{30}{77}$ × 0.06 = $ $826.22\frac{26}{77}$; $ $826.22\frac{26}{77}$ — $ 451.20 = $ $375.02\frac{26}{77}$, Ans.

(4.) 2538 × 4 × 0.04 = $ 406.08; 2538 × 231 = 586278; 586278 ÷ 282 = 2079; 2079 × 4 × 0.06 = $ 498.96; $ 498.96 — $ 406.08 = $ 92.88 gain, Ans.

(5.) 385 × 4.00 = $ 1540.00; 385bu. × $268\frac{4}{5}$ = 103488; 103488 ÷ 231 = 448bu.; 448 × 32 × 0.20 = $ 2867.20; $ 2867.20 — $ 1540.00 = $ 1327.20 gain, Ans.

(6.) 1000 × 1.05 = $ 1050.00; 1000bu. × $268\frac{4}{5}$ = 268800; 268800 ÷ 231 = $1163\frac{7}{11}$bu.; $1163\frac{7}{11}$ × 1.12 = $ 1303.27; $ 1303.27 — $ 1050.00 = $ 253.27 gain, Ans.

(7.) 30 × 1728 = 51840 cubic inches; 51840 ÷ 282 = $183\frac{39}{47}$gal.; 51840 ÷ $268\frac{4}{5}$ = $192\frac{6}{7}$gal.; $192\frac{6}{7}$ — $183\frac{39}{47}$ = $9\frac{9}{329}$gal., Ans.

(8.) 365 × 4 × 0.05 = $ 73.00; 365 × 231 = 84315; 84315 ÷ 282 = 299gal.; 299 × 4 × 0.05 = $ 59.80; $ 73.00 — $ 59.80 = $ 13.20, Ans.

TIME.

(Art. 142, p. 106.)

(1.) 365da. 5h. 48min. 49sec.
24
1465
730
8765h.
60
525948m.
60
31556929sec., Ans.

(2.)
60)31556929sec.
60)525948m. 49sec.
24)8765h. 48m.
365da. 5h.

Ans. 365da. 5h. 48m. 49sec.

(3.) 2348yrs. + 1855yrs. = 4203yrs.; 365da. 6h. = 31557600 sec.; 31557600sec. × 4203 = 132636592800sec., Ans.

(4.) 31556929 seconds in a solar year; therefore 74726807872 ÷ 31556929 = 2368 years, Ans.

(5.) June 13da. + July 31 + August 31 + September 30 + October 31 + November 30 + December 31 + January 31 + February 16 + 365 + 365 = 974da. = 2 yrs. 244da., Ans.

CIRCULAR MEASURE.

(Art. 143, p. 107.)

(1.) 27S. 27° 43′
30
837°
60
50263′, Ans.

(2.)
60)50263′
30)837° 43′
27S. 27°
Ans. 27S. 27° 43′.

(3.) 44 × 30 = 1320°; 1320° + 18° = 1338°; 1338 × 60 = 80280′; 80280′ + 57′ = 80337′; 80337 × 60 = 4820220″; 4820220″ + 23″ = 4820243″, Ans.

(4.) 4820243 ÷ 60 = 80337′ 23″; 80337 ÷ 60 = 1338° 57′; 1338 ÷ 30 = 44S. 18°; 44S. 18° 57′ 23″, Ans.

(5.) 360 × 60 = 21600′; 21600 × 60 = 1296000, Ans.

(6.) 1296000 ÷ 60 = 21600′; 21600 ÷ 60 = 360°, Ans.

MISCELLANEOUS.

(ART. 144, p. 108.)

(1.) $4 \times 5 + 4 = 24$; $24 \times 2 + 1 = 49$; $49 \times 20 + 10 = 990$; $990 \times 24 = 23760$ sheets, Ans.

(2.) $23760 \div 24 = 990$qr.; $990 \div 20 = 49$rm. 10qr.; $49 \div 2 = 24$bun. 1rm.; $24 \div 5 = 4$ba. 4bun.; 4ba. 4bun. 1rm. 10qr., Ans.

(3.) $10 \times 8 + 6 = 86$; $86 \times 21\frac{1}{2} + 8 = 1857$; $1857 \times 14 = 25998$lbs.

(4.) $25998 \div 14 = 1857$; $1857 \div 21\frac{1}{2} = 86$ pigs 8 stone; $86 \div 8 = 10$ fother 6p.; 10 fother 6 pigs 8 stone, Ans.

(5.) $56 \times 12 \times 0.23 = \$154.56$, Ans.

(6.) $\$22.00 \div 200 = \$0.11$, Ans.

(7.) $302 \times 8 \times 0.30 = \$724.80$, Ans.

(8.) $\$4.00 \div 100 = \$0.04$; $50.24 \div .04 = 1256$ pounds.

(9.) $12 \times 12 \times 12 \times \$0.045 = \$77.76$.

(10.) $56 \times 2 = 112$lb.; $112 \times 0.20 = \$22.40$; 196lb. $\div 2 = 98$lb.; $98 \times 0.05 = \$4.90$; $\$22.40 - \$4.90 = \$17.50$, Ans.

MISCELLANEOUS EXAMPLES IN REDUCTION.

(PAGE 109.)

(1.) $57 \times 20 + 15 = 1155$s.; $1155 \times 12 = 13860$d.; $13860 \times 2 = 27720$ half-pence, Ans.

(2.) $59 \times 12 = 708$oz.; $708 \times 20 + 13 = 14173$pwt.; $14173 \times 24 + 15 = 340167$gr., Ans.

(3.) $340167 \div 24 = 14173$pwt. 15 gr.; $14173 \div 20 = 708$oz. 13pwt.; $708 \div 12 = 59$lb.; 59lb. 13pwt. 15gr., Ans.

(4.) $761 \times 4 = 3044$qr.; $3044 \div 5 = 608$E.E. 4qr., Ans.

(5.) $61 \times 3 = 183$qr.; $183 \div 4 = 45$yd. 3qr., Ans.

(6.) $63 \times 4 = 252$qt.; $252 \times 2 = 504$pt.; $504 \div 3 = 168$ bottles, Ans.

(7.) 2ft. 8in. = 32in.; $15 \times 8 \times 40 \times 16\frac{1}{2} \times 12 = 950400$in.; $950400 \div 32 = 29700$ times, Ans.

(8.) 2oz. 12pwt. = 52pwt.; 5lb. 2oz. 8pwt. = 1248pwt.; $1248 \div 52 = 24$ spoons, Ans.

(9.) 14ft. 9in. = 177in.; 436 × 8 × 40 × $16\frac{1}{2}$ × 12 = 27624960in.; 27624960 ÷ 177 = $156073\frac{39}{177}$ times, Ans.

(10.) 123 × 78 × 4 = 38376 ears; 38376 ÷ 8 = 4797qt.; 4797qt. ÷ 8 = 599pk. 5qt.; 599pk. ÷ 4 = 149bu. 3pk. 5qt., Ans.

(11.) 5yd. 2qr. 3na. = 91na.; 182 × 4 × 4 = 2912na.; 2912 ÷ 91 = 32 suits, Ans.

(12.) 5pwt. 10gr. = 130gr.; 3lb. 1oz. 2pwt. 2gr. = 17810gr.; 17810 ÷ 130 = 137 rings, Ans.

(13.) 18in. ÷ 3 = 6in.; 6in. × 4 = 24in.; 56ft. × 60 = 3360ft.; 3360 × 144. = 483840in.; 483840 ÷ 24 = 20160 shingles, Ans.

(14.) 56 × 25 × 2 = 2800sq. ft.; 2800 × 6 = 16800 shingles, Ans.

(15.) 22m. 3fur. 17rd. = 7177rd.; 25000 × 320 = 8000000rd.; 8000000 ÷ 7177 = $1114\frac{4822}{7177}$da., Ans.

(16.) 7lb. 10oz. = 122oz.; 10cwt. 3qr. 16lb. = 1091lb.; 1091. × 16 = 17456oz.; 17456 ÷ 122 = $143\frac{5}{61}$ weeks, Ans.

(17.) 7 × 63 = 441gal.; 441 × 4 = 1764qt.; 1764 × 0.09 = $ 158.76, Ans.

(18.) 15 × 54 = 810gal.; 810 × 4 = 3240qt.; 3240 × 2 = 6480pt.; 6480 × 0.03 = $ 194.40, Ans.

(19.) 73 × 32 = 2336qt.; 2336 × 0.02 = $ 46.72, Ans.

(20.) 29 × 57 = 1653yd.; 1653 × 0.15 = $ 247.95, Ans.

(21.) 120 × $4\frac{1}{2}$ × 4 = 2160 cubic feet; $ 4.00 ÷ 128 = $ 0.03125; 0.03125 × 2160 = $ 67.50, Ans.

(22.) 33 × 18 = 594sq. ft.; 594 ÷ 9 = 66sq. yd.; $ 0.20 × 66 = $ 13.20, Ans.

(23.) 20 × 25 = 500 pills; 6 + 5 + 4 = 15gr.; 500 × 15 = 7500gr., Ans.

(24.) 87 × 63 = 5481gal.; 5481 × 0.33 = $ 1808.73, Ans.

(25.) 128ft. ÷ 2 = 64ft.; 64 × 880 = 56320oz.; 128 × 480 = 61440oz.; 56320oz. + 61440oz. = 117760oz.; 117760 ÷ 16 = 7360lb., Ans.

(26.) 24 × 0.43 = $ 10.32; $ 1548.00 ÷ 10.32 = $ 150.00, Ans.

(27.) From 1837 to 1857 there are 5 leap years, viz., 1840, 1844, 1848, 1852, and 1856. Therefore, $ 10.00 × 5 = $ 50.00; $ 200.00 — $ 50.00 = $ 150.00, Ans.

(28.) 1,000,000 × 25⅘ = 25,800,000gr.; 25,800,000 ÷ 7000 = 3685$\frac{5}{7}$lb., Ans.

(29.) 188 × 0.50 = $ 94.00; 188gal. ÷ 2 = 94gal.; 94 × 231 = 21714; 21714 ÷ 282 = 77gal.; 77 × 4 = 308qt.; 308 × 0.12½ = $ 38.50; 94 × 4 = 376qt.; 376 × 0.14 = $ 52.64; $ 52.64 + $ 38.50 = $ 91.14; $ 94.00 — $ 91.14 = $ 2.86, Ans.

ADDITION OF COMPOUND NUMBERS.

(Art. **145**, p. **112**.)

2. 324T. 15cwt. 2qr. 15lb. 12oz. 9dr.
3. 233cwt. 1qr. 6lb. 4oz. 5dr.
4. 209lb. 7oz. 15pwt. 8gr.
5. 385lb. 7oz. 5pwt. 10gr.
6. 183℔ 6℥ 3ʒ 1℈ 19gr.
7. 246℔ 10℥ 3ʒ 2℈ 2gr.
8. 195deg. 55m. 5fur. 24rd. 1ft. 1in.
9. 317m. 3fur. 18rd. 4yd. 2ft. 3in.
10. 229yd. 3qr. 3na. 1¼in.
11. 2513E.E. 3qr. 2na. 0in.
12. 2038A. 1R. 13p. 2ft. 95in.
13. 317A. 0R. 24p. 13¾ft.
14. 160m. 0fur. 1ch. 1p. 12l.
15. 194m. 6fur. 9ch. 0p. 12l.
16. 335T. 23ft. 1173in.
17. 378C. 96ft. 1460in.
18. 354tu. 0hhd. 30gal. 1qt.
19. 286hhd. 42gal. 1qt. 1pt.
20. 784hhd. 24gal. 2qt. 0pt.

21. 298bu. 0pk. 4qt. 1pt.
22. 128bu. 2pk. 7qt.
23. 227y. 7m. 16da. 21h. 28min. 2sec.
24. 240w. 1da. 23h. 33min. 8sec.
25. 5S. 10° 35′ 16″.
26. 11S. 2° 22′ 42″.
27. 20£. 17s. 10d.
28. 5lb. 9oz. 4pwt. 20gr.
29. 7℔ 10℥ 3ʒ 1℈ 4gr.
30. 117cwt. 1qr. 22lb.
31. 175yd. 2qr. 2na.
32. 1060m. 5fur. 16rd. 5yd. 1ft.
33. 588A. 1R. 31p.
34. 7S. 29° 33′ 56″.

SUBTRACTION OF COMPOUND NUMBERS.

(Art. **146**, p. **116**.)

1. 435£. 16s. 3d.
2. 67£. 18s. 5¾d.
3. 435£. 15s. 9¼d.
4. 51T. 18cwt. 2qr. 21lb. 9oz. 8dr.
5. 53cwt. 3qr. 20lb. 14oz.
6. 54lb. 4oz. 14pwt. 19gr.
7. 38lb. 7oz. 14pwt. 13gr.
8. 52℔ 6℥ 3ʒ 1℈ 15gr.
9. 5℔ 5ʒ 0ʒ 1℈ 17gr.
10. 6m. 7fur. 38rd. 2ft. 11in.
11. 9deg. 4m. 5fur. 37rd. 4yd. 1ft. 4in.
12. 48yd. 2qr. 2na. 1¼in.
13. 31E.E. 4qr. 2na.
14. 38A. 2R. 5p. 264ft. 33in.
15. 3A. 1R. 38p. 18yd. 7ft. 36in.
16. 1m. 7fur. 1ch. 2p. 23l.
17. 12m. 6fur. 8ch. 0p. 24l.

18. 30T. 37ft. 1620in.
19. 163C. 53ft. 1289in.
20. 68tu. 1hhd. 53gal. 3qt.
21. 577hhd. 52gal.
22. 17ch. 35bu.
23. 12bu. 3pk. 4qt.
24. 4mo. 23da. 20h. 45min. 42sec.
25. 28y. 1mo. 15da. 21h. 48min. 5sec.
26. 9S. 2° 56′ 52″.
27. 2S. 27° 21′ 54″.
28. 67A. 0rd. 38p. 265¼ft.
29. 99£. 16s. 5½d.
30. 24956m. 7fur. 22rd. 15ft. 9in.
31. 4C. 50ft.
32. 11yd. 1qr. 3na.
33. 33cwt. 2q. 20lb.
34. 54gal. 0qt. 1pt.
35. 6A. 2R. 32p.
36. 12° 11′ 27″.
37. 86bu. 1pk. 0qt. 1pt.
38. 1T. 19ft. 1418in.

MULTIPLICATION OF COMPOUND NUMBERS.

(Art. **147,** p. **119.**)

2. 19£. 10s. 4d.
3. 557T. 19cwt. 1qr. 11lb. 15oz. 7dr.
4. 39lb. 10oz. 13pwt. 3gr.
5. 261lb. 11oz. 0pwt. 0gr.
6. 427℔ 10℥ 0ʒ 2℈ 14gr.
7. 858deg. 44m. 4fur. 6rd. 7ft. 0in.
8. 215m. 7fur. 9ch. 3p. 1l.
9. 181A. 3R. 11p. 6yd. 4ft. 41in.
10. 57yd. 2qr. 3na.
11. 123tu. 3hhd. 36gal. 3qt.
12. 209hhd. 55gal. 3qt. 0pt. 1gi.

13. 118bu. 1pk. 5qt.
14. 110y. 343d. 3h. 41m. 12sec.
15. 149deg. 9m. 5fur. 13rd. 3yd. 1ft.
16. 214A. 3R. 12p.
17. 3d. 16h. 54m.
18. 29bu. 0p. 0qt.

(Art. **148**, p. **121.**)

2. 56T. 14cwt. 3qr. 15lb.
3. 5£. 4s. 3d.
4. 10£. 8s. 3d.
5. 12£. 13s. 0d.
6. 1290£. 4s. 0d.
7. 32lb. 9oz. 15pwt.
8. 746m. 5fur.
9. 15° 0′ 0″.
10. 1275A. 2R. 16p. 22yd. 8ft. 32in.
11. 2y. 68d. 19h. 54m.

(Art. **149.** p. **122.**)

2. 89hhd. 52gal. 3qt. 1pt.
3. 179bu. 3pk. 5qt. 0pt. 1gi.
4. 275ch. 19bu. 2pk.
5. 23£. 11s. 5¼d.
6. 3419A. 2R. 23p.
7. 3375yd. 3qr. 1na. 0¼in.
8. 44hhd. 52gal. 2qt. 1pt.
9. 6357m. 5fur. 14rd. 11½ft.
10. 175gal. 2qt. 0pt.
11. 15T. 38ft. 1074in.
12. 132C. 2ft.

DIVISION OF COMPOUND NUMBERS.

(Art. 150, p. 123.)

2. 2£. 8s. 9½d.
3. 61T. 19cwt. 3qr. 17lb. 15oz. 15dr.
4. 7lb. 11oz. 14pwt. 15gr.
5. 32lb. 8oz. 17pwt. 12gr.
6. 38℔ 10℥ 5ʒ 2℈ 14gr.
7. 71deg. 38m. 2fur. 13rd. 14ft. 4in.
8. 17m. 7fur. 9ch. 3p. 23l.
9. 16A. 2R. 4p. 19yd. 7ft. 79in.
10. 10yd. 3qr. 3na.
11. 17tun. 2hhd. 50gal. 1qt.
12. 29hhd. 61gal. 3qt. 1pt. 3gi.
13. 19bu. 2pk. 7qt. 1pt.
14. 13y. 316d. 15h. 27m. 39sec.
15. 16deg. 39m. 3fur. 39rd. 5yd. 2ft.
16. 23A. 3R. 19½p.
17. 7h. 24m. 30sec.
18. 3bu. 2pk. 4qt.

(Art. 151, p. 124.)

2. 12cwt. 2qr. 11lb.
3. 0£. 5s. 9½d.
4. 0£. 9s. 11d.
5. 0£. 11s. 6d.
6. 13£. 8s. 9½d.
7. 3oz. 5pwt. 15gr.
8. 24m. 7fur. 4rd.
9. 0° 15′.
10. 39A. 3R. 17p. 30yd. 8ft. 100in.
11. 2d. 5h. 17m. 19sec.

(Art. 152, p. 125.)

2. 2hhd. 19gal 0qt. 1pt.
3. 3bu. 1pk. 4qt. 1pt. 1gi.

4. 16ch. 7bu. 2pk. 0qt. 0pt.
5. 0£. 8s. $3\frac{1}{4}$d.
6. 117A. 3R. 27p.
7. 37yd. 3qr. 2na. 2in.
8. 47gal. 3qt. 1pt.
9. 17m. 3fur. 13rd. 14ft.
10. 3gal. 1qt. 1pt.
11. 37ft. 978in.
12. 7C. 98ft.
13. 4T. 15cwt. 2qr. $10\frac{129}{146}$lb.
14. 19m. 3fur. 39rd. 13ft. $2\frac{26}{47}$in.
15. 24A. 3R. $6\frac{23}{29}$p.
16. 9A. 1R. 19p. $139\frac{86}{87}$ft.
17. 13A. 0R. 27p. 3yd. 0ft. $45\frac{9}{11}$in.

PRINCIPLES AND APPLICATIONS.

(Art. **153**, p. **127**.)

(2.)

y.	mo.	d.
1857	3	19
1854	5	3
2	10	16

(3.)

y.	mo.	d.
1857	5	12
1856	9	26
0	7	16

(4.)

y.	mo.	d.
1776	6	4
1620	11	22
155	6	12

(5.)

y.	mo.	d.
1799	11	14
1732	1	22
67	9	22

(Art. **154**, p. **128**.)

2. From Dec. to Feb. = 2 mo., $31 \times 2 = 62$ days.
For Nov. 14d., Feb. 12d., $16 + 12 = \underline{28}$
90 days.

3. From July to May = 10mo., 10 × 31 = 310 days.
For June 13 days, 13
323
For Sept. 1d., Nov. 1d., Feb. 3d., April 1, 6
317 days.

4. From April to May = 1mo., 30 days.
For March 28 days, May 2 days, 30
60 days.

5. From Dec. to April = 4mo., 4 × 31 = 124 days.
For Nov. 27 days, April 3 days, 30
154
For Feb. 3 days, 3
151 days.

6. From Feb. to Dec. = 10 mo., 10 × 31 = 310 days.
For Jan. 25 days, Dec. 10 days, 35
345
For Feb. 2 days, April 1, June 1, Sept. 1, Nov. 1, 6
Ans. 339 days.

(Art. 155, p. 129.)

2. From April 2 to July 4 = 93 days.
93 ÷ 7 = 13 weeks, 2 days.
2 days after Wednesday = Friday, Ans.

3. From Jan. 1 to June 17 = 168 days.
168 ÷ 7 = 24 weeks.
Tuesday, Ans.

4. From Jan. 1 to Dec. 25 = 358 days.
358 ÷ 7 = 51 weeks, 1 day.
1 day before Tuesday = Monday, Ans.

5. From Nov. 4 to Feb. 1 = 89 days.
89 ÷ 7 = 12 weeks, 5 days, therefore February will commence 5 days after Tuesday = Sunday. If Sunday be the 1st day, Monday will be the 2nd and 9th days.

6. From Dec. 20 to March 1 = 72 days, it being *leap year.* 90 − 72 = 18 days; the note will therefore become due March 19. 91 ÷ 7 = 13 weeks. As there is no remainder, and as the note was given on Thursday, it will be payable on Wednesday, March 19.

NOTE.—In reckoning the interest on any note, it is not customary to charge interest on the day the note is paid, but it is reckoned on the day it is given.

DIFFERENCE OF LATITUDE.

(ART. **157**, p. **130**.)

2. Latitude of Quebec = 46° 48′ N.
Latitude of New Orleans = 29° 57′ N.
Difference of Latitude = 16° 51′, Ans.

3. Latitude of Washington City = 38° 53′ N.
Latitude of Cape Horn = 55° 58′ S.
Difference of Latitude = 94° 51′, Ans.

4. Latitude of Valparaiso = 33° 2′ S.
Latitude of San Francisco = 37° 48′ N.
Difference of Latitude = 70° 50′, Ans.

5. Latitude of New York City = 40° 42′ N.
Difference of Latitude N. = 17° 33′ N.
Latitude of Havana = 23° 9′ N., Ans.

6. Latitude of Mobile = 30° 41′ N.
Difference of Latitude = 9° 15′
Latitude of Philadelphia = 39° 56′ N., Ans.

DIFFERENCE OF LONGITUDE.

(Art. **159.** p. **131.**)

2. Longitude of Washington City = 77° 16′ W.
Longitude of Paris = 2° 20′ E.
Difference of Longitude = 79° 36′, Ans.

3. Longitude of Cape Flattery = 124° 43′ W.
Longitude of St. Croix River = 67° 2′ W.
Difference of Longitude = 57° 41′, Ans.

4. Longitude of Sacramento City = 120° 0′ W.
Longitude of Raleigh = 78° 48′ W.
Difference of Longitude = 41° 12′, Ans.

5. Longitude of Fort Leavenworth = 94° 44′ W.
Longitude of Hartford, Ct. = 72° 40′ W.
Difference of Longitude = 22° 4′, Ans.

6. Longitude of Honolulu = 157° 52′ W.
Longitude of Canton = 113° 14′ E.
271° 6′
360° — 271° 6′ = 88° 54′, Ans.

LONGITUDE AND TIME.

(Art. **160,** p. **132.**)

2. Longitude of Galveston = 14° 43′
4
Difference of time = 0h. 58m. 52sec.
12h. 0m. 0sec.
12h. 58m. 52sec.

3. Longitude of Valparaiso = 71° 37′ W.
Longitude of Rome = 20° 30′ E.
92° 7′
4
Difference of time = 6h. 8m. 28sec.
11h. 15m. 0s. A.M.
Time at Rome = 5h. 23m. 28s. P.M.

4. Longitude of Jerusalem = 35° 32′ E.
Longitude of Baltimore = 76° 37′ W.
112° 9′
4
Difference of time = 7h. 28m. 36sec.
Time at Jerusalem = 9h. 0m. 0s. A. M.
7h. 28m. 36s. A. M.
Time at Baltimore = 1h. 31m. 24s. A. M., Ans.

(Art. **161**, p. **133.**)

(2.) 29m. ÷ 4 = 7° 15′
36sec. ÷ 4 = 0° 9′
7° 24′

(3.) 2h. × 15 = 30°
20m. ÷ 4 = 5°
40sec. ÷ 4 = 0° 10′
85° 10′

(4.) 1h. × 15 = 15° 0′
20sec. ÷ 4 = 0° 5′
15° 5′
75° 10′
90° 15′ W.

(5.) 53m. ÷ 4 = 13° 15′
30sec. ÷ 4 = 0° 7′ 30″
13° 22′ 30″

(6.) 1h. × 15 = 15° 0′
37m. ÷ 4 = 9° 15′
12sec. ÷ 4 = 0° 3′
Take 24° 18′
From 93° 5′
68° 47′ West.

(7.) 7h. × 15 = 105° 0′
15m. ÷ 4 = 3° 45′
40sec. ÷ 4 = 0° 10′
108° 55′
30° 0′
78° 55′ West.

$$\begin{array}{rcrcr} (8.)\ 4\text{h.} & \times & 15 & = & 60^\circ\ 0' \\ 44\text{m.} & \div & 4 & = & 11^\circ\ 0' \\ 32\text{sec.} & \div & 4 & = & 0^\circ\ 8' \\ \cline{5-5} & & & & 71^\circ\ 8' \end{array}$$

MISCELLANEOUS EXAMPLES.

(Page **134.**)

$$\begin{array}{lr} (1.) & 57\ 650\ 000 \\ & 263\ 517\ 496 \\ & 626\ 400\ 000 \\ & 100\ 000\ 000 \\ & 1\ 445\ 000 \\ & 1\ 500\ 000 \\ \cline{2-2} 33) & 1050\ 512\ 496 \\ \cline{2-2} & 31{,}833\ 712 \end{array}$$

(2.) $755 - 125 = 630$; $630 \div 2 = 315$; $315 + 48 = 363$ bushels in the second; $315 - 48 = 267$ bushels in the third.

(3.) $30 \times 3 = 90$ miles A travels. He has therefore travelled round the island three times. $30 \times 5 = 150$; $150 \div 30 = 5$; B has travelled round the island five times. A and B are therefore together.

(4.) \$$1750 \times 2 =$ \$$3500$; \$$103 \times 19 =$ \$$1957$; \$$3500 +$ \$$1957 +$ \$$113 =$ \$$5570$, Ans.

(5.) $960 \div 2 = 480$; $480 \div 12 = 40$; $40 \div 2 = 20$ bushels each for those who left; $480 \div 8 = 60$; $60 \div 2 = 30$; $30 + 20 = 50$ bushels for each who remained.

(6.) $652.00 \div .50 = 1304$ sheep, Ans.

(7.) $27 \times 18 = 486$sq. ft.; $486 \div 9 = 54$yd.; $54 \times 2.25 =$ \$ 121.50, Ans.

(8.) $365 \times 2\frac{1}{2} = 912\frac{1}{2}$h.; $912\frac{1}{2}$h. $\times 20 = 18250$h. in 20 years; $18250 \div 12 = 1520$da. 10h. $=$ 4y. 60da. 10h., Ans.

(9.) 7C. 76ft. 1671in. + 16C. 28ft. 56in. + 29C. 127ft. 1000in. + 29C. 10ft. 1216in. = 82C. 115ft. 487in.; 82C. 115ft. 487in. — 45C. 96ft. 0in. = 37C. 19ft. 487in., Ans.

(10.) $42^\circ\ 21' + 1^\circ\ 15' = 43^\circ\ 36'$; $43^\circ\ 36' - 10^\circ\ 40' = 32^\circ\ 56'$, Ans.

(11.) 2000lb. $\times 2 =$ 4000lb.; $4000 \div 54 = 74\frac{2}{27}$ cubic feet, Ans.

(13.) \$ $295{,}300{,}000 \div 250{,}000 =$ \$ 1181.20; \$ $295{,}300{,}000 \div 4 =$ \$ $73{,}825{,}000$, Ans.

(14.) $216 \times 1728 = 373248$in.; $373248 \div 231 = 1615\frac{61}{77}$gal.; $1615\frac{61}{77} \div 63 =$ 25hhd. $40\frac{61}{77}$gal., Ans.

(15.) Ans. 11th day of May; 131st day of the year.

(16.) 18T. 17cwt. 3qr. 20lb. = 37795lb.; $37795 \times 0.04 = \$\,1511.80$; 4T. 6cwt. 1qr. 14lb. = 8639lb.; $8639 \times 0.05 = \$\,431.95$; 7T. 1cwt. 3qr. 10lb. = 14185lb.; $14185 \times 0.06 = \$\,851.10$; 8639lb. + 14185lb. = 22824lb.; 37795lb. — 22824lb. = 14971lb.; 14971lb. ÷ 4 = $3742\frac{3}{4}$lb.; $3742\frac{3}{4} \times 0.06 = \$\,224.56\frac{1}{2}$; 14971lb. — $3742\frac{3}{4}$lb. = $11228\frac{1}{4}$lb.; $11228\frac{1}{4}$lb. ÷ 2 = $5614\frac{1}{8}$lb.; $5614\frac{1}{8} \times 0.10 = \$\,561.41\frac{1}{4}$; $5614\frac{1}{8} \times 0.12 = \$\,673.69\frac{1}{2}$; $\$\,673.69\frac{1}{2} \div 2 = \$\,336.84\frac{3}{4}$; $\$\,431.95 + \$\,851.10 + \$\,224.56\frac{1}{2} + \$\,561.41\frac{1}{4} + \$\,336.84\frac{3}{4} = \$\,2405.87\frac{1}{2}$; $\$\,2405.87\frac{1}{2} - \$\,1511.80 = \$\,894.07\frac{1}{2}$, Ans.

(17.) 3m. 7fur. 29rd. 15ft. + 5m. 0fur. 10rd. $1\frac{1}{2}$ft. = 9m.; 9m. × $6\frac{1}{2}$ = $58\frac{1}{2}$m., the distance both travel each day; $2691 \div 58\frac{1}{2}$ = 46da., the number of days they travel before they meet. 46 days from the 1st of January, omitting 8 Sabbaths, as in duty bound, will be February 23d, 1857, at 2h. 30min. P. M. Swift will have travelled 1186m. 4fur. 22rd. 13ft. 6in.; and Fleet 1504m. 3fur. 17rd. 3ft. As January begins on Thursday, the 23d of February will be on Monday, Ans.

EXAMPLES BY ANALYSIS.

(PAGE **136.**)

(1.) $\$\,8.75 \div 7 = \$\,1.25$; $\$\,1.25 \times 20 = \$\,25.00$, Ans.

(2.) $\$\,85.00 \div 5 = \$\,17.00$; $\$\,17.00 \times 17 = \$\,289.00$, Ans.

(3.) $\$\,0.75 \div 3 = \$\,0.25$; $\$\,0.25 \times 37 = \$\,9.25$, Ans.

(4.) $\$\,1.92 \div 4 = \$\,0.48$; $\$\,0.48 \times 37 = \$\,17.76$, Ans.

(5.) $\$\,1.08 \div 12 = \$\,0.09$; $\$\,0.09 \times 25 = \$\,2.25$, Ans.

(6.) $\$\,63.00 \div 9 = \$\,7.00$; $\$\,7.00 \times 27 = \$\,189.00$, Ans.

(7.) 20m. ÷ 5 = 4m.; 28 ÷ 4 = 7h., Ans.

(8.) 49 crackers ÷ 7 = 7 crackers; 7 crackers × 19 = 133 crackers, Ans.

(9.) $ 20 ÷ 5 = $ 4; 40 × 4 = $ 160.00, Ans.

(10.) $ 0.36 ÷ 3 = $ 0.12; $ 0.12 × 37 = $ 4.44, Ans.

(11.) $ 0.72 ÷ 9 = $ 0.08; $ 0.08 × 37 = $ 2.96, Ans.

(12.) 2£. 17s. 5d. ÷ 52 = 0£. 1s. $1\frac{1}{4}$d; 0£. 1s. $1\frac{1}{4}$d. × 76 = 4£. 3s. 11d., Ans.

(13.) 4£. 3s. 11d. ÷ 76 = 0£. 1s. $1\frac{1}{4}$d.; 0£. 1s. $1\frac{1}{4}$d. × 52 = 2£. 17s. 5d., Ans.

(14.) 17m. 4fur. 28rd. ÷ 6 = 2m. 7fur. 18rd.; 2m. 7fur. 18rd. × 100 = 293m. 1fur., Ans.

(15.) 41bu. 3pk. 4qt. 1pt. ÷ 7 = 5bu. 3pk. 7qt. 1pt.; 5bu. 3pk. 7qt. 1pt. × 12 = 71bu. 3pk. 2qt.; 71bu. 3pk. 2qt. × 7 = 502bu. 2pk. 6qt., Ans.

(16.) 39A. 2R. 5p. 8yd. 6ft. 108in. ÷ 8 = 4A. 3R. 30p. 20yd.; 4A. 3R. 30p. 20yd. × 9 = 44A. 1R. 35p. 28yd. 6ft. 108in.; 44A. 1R. 35p. 28yd. 6ft. 108in. × 8 = 355A. 3R. 7p. 18yd. 2ft. 36in., Ans.

(17.) 63gal. 3qt. ÷ 4 = 15gal. 3qt. 1pt. 2gi.; 15gal. 3qt. 1pt. 2gi. × 37 = 589gal. 2qt. 1pt. 2gi., Ans.

(18.) 5yd. 3qr. 1na. ÷ 4 = 1yd. 1qr. $3\frac{1}{4}$na.; 1yd. 1qr. $3\frac{1}{4}$na. × 36 = 52yd. 1qr. 1na., Ans.

(19.) 11T. 3cwt. 2qr. ÷ 4 = 2T. 15cwt. 3qr. 12lb. 8oz.; 2T. 15cwt. 3qr. 12lb. 8oz. × 23 = 64T. 5cwt. 0qr. 12lb. 8oz., Ans.

(20.) 286da. 4h. 33min. × 12 = 3434da. 6h. 36min., the time one man would dig it; 3434da. 6h. 36min. ÷ 72 = 47da. 16h. 45min. 30sec., time 72 men would dig it, Ans.

(21.) 27yd. 1qr. ÷ 21 = 1yd. 1qr. 0na. $1\frac{5}{7}$in; 1yd. 1qr. 0na. $1\frac{5}{7}$in. × 11 = 14yd. 1qr. 0na. $0\frac{6}{7}$in., Ans.

(22.) 174m. 0fur. 26rd. ÷ 7 = 24m. 6fur. 38rd.; 24m. 6fur. 38rd. × 10 = 248m. 5fur. 20rd., Ans.

(23.) 4 × 60 = 240 pair; $ 192.00 ÷ 240 = $ 0.80; $ 192.00 ÷ 4 = $ 48.00; $ 48.00 × 25 = $ 1200.00, Ans.

(24.) 3A. 2R. 20p. ÷ 4 = 0A. 3R. 25p., cost of one hogshead; 0A. 3R. 25p. × 30 = 27A. 0R. 30p., Ans.

(25.) 20deg. 49m. 5fur. 35rd. 5yd. 0ft. 3in. ÷ 9 = 2deg. 6m. 7fur. 6rd. 4yd. 1ft. 11in.

20deg. 49m. 5fur. 35rd. 5yd. 0ft. 3in.
10
207deg. 12⅚m. 2fur. 39rd. ½yd. 2ft. 6in.
½yd. = 1ft. 6in.
207deg. 12⅚m. 2fur. 39rd. 1yd. 1ft. 0in.
⅚m. = 6fur. 26rd. 3yd. 2ft. 0in.
207deg. 13m. 1fur. 25rd. 5yd. 0ft. 0in., Ans.

FACTORING.

(Page 141.)

(2.)
3)105
5)35
7)7
1
Ans. 3, 5, 7.

(3.)
2)220
2)110
11)55
5)5
1
Ans. 2, 5, 11.

(4.)
2)936
2)468
2)234
3)117
3)39
13)13
1
Ans. 2, 3, 13.

(5.)
3)1953
3)651
7)217
31)31
1
Ans. 3, 7, 31.

(6.)
2)12462
3)6231
31)2077
67)67
1
Ans. 2, 3, 31, 67.

(7.)
11)19987
23)1817
79)79
1
Ans. 11, 23, 79.

(8.)
3)225
3)75
5)25
5)5
1
Ans. 3, 5.

3)435
5)145
29

2)540
2)270
3)135
3)45
3)15
5)5
1

(9.) 2)960
2)480
2)240
2)120
2)60
2)30
3)15
5)5
1

2)1568
2)784
2)392
2)196
2)98
7)49
7)7
1

2)5824
2)2912
2)1456
2)728
2)364
2)182
7)91
13)13
1

Ans. 2.

(10.) 2)2340
2)1170
3)585
3)195
5)65
13)13
1

2)11934
3)5967
3)1989
3)663
13)221
17)17
1

3)12987
3)4329
3)1443
13)481
37)37
1

3)14859
3)4953
13)1651
127)127
1

Ans. 3, 3, 13.

(11.) 3)105
5)35
7)7
1

Ans. 3, 5, 7.

DIVISORS OR MEASURES.

(Art. **194**, p. **145**.)

(2.) $72 = 2 \times 2 \times 2 \times 3 \times 3.$

$$\text{Divisors.}\begin{cases} 1,\ 2,\ 4,\ 8 = 2 \times 2 \times 4. \\ 3,\ 6,\ 12,\ 24 = (2 \times 2 \times 2) \times 3. \\ 9,\ 18,\ 36,\ 72 = (2 \times 2 \times 2) \times (3 \times 3). \end{cases}$$

(3.) $105 = 3 \times 5 \times 7.$

Exponent of 3 is $1 + 1 = 2$.
Exponent of 5 is $1 + 1 = 2$.
Exponent of 7 is $1 + 1 = 2$.
$2 \times 2 \times 2 = 8$, the number of divisors.

4. $1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$.

2 (the exponent of factor 2) $+ 1 = 3$; $2 + 1 = 3$; $2+1 = 3$; $3 \times 3 \times 3 = 27$, Ans.

5. $3528 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 =$

3 (the exponent of the factor 2) $+ 1 = 4$; $2 + 1 = 3$; $2 + 1 = 3$; $4 \times 3 \times 3 = 36$, Ans.

6. $5880 = 2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 7$.

$3 + 1 = 4$; $1 + 1 = 2$; $1 + 1 = 2$; $2 + 1 = 3$; $4 \times 2 \times 2 \times 3 = 48$, Ans.

COMMON DIVISORS.

(Art. **198**, p. **146**.)

2. $51 = 3 \times 17$.
$153 = 3 \times 3 \times 17$.
$255 = 3 \times 5 \times 17$.

The prime factors which are common are 1, 3, 17. Their products are found as in Art. **194**.

Divisors 1, 3, 17, 51 = Ans. 1, 3, 17, 51.

3. $180 = 2 \times 2 \times 3 \times 3 \times 5$.
$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$.
2, 2, 3, 3, 5, are common. Hence,
1, 2, 4,
3, 6, $12 = (2 \times 2) \times 3$.
5, 10, $20 = (2 \times 2) \times 5$.
9, 18, $36 = (2 \times 2) \times (3 \times 3)$.
15, 30, $60 = (2 \times 2) \times (3 \times 5)$.
45, 90, $180 = (2 \times 2) \times (3 \times 3 \times 5)$.

Ans. 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, [60, 90, 180.

4. $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$.
$6075 = 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$.
$8100 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$.

The factors $3 \times 3 \times 3 \times 3 \times 5 \times 5$ are common.

The exponent of 3 is $4 + 1 = 5$; the exponent of 5 is $2 + 1 = 3$; and $5 \times 3 = 15$, Ans.

5. $4500 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5.$
$9000 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5.$

The *common* prime factors are $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$. Their exponents are $2 + 1 = 3$; $2 + 1 = 3$; $3 + 1 = 4$; and $3 \times 3 \times 4 = 36$, Ans.

(ART. **200**, p. **148**.)

THE GREATEST COMMON DIVISOR OR MEASURE.

2.	56	6.	35	10.	15
3.	32	7.	1	11.	78
4.	57	8.	1	12.	4 acre lots.
5.	1	9.	12	13.	6 bushels.

LEAST COMMON MULTIPLE.

(ART. **203**, p. **151**.)

(2.) $3 \quad \cancel{13} \quad 37 \quad 91$

$3 \times 37 \times 91 = 10101.$

(3.)
$$\begin{array}{r|rrrrr} 3 & 9 & 14 & 30 & 35 & 47 \\ \hline 5 & 3 & 14 & 10 & 35 & 47 \\ \hline 2 & 3 & 14 & 2 & 7 & 47 \\ \hline 7 & 3 & 7 & 1 & 7 & 47 \\ \hline & 3 & 1 & 1 & 1 & 47 \end{array}$$

$47 \times 3 \times 7 \times 2 \times 5 \times 3 = 29610$, Ans.

(4.)
$$\begin{array}{ccccc|c} \cancel{6} & \cancel{8} & \cancel{10} & 18 & 20 & 24 \\ \hline & & & 3 & 5 & \end{array}$$

$3 \times 5 \times 24 = 360$, Ans.

(5.)
$$\begin{array}{ccc|c} 14 & \cancel{19} & 38 & 57 \\ \hline 14 & & \cancel{2} & \end{array}$$

$14 \times 57 = 798$, Ans.

(6.)
$$\begin{array}{ccc|c} 20 & 36 & 48 & 50 \\ \hline \cancel{2} & 18 & 24 & \\ \cline{1-2} & 3 & & \end{array}$$

$3 \times 24 \times 50 = 3600$, Ans.

(7.)
$$\begin{array}{cccc|c} \cancel{15} & \cancel{25} & 35 & 45 & 100 \\ \hline & & 7 & 9 & \end{array}$$

$7 \times 9 \times 100 = 6300$, Ans.

(8.)
$$\begin{array}{cccc|c} \cancel{100} & \cancel{200} & 300 & 400 & 575 \\ \hline & & 12 & 16 & \\ \cline{3-3} & & 3 & & \end{array}$$

$3 \times 16 \times 575 = 27600$, Ans.

(9.)
$$\begin{array}{r|l} \cancel{1}\ \ \cancel{2}\ \ \cancel{3}\ \ \cancel{4}\ \ 5\ \ 6\ \ 8 & 9 \\ \hline 5\ \ \cancel{2}\ \ 8 & \end{array}$$

$5 \times 8 \times 9 = 360$; $2520 \div 360 = 7$, Ans.

(10.)
$$\begin{array}{r|l} \cancel{18}\ \ \cancel{24}\ \ 36\ \ 20\ \ 48 & 126 \\ \hline \cancel{2}\ \ \begin{array}{c|l} 10 & 8 \\ \hline 5 & \end{array} & \end{array}$$

$5 \times 8 \times 126 = 5040$, Ans.

(11.)
$$\begin{array}{r|l} \cancel{4}\ \ \cancel{6}\ \ 10 & 12 \\ \hline 5 & \end{array}$$

$5 \times 12 = 60$qt., Ans.

(12.)
$$\begin{array}{r|l} 50\ \ 40 & 75 \\ \hline \cancel{2}\ \ 8 & \end{array}$$

$8 \times 75 = 600$, Ans.

MISCELLANEOUS EXAMPLES.

(Page **152.**)

(1.) 7)6174
7)882
7)126
2)18
3)9
3)3
1 Ans. 3 times.

(2.) 5)5775
5)1155
7)231
3)33
11)11
1 11, Ans.

(3.) 3)19929
7)6643
13)949
73)73
1

$73 \times 13 \times 7 = 6643$, Ans.

(4.) 2)2338
7)1169
167)167
1

$$\left.\begin{array}{l} 2338 \div 2 = 1169 \\ 2338 \div 7 = 334 \end{array}\right\} \text{Ans.}$$

(5.)
$$\begin{array}{r} 5\underline{)17385} \\ 3\underline{)3477} \\ 19\underline{)1159} \\ 61\underline{)61} \\ 1 \end{array}$$

Ans. 3, 5, 19, 61.

(6.)
$$\begin{array}{r} 2\underline{)3000} \\ 3\underline{)1500} \\ 5\underline{)500} \\ 2\underline{)100} \\ 2\underline{)50} \\ 5\underline{)25} \\ 5\underline{)5} \\ 1 \end{array}$$

$$\left.\begin{array}{l} 5 \times 5 \times 5 \times 3 \times 2 \times 2 = 1500 \\ 5 \times 5 \times 5 \times 2 \times 2 \times 2 = 1000 \\ 5 \times 5 \times 5 \times 3 \times 2 \quad = 750 \end{array}\right\} \text{Bins.}$$

1, 2, 3 = Bags.
Ans.

(7.)
$$\begin{array}{l} 136)152(1 \\ \quad\underline{136} \\ \quad\;16)136(8 \\ \qquad\quad\underline{128} \\ \qquad\qquad 8)16(2 \\ \qquad\qquad\quad\underline{16} \end{array}$$

$152 \div 8 = 19$ classes of ladies.
$136 \div 8 = 17$ classes of gentlemen.
Ans.

8. The velocity of the hands of a clock is as the times in which they perform one revolution. The second-hand performs a revolution in 60 seconds, the minute-hand in 3600 seconds, and the hour-hand in 43200 seconds.

$$\begin{array}{rrr} 60)43200 & 3600 & 60 \\ \hline 60)\ 720 & 60 & 1 \\ \hline 12 & 1 & 1 \end{array}$$

$60 \times 60 \times 12 = 43200$ seconds = 12 hours, Ans.

9. Greatest common divisor of 348, 372, 444, 492 = 12; $348 \div 12 = 29$ rails; $372 \div 12 = 31$ rails; $444 \div 12 = 37$ rails; $492 \div 12 = 41$ rails; $29 + 31 + 37 + 41 = 138$ rails; $138 \times 5 = 690$ rails; 12ft. + 9in. = 12ft. 9in., length of each rail, Ans.

10. 3A. 2R. 1p. = 561p.; 5A. 3R. 15p. = 935p.; 8A. 0R. 29p. = 1309p.; 12A. 3R. 17p. = 2057p.; 15A. 0R. 31p. = 2431p.; 1309 ÷ 187 = 7; 2057 ÷ 187 = 11; 187p. is therefore the common divisor, and each field will contain 187p. = 1A. 0R. 27p., Ans.

$$\begin{array}{l} 561)935(1 \\ \quad\;\; \underline{561} \\ \quad\;\; 374)561(1 \\ \qquad\quad\;\; \underline{374} \\ \qquad\quad\;\; 187)374(2 \\ \qquad\qquad\quad\; \underline{374} \end{array}$$

(11.) $2772 = 2 \times 2 \times 3 \times 3 \times 7 \times 11$

$12 = 2 \times 2 \times 3$

$$\left.\begin{array}{l} 12 \times 3 = 36 \\ 12 \times 7 = 84 \\ 12 \times 11 = 132 \end{array}\right\} \text{Ans.}$$

(12.) $3 + 1 = 4$; $5 + 1 = 6$; $7 + 1 = 8$

$$\begin{array}{r|lll} 2 & 4 & 6 & 8 \\ \hline 2 & 2 & 3 & 4 \\ \hline & 1 & 3 & 2 \end{array}$$

$2 \times 2 \times 3 \times 2 = 24$; $24 - 1 = 23$ days, Ans.

COMMON FRACTIONS.

REDUCTION.

(Art. **220**, p. **156.**)

2.	$\frac{4}{21}$	5.	$\frac{7}{54}$	8.	$\frac{1}{7}$	11.	$\frac{41}{61}$
3.	$\frac{3}{4}$	6.	$\frac{3}{35}$	9.	$\frac{416}{875}$	12.	$\frac{408}{509}$
4.	$\frac{3}{7}$	7.	$\frac{81}{422}$	10.	$\frac{811}{1116}$	13.	$\frac{41}{152}$

(Art. **221**, p. **157.**)

2.	$11\frac{2}{15}$	5.	$3\frac{134}{161}$	8.	125
3.	$14\frac{7}{116}$	6.	$111\frac{1}{9}$	9.	1
4.	$7\frac{12}{17}$	7.	$91\frac{28}{45}$		

(Art. **222**, p. **157.**)

3.	$\frac{60}{4}$	7.	$\frac{100}{1}$	10.	$\frac{7107}{61}$	13.	$\frac{7}{1}$
4.	$\frac{2577}{16}$	8.	$\frac{85}{17}$	11.	$\frac{69691}{97}$	14.	$\frac{247}{13}$
5.	$\frac{10146}{59}$	9.	$\frac{2554}{97}$	12.	$\frac{20000}{199}$	15.	$\frac{465}{4}$
6.	$\frac{11}{1}$						

(Art. **223**, p. **158.**)

3. $\frac{2}{3}\times\frac{5}{6}\times\frac{7}{8}\times\frac{11}{12}=\frac{770}{1728}=\frac{385}{864}$, Ans.

4. $\frac{\cancel{6}}{7}\times\frac{11}{\underset{5}{\cancel{15}}}\times\frac{\cancel{3}}{5}\times\frac{1}{\underset{3}{\cancel{18}}}=\frac{.11}{525}$, Ans.

5. $\frac{\overset{1}{\cancel{4}}}{7}\times\frac{5}{\underset{2}{\cancel{8}}}\times\frac{3}{8}\times\frac{11}{13}=\frac{165}{1456}$, Ans.

6. $\frac{6}{11}\times\frac{6}{\underset{1}{\cancel{7}}}\times\frac{1}{4}\times\frac{\overset{3}{\cancel{21}}}{1}=\frac{108}{44}=2\frac{5}{11}$, Ans.

7. $\frac{7}{11}\times 15\frac{7}{8}\times 5\frac{7}{10}\times 100=\frac{7}{11}\times\frac{127}{8}\times\frac{57}{\underset{1}{\cancel{10}}}\times\frac{\overset{10}{\cancel{100}}}{1}=$

$[\frac{506730}{88}=5758\frac{13}{44}$, Ans.

8. $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}}\times\frac{\overset{1}{\cancel{4}}}{7}\times\frac{11}{\underset{8}{\cancel{24}}}=\frac{11}{56}$, Ans.

9. $\frac{7}{\cancel{11}}\times\frac{\cancel{11}}{\cancel{25}}\times\frac{\cancel{25}}{31}\times\frac{7.75}{1}=\frac{54.25}{31}=\$1.75$, Ans.

10. $\frac{4}{9}\times\frac{9}{17}\times\frac{17}{18}\times 3\frac{3}{5}=\frac{4}{\cancel{9}}\times\frac{\cancel{9}}{\cancel{17}}\times\frac{\cancel{17}}{\cancel{18}}\times\frac{\cancel{18}}{5}=\frac{4}{5}$ gal., Ans.

11. $\frac{1}{\underset{2}{\cancel{4}}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{4}=\frac{1}{8}$, Ans.

$$12.\ \frac{\cancel{4}}{\cancel{7}} \times \frac{9}{\underset{5}{\cancel{20}}} \times \frac{\overset{3}{\cancel{15}}}{16} \times \frac{\overset{2}{\cancel{14}}}{\cancel{17}} \times \frac{\overset{2}{\cancel{34}}}{1} = \frac{108}{16} = \$6.75,\ \text{Ans.}$$

COMMON DENOMINATORS.

(Art. **226**, p. **160.**)

2. $\frac{54}{72}$, $\frac{40}{72}$, $\frac{63}{72}$, $\frac{30}{72}$.
3. $\frac{720}{1320}$, $\frac{704}{1320}$, $\frac{1254}{1320}$, $\frac{275}{1320}$.
4. $\frac{12}{84}$, $\frac{18}{84}$, $\frac{32}{84}$, $\frac{15}{84}$.
5. $\frac{32}{42}$. $\frac{24}{42}$, $\frac{33}{42}$, $\frac{21}{42}$.
6. $\frac{210}{308}$, $\frac{84}{308}$, $\frac{176}{308}$, $\frac{231}{308}$.
7. $\frac{30}{60}$, $\frac{40}{60}$. $\frac{45}{60}$, $\frac{48}{60}$.
8. $\frac{594}{693}$, $\frac{308}{693}$, $\frac{315}{693}$, $\frac{441}{693}$.
9. $\frac{90}{120}$, $\frac{105}{120}$, $\frac{20}{120}$, $\frac{456}{120}$.
10. $\frac{70}{168}$, $\frac{63}{168}$, $\frac{96}{168}$, $\frac{798}{168}$.
11. $\frac{24}{42}$, $\frac{15}{42}$, $\frac{22}{42}$, $\frac{21}{42}$.
12. $\frac{224}{252}$, $\frac{147}{252}$, $\frac{238}{252}$, $\frac{24}{252}$.
13. $\frac{570}{600}$, $\frac{140}{600}$, $\frac{165}{600}$, $\frac{12}{600}$.
14. $\frac{210}{240}$, $\frac{140}{240}$, $\frac{105}{240}$, $\frac{84}{240}$.
15. $\frac{14}{63}$, $\frac{441}{63}$, $\frac{504}{63}$, $\frac{324}{63}$.
16. $\frac{80}{120}$, $\frac{96}{120}$, $\frac{105}{120}$.
17. $\frac{360}{630}$, $\frac{560}{630}$, $\frac{189}{630}$.
18. $\frac{546}{1001}$, $\frac{572}{1001}$, $\frac{616}{1001}$.
19. $\frac{364}{624}$, $\frac{192}{624}$, $\frac{4836}{624}$.
20. $\frac{1485}{2295}$, $\frac{1020}{2295}$, $\frac{612}{2295}$.
21. $\frac{2040}{2295}$, $\frac{540}{2295}$, $\frac{26316}{2295}$.
22. $\frac{21}{42}$, $\frac{28}{42}$, $\frac{24}{42}$, $\frac{336}{42}$.
23. $\frac{528}{1188}$, $\frac{756}{1188}$, $\frac{6039}{1188}$.
24. $\frac{24}{48}$, $\frac{36}{48}$, $\frac{8}{48}$, $\frac{816}{48}$.
25. $\frac{110}{120}$, $\frac{576}{120}$, $\frac{2580}{120}$.
26. $\frac{12012}{14014}$, $\frac{5096}{14014}$, $\frac{5390}{14014}$,
[$\frac{8008}{14014}$, $\frac{7007}{14014}$.
27. $\frac{28506816}{17843552}$, $\frac{37088064}{17843552}$,
[$\frac{722813}{17843552}$.

ADDITION OF COMMON FRACTIONS.

(Art. **228**, p. **162.**)

3. $3\frac{10}{17}$.
4. $2\frac{10}{23}$.
5. $1\frac{40}{87}$.
6. $1\frac{11}{72}$.
7. $2\frac{426}{631}$.
8. $2\frac{43}{72}$.
9. $2\frac{29}{88}$.
10. $2\frac{199}{360}$.
11. 1.
12. $3\frac{19}{60}$.
13. $1\frac{83}{140}$.
14. $6\frac{349}{504}$.
15. $9\frac{21}{22}$.
16. $6\frac{1}{3}$.
17. $17\frac{1}{20}$.
18. $9\frac{115}{252}$.
19. $18\frac{311}{360}$.
20. $45\frac{1}{126}$.
21. $40\frac{4}{15}$.
22. $1068\frac{37}{60}$.
23. $12\frac{691}{693}$.
24. $13\frac{31}{44}$.

SUBTRACTION OF COMMON FRACTIONS

(Art. **230**, p. **165**.)

3.	$\frac{7}{17}$.	11.	$\frac{7}{40}$.	19	$\frac{8}{25}$.
4.	$\frac{9}{19}$.	12.	$\frac{269}{1728}$.	20.	$\frac{33}{100}$.
5.	$\frac{15}{37}$.	13.	$\frac{13}{250}$.	21.	$\frac{5}{54}$.
6.	$\frac{2}{39}$.	14.	$\frac{19}{34}$	22.	$\frac{3}{56}$.
7.	$\frac{5}{29}$	15.	$\frac{17}{60}$.	23.	$1\frac{9}{28}$.
8.	$\frac{15}{37}$.	16.	$\frac{124}{323}$.	24.	$1\frac{11}{12}$.
9.	$\frac{1}{4}$.	17.	$\frac{33}{38}$.	25.	$\frac{1}{12}$.
10.	$\frac{5}{36}$.	18.	$\frac{92}{231}$.		

(Art. **231**, p. **166**.)

2.	$26\frac{5}{8}$.	6.	$15\frac{129}{237}$.	10.	$77\frac{9}{23}$.
3.	$11\frac{1}{4}$.	7.	$16\frac{1}{4}$.	11.	$680\frac{64}{79}$.
4.	$670\frac{9}{50}$.	8.	$5\frac{4}{7}$.	12.	$30\frac{1}{9}$.
5.	$368\frac{22}{39}$.	9.	$18\frac{4}{7}$.	13.	$31\frac{8}{19}$.

(Art. **232**, p. **167**.)

(2.)

$$\begin{array}{rcl} 18\frac{3}{8} &=& 18\frac{27}{72} \\ 9\frac{8}{9} &=& \underline{9\frac{64}{72}} \\ && 8\frac{35}{72}, \text{ Ans.} \end{array}$$

(3.)

$$\begin{array}{rcl} 73\frac{4}{5} &=& 73\frac{48}{60} \\ 16\frac{11}{12} &=& \underline{16\frac{55}{60}} \\ && 56\frac{53}{60}, \text{ Ans.} \end{array}$$

(4.)

$$\begin{array}{rcl} 67\frac{1}{3} &=& 67\frac{3}{9} \\ 16\frac{2}{3} &=& \underline{16\frac{6}{9}} \\ && 50\frac{2}{3}, \text{ Ans.} \end{array}$$

(5.)

$$\begin{array}{rcl} 29\frac{18}{19} &=& 29\frac{162}{171} \\ 15\frac{4}{9} &=& \underline{15\frac{76}{171}} \\ && 14\frac{86}{171}, \text{ Ans.} \end{array}$$

(6.)

$$\begin{array}{rcl} 144\frac{3}{4} &=& 144\frac{39}{52} \\ 99\frac{11}{13} &=& \underline{99\frac{44}{52}} \\ && 44\frac{47}{52}, \text{ Ans.} \end{array}$$

(7.)

$$\begin{array}{rcl} 171\frac{15}{22} &=& 171\frac{150}{220} \\ 91\frac{9}{10} &=& \underline{91\frac{198}{220}} \\ && 79\frac{43}{55}, \text{ Ans.} \end{array}$$

(8.)

$$\begin{array}{rcl} 101\frac{7}{8} &=& 101\frac{35}{40} \\ 93\frac{4}{5} &=& \underline{93\frac{32}{40}} \\ && 8\frac{3}{40}, \text{ Ans.} \end{array}$$

(9.)

$$\begin{array}{rcl} 165\frac{1}{2} &=& 165\frac{8}{16} \\ 98\frac{7}{8} &=& \underline{98\frac{14}{16}} \\ && 66\frac{5}{8}, \text{ Ans,} \end{array}$$

(10.)

$$\begin{array}{rcl} 77\frac{11}{12} &=& 77\frac{220}{240} \\ 19\frac{19}{20} &=& \underline{19\frac{228}{240}} \\ && 57\frac{232}{240}, \text{ Ans.} \end{array}$$

(11.)	(12.)	(13.)
$84\frac{23}{24} = 84\frac{276}{288}$	$671\frac{111}{721} = 671\frac{9213}{59843}$	$17\frac{123}{457} = 17\frac{87453}{324927}$
$15\frac{11}{12} = 15\frac{264}{288}$	$183\frac{69}{83} = 183\frac{49749}{59843}$	$8\frac{234}{711} = 8\frac{106938}{324927}$
$69\frac{12}{288}$, Ans.	$487\frac{19307}{59843}$, Ans.	Ans. $8\frac{305442}{324927}$

14. $63 - 7\frac{9}{11} = 55\frac{2}{11}$ gal., Ans.

15. $30 - 5\frac{7}{12} = 24\frac{5}{12}$ days, Ans.

16 $17\frac{5}{8} + 28\frac{7}{12} = 46\frac{5}{24}$; $144 - 46\frac{5}{24} = 97\frac{19}{24}$ lb, Ans.

17. $34 - 9\frac{7}{8} = 24\frac{1}{8}$ yd., Ans.

18. $40 \times \frac{2}{17} = 4\frac{12}{17}$; $40 \times \frac{2}{11} = 7\frac{3}{11}$; $4\frac{12}{17} + 7\frac{3}{11} = 11\frac{183}{187}$; $40 - 11\frac{183}{187} = 28\frac{4}{187}$ miles, Ans.

19. $\frac{1}{4}$ of a square yard $= \frac{1}{4}$ of 9 feet $= 2\frac{1}{4}$ square feet. 1 yard $=$ 3 feet; $\frac{1}{6}$ of 3 feet $=$ 6 inches. 6 inches squared $=$ 36 square inches $= \frac{1}{4}$ of a square foot. $2\frac{1}{4}$ feet $- \frac{1}{4}$ foot $=$ 2 square feet, Ans.

MISCELLANEOUS EXAMPLES IN ADDITION AND SUBTRACTION OF FRACTIONS.

(PAGE 168.)

1. $\frac{3}{4}, \frac{1}{2}, \frac{7}{8} = \frac{6}{8}, \frac{4}{8}, \frac{7}{8}$; $6 + 4 + 7 = 17$; $17 \div 8 = 2\frac{1}{8}$, Ans.

2. $\frac{5}{8} + \frac{1}{2} + \frac{1}{3} + \frac{5}{6} = \frac{15}{24} + \frac{12}{24} + \frac{8}{24} + \frac{20}{24} = \frac{15+12+8+20}{24} = \frac{55}{24} = \$2\frac{7}{24}$, Ans.

3. $20\frac{7}{10}, 19\frac{1}{2}, 22\frac{1}{16} = 20\frac{56}{80}, 19\frac{40}{80}, 22\frac{5}{80}$; $20\frac{56}{80} + 19\frac{40}{80} + 22\frac{5}{80} = 62\frac{21}{80}$ miles, Ans.

4. $6\frac{1}{2}, 19\frac{4}{5}, 3\frac{3}{4} = 6\frac{10}{20}, 19\frac{16}{20}, 3\frac{15}{20}$; $6\frac{10}{20} + 19\frac{16}{20} + 3\frac{15}{20} = 30\frac{1}{20}$ tons, Ans.

5. $\frac{1}{5}, \frac{1}{3} = \frac{3}{15}, \frac{5}{15}$; $\frac{3}{15} + \frac{5}{15} = \frac{8}{15}$; $1 - \frac{8}{15} = \frac{15}{15} - \frac{8}{15} = \frac{7}{15}$, Ans.

6. $1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} = \frac{1}{3}$; $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$; $\frac{2}{3} + \frac{1}{12} = \frac{8}{12} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$; $1 - \frac{3}{4}$; $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$, Ans.

7. $11\frac{5}{8}, 9\frac{1}{2} = 11\frac{5}{8}, 9\frac{4}{8}$; $11\frac{5}{8} + 9\frac{4}{8} = 21\frac{1}{8}$; $31\frac{1}{4}, 21\frac{1}{8} = 31\frac{2}{8}, 21\frac{1}{8}$; $31\frac{2}{8} - 21\frac{1}{8} = 10\frac{1}{8}$ yards, Ans.

8. $4\frac{3}{7}$, 11, $26\frac{1}{2} = 4\frac{6}{14}$, 11, $26\frac{7}{14}$; $4\frac{6}{14} + 11 + 26\frac{7}{14} = 41\frac{13}{14}$; $84\frac{3}{8} - 41\frac{13}{14} = 42\frac{25}{56}$, $+ 3\frac{3}{4} = 46\frac{11}{56}$; $46\frac{11}{56} - 13\frac{8}{11} = 46\frac{121}{616} - 13\frac{448}{616} = 32\frac{289}{616}$; $32\frac{289}{616} + 3\frac{1}{2} = 32\frac{289}{616} + 3\frac{308}{616} = 35\frac{597}{616}$ gallons, Ans.

9. $19\frac{3}{4}$, $36\frac{1}{3}$, $33\frac{5}{8} = 19\frac{18}{24}$, $36\frac{8}{24}$, $33\frac{15}{24} = 19\frac{18}{24} + 36\frac{8}{24} + 33\frac{15}{24} = 89\frac{17}{24}$; $89\frac{17}{24} - 71\frac{3}{8} = 89\frac{17}{24} - 71\frac{9}{24} = 18\frac{1}{3}$ yards, Ans.

MULTIPLICATION OF COMMON FRACTIONS.

(ART. **235**, p. **171**.)

5. $12 \times \frac{5}{7} = \frac{12}{1} \times \frac{5}{7} = \frac{60}{7} = 8\frac{4}{7}$, Ans.

6. $\frac{2}{7} \times 12 = \frac{2}{7} \times \frac{12}{1} = \frac{24}{7} = 3\frac{3}{7}$, Ans.

7. $\frac{14}{17} \times \frac{17}{42} = \frac{\overset{1}{\cancel{14}}}{\cancel{17}} \times \frac{\cancel{17}}{\underset{3}{\cancel{42}}} = \frac{1}{3}$, Ans.

8. $\frac{3}{7} \times \frac{19}{20} = \frac{57}{140}$, Ans.

9. $\frac{4}{5} \times \frac{6}{11} = \frac{24}{55}$, Ans.

10. $\frac{7}{12} \times \frac{11}{16} = \frac{77}{192}$, Ans.

11. $\frac{8}{11} \times \frac{4}{7} = \frac{32}{77}$, Ans.

12. $\frac{3}{4} \times \frac{15}{16} = \frac{45}{64}$, Ans.

13. $\frac{16}{17} \times \frac{11}{12} = \frac{\overset{4}{\cancel{16}}}{17} \times \frac{11}{\underset{3}{\cancel{12}}} = \frac{44}{51}$, Ans.

14. $\frac{9}{10}\times\frac{14}{1}=\frac{9}{\underset{5}{\cancel{10}}}\times\frac{\overset{7}{\cancel{14}}}{1}=\frac{63}{5}=12\frac{3}{5}$, Ans.

15. $13\times\frac{4}{7}=\frac{13}{1}\times\frac{4}{7}=\frac{52}{7}=7\frac{3}{7}$, Ans.

16. $16\times\frac{3}{19}=\frac{16}{1}\times\frac{3}{19}=\frac{48}{19}=2\frac{10}{19}$, Ans.

17. $11\times\frac{4}{7}=\frac{11}{1}\times\frac{4}{7}=\frac{44}{7}=6\frac{2}{7}$, Ans.

18. $\frac{9}{10}\times\frac{14}{1}=\frac{9}{\underset{5}{\cancel{10}}}\times\frac{\overset{7}{\cancel{14}}}{1}=\frac{63}{5}=12\frac{3}{5}$, Ans.

19. $\frac{8}{9}\times\frac{19}{1}=\frac{152}{9}=16\frac{8}{9}$, Ans.

20. $\frac{9}{11}\times\frac{22}{27}=\frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{11}}}\times\frac{\overset{2}{\cancel{22}}}{\underset{3}{\cancel{27}}}=\frac{2}{3}$, Ans.

21. $\frac{4}{9}\times\frac{18}{19}=\frac{4}{\underset{1}{\cancel{9}}}\times\frac{\overset{2}{\cancel{18}}}{19}=\frac{8}{19}$, Ans.

22. $\frac{3}{19}\times\frac{38}{39}=\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{19}}}\times\frac{\overset{2}{\cancel{38}}}{\underset{13}{\cancel{39}}}=\frac{2}{13}$, Ans.

23. $\frac{7}{9}\times\frac{9}{11}=\frac{7}{\cancel{9}}\times\frac{\cancel{9}}{11}=\frac{7}{11}$, Ans.

24. $\frac{1}{99}\times\frac{99}{100}=\frac{1}{\cancel{99}}\times\frac{\cancel{99}}{100}=\frac{1}{100}$, Ans.

25. $\frac{3}{7}\times\frac{7}{11}\times\frac{11}{24}+100=\frac{\overset{1}{\cancel{3}}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{11}}\times\frac{\cancel{11}}{\underset{8}{\cancel{24}}}\times\frac{100}{1}=\frac{100}{8}=12\frac{1}{2}$, [Ans.

26. $\frac{1}{2}\times\frac{2}{3}\times\frac{7}{8}\times\frac{11}{1}=\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{3}\times\frac{7}{8}\times\frac{11}{1}=\frac{77}{24}=3\frac{5}{24}$, Ans.

27. $\frac{7}{12}\times\frac{17}{1}=\frac{119}{12}=\$9\frac{11}{12}$, Ans.

28. $\frac{9}{20}\times\frac{37}{1}=\frac{333}{20}=\$16\frac{13}{20}$, Ans.

29. $\frac{5}{\cancel{8}}\times\frac{\cancel{8}}{1}\times\frac{7}{1}=\$35$, Ans.

30. $161\frac{11}{15}\times 19\frac{11}{28}=\frac{2426}{15}\times\frac{543}{28}=\frac{1317318}{420}=3136\frac{33}{70}$, Ans.

31. $\frac{3}{7}\times 8\frac{3}{4}=\frac{3}{7}\times\frac{35}{4}=\frac{105}{28}=3\frac{3}{4}$, Ans.

32. $\frac{9}{10}\times 17\frac{3}{11}=\frac{9}{10}\times\frac{190}{11}=\frac{1710}{110}=15\frac{6}{11}$, Ans.

33. $\frac{8}{9}\times 71\frac{4}{5}=\frac{8}{9}\times\frac{359}{5}=\frac{2872}{45}=63\frac{37}{45}$, Ans.

34. $\frac{3}{4}\times 9\frac{1}{4}\times\frac{2}{3}\times 17=\frac{\cancel{3}}{4}\times\frac{37}{\underset{2}{\cancel{4}}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{17}{1}=\frac{629}{8}=78\frac{5}{8}$, [Ans.

35. $\frac{9}{10}\times 7\times\frac{11}{15}\times 87\frac{3}{11}=\frac{\overset{3}{\cancel{9}}}{\cancel{10}}\times\frac{7}{1}\times\frac{\cancel{11}}{\underset{5}{\cancel{15}}}\times\frac{\overset{96}{\cancel{960}}}{\cancel{11}}=\frac{2016}{5}=$ [$403\frac{1}{5}$, Ans.

36. $8\times\frac{7}{9}=\frac{8}{1}\times\frac{7}{9}=\frac{56}{9}=6\frac{2}{9}$, Ans.

37. $12\times\frac{5}{7}=\frac{12}{1}\times\frac{5}{7}=\frac{60}{7}=8\frac{4}{7}$, Ans.

38. $15 \times \frac{6}{11} = \frac{15}{1} \times \frac{6}{11} = \frac{90}{11} = 8\frac{2}{11}$, Ans.

39. $\frac{7}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{11} = \frac{7}{22}$, Ans.

40. $3\frac{7}{9} \times 10\frac{4}{7} = \frac{34}{9} \times \frac{74}{7} = \frac{2516}{63} = 39\frac{59}{63}$, Ans.

41. $\frac{2}{3} \times 7\frac{1}{4} \times \frac{7}{8} \times 11\frac{3}{4} = \frac{\cancel{2}}{3} \times \frac{29}{\underset{2}{\cancel{4}}} \times \frac{7}{8} \times \frac{47}{4} = \frac{9541}{192} = 49\frac{133}{192}$, [Ans.

42. $\frac{2}{7} \times 9 \times \frac{3}{5} \times 17 = \frac{2}{7} \times \frac{9}{1} \times \frac{3}{5} \times \frac{17}{1} = \frac{918}{35} = 26\frac{8}{35}$, Ans.

43. $\frac{4}{7} \times 8\frac{3}{10} \times \frac{4}{7} \times 9\frac{1}{4} = \frac{4}{7} \times \frac{83}{10} \times \frac{\cancel{4}}{7} \times \frac{37}{\cancel{4}} = \frac{12284}{490} = 25\frac{17}{245}$, [Ans.

(ART. **236**, p. **172.**)

(3.)

$$\begin{array}{rr} & 8\frac{3}{5} \\ & 7 \\ \cline{2-2} \frac{3}{5} \times 7 = & 4\frac{1}{5} \\ 7 \times 8 = & 56 \\ \cline{2-2} & 60\frac{1}{5} \end{array}$$

(4.)

$$\begin{array}{rr} & 3\frac{1}{6} \\ & 17 \\ \cline{2-2} \frac{1}{6} \times 17 = & 2\frac{5}{6} \\ 3 \times 17 = & 41 \\ \cline{2-2} & 43\frac{5}{6} \end{array}$$

(5.)

$$\begin{array}{rr} & 8\frac{3}{7} \\ & 13 \\ \cline{2-2} \frac{3}{7} \times 13 = & 5\frac{4}{7} \\ 8 \times 13 = & 104 \\ \cline{2-2} & 109\frac{4}{7} \end{array}$$

(6,)

$$\begin{array}{rr} & 13\frac{8}{11} \\ & 37 \\ \cline{2-2} \frac{8}{11} \times 37 = & 26\frac{10}{11} \\ 37 \times 13 = & 481 \\ \cline{2-2} & 507\frac{10}{11} \end{array}$$

(7.)

$$\begin{array}{rr} & 11\frac{6}{7} \\ & 8 \\ \cline{2-2} \frac{6}{7} \times 8 = & 6\frac{6}{7} \\ 8 \times 11 = & 88 \\ \cline{2-2} & 94\frac{6}{7} \end{array}$$

(8.)

$$\begin{array}{rr} & 7\frac{6}{11} \\ & 5 \\ \cline{2-2} \frac{6}{11} \times 5 = & 2\frac{8}{11} \\ 5 \times 7 = & 35 \\ \hline & \$0.37\frac{8}{11} \end{array}$$

(9.)

$$\begin{array}{rr}
 & 23\frac{7}{12} \\
 & 6 \\ \cline{2-2}
\frac{7}{12} \times 6 = & 3\frac{1}{2} \\
6 \times 23 = & 138 \\ \cline{2-2}
 & \$141\frac{1}{2}
\end{array}$$

(10.)

$$\begin{array}{rr}
 & 8\frac{3}{8} \\
 & 5 \\ \cline{2-2}
\frac{3}{8} \times 5 = & 1\frac{7}{8} \\
8 \times 5 = & 40 \\ \cline{2-2}
 & \$41\frac{7}{8}
\end{array}$$

(11.)

$$\begin{array}{rr}
 & 6\frac{3}{8} \\
 & 9 \\ \cline{2-2}
\frac{3}{8} \times 9 = & 3\frac{3}{8} \\
9 \times 6 = & 54 \\ \cline{2-2}
 & \$57\frac{3}{8}
\end{array}$$

(12.)

$$\begin{array}{rr}
 & \$6.37\frac{1}{2} \\
 & 12 \\ \cline{2-2}
\frac{1}{2} \times 12 = & 6 \\
12 \times 6.37 = & 76.44 \\ \cline{2-2}
 & \$76.50
\end{array}$$

(13.)

$$\begin{array}{rr}
 & 9\frac{3}{8} \\
 & 11 \\ \cline{2-2}
\frac{3}{8} \times 11 = & 4\frac{1}{8} \\
11 \times 9 = & 99 \\ \cline{2-2}
 & \$103\frac{1}{8}
\end{array}$$

(14.)

$$\begin{array}{rr}
 & 4\frac{3}{8} \\
 & 1.75 \\ \cline{2-2}
\frac{3}{8} \times 1.75 = & 65\frac{5}{8} \\
1.75 \times 4 = & 700 \\ \cline{2-2}
 & \$7.65\frac{5}{8}
\end{array}$$

(15.)

$$\begin{array}{rr}
 & 11\frac{7}{8} \\
 & 7 \\ \cline{2-2}
\frac{7}{8} \times 7 = & 6\frac{1}{8} \\
7 \times 11 = & 77 \\ \cline{2-2}
 & \$83\frac{1}{8}
\end{array}$$

(16.)

$$\begin{array}{rr}
 & \$10\frac{5}{8} \\
 & 9 \\ \cline{2-2}
\frac{5}{8} \times 9 = & 5\frac{5}{8} \\
9 \times 10 = & 90 \\ \cline{2-2}
 & \$95\frac{5}{8}
\end{array}$$

(17.)

$$\begin{array}{rr}
 & \$3\frac{1}{8} \\
 & 5 \\ \cline{2-2}
\frac{1}{8} \times 5 = & 0\frac{5}{8} \\
5 \times 3 = & 15 \\ \cline{2-2}
 & \$15\frac{5}{8}
\end{array}$$

(18.)

$$\begin{array}{rr}
 & \$7.62\frac{1}{2} \\
 & 15 \\ \cline{2-2}
\frac{1}{2} \times 15 = & 7\frac{1}{2} \\
15 \times 7.62 = & 114.30 \\ \cline{2-2}
 & \$114.37\frac{1}{2}
\end{array}$$

(19.)

$$\begin{array}{rr}
 & \$8.37\frac{1}{2} \\
 & 40 \\ \cline{2-2}
\frac{1}{2} \times 40 = & 20 \\
40 \times 8.37 = & 334.80 \\ \cline{2-2}
 & \$335.00
\end{array}$$

DIVISION OF COMMON FRACTIONS.

(Art. **239**, p. **175**.)

6 $\frac{7}{11} \div 18 = \frac{7}{11} \times \frac{1}{18} = \frac{7}{198}$, Ans.

7. $\frac{4}{9} \div \frac{7}{8} = \frac{4}{9} \times \frac{8}{7} = \frac{32}{63}$, Ans.

8. $18 \div \frac{7}{11} = \frac{18}{1} \times \frac{11}{7} = \frac{198}{7} = 28\frac{2}{7}$, Ans.

9. $\frac{5}{27} \div \frac{5}{3} = \frac{5}{27} \times \frac{3}{5} = \frac{15}{135} = \frac{1}{9}$, Ans.

10. $\frac{16}{21} \div \frac{2}{3} = \frac{8}{7} = 1\frac{1}{7}$, Ans.

11. $\frac{15}{22} \div 28 = \frac{15}{22} \times \frac{1}{28} = \frac{15}{616}$, Ans.

12. $\frac{1}{17} \div 27 = \frac{1}{17} \times \frac{1}{27} = \frac{1}{459}$, Ans.

13. $\frac{2}{15} \div 128 = \frac{\overset{1}{\cancel{2}}}{15} \times \frac{1}{\underset{64}{\cancel{128}}} = \frac{1}{960}$, Ans.

14. $\frac{11}{17} \div 98 = \frac{11}{17} \times \frac{1}{98} = \frac{11}{1666}$, Ans.

15. $\frac{14}{23} \div 19 = \frac{14}{23} \times \frac{1}{19} = \frac{14}{437}$, Ans.

16. $\frac{5}{6} \div 167 = \frac{5}{6} \times \frac{1}{167} = \frac{5}{1002}$, Ans.

17. $\frac{16}{29} \div 49 = \frac{16}{29} \times \frac{1}{49} = \frac{16}{1421}$, Ans.

18. $\frac{1}{15} \div 15 = \frac{1}{15} \times \frac{1}{15} = \frac{1}{225}$, Ans.

19. $27 \div \frac{1}{17} = \frac{27}{1} \times \frac{17}{1} = 459$, Ans.

20. $128 \div \frac{2}{15} = \frac{\overset{64}{\cancel{128}}}{1} \times \frac{15}{\underset{1}{\cancel{2}}} = 960$, Ans.

21. $98 \div \frac{11}{17} = \frac{98}{1} \times \frac{17}{11} = \frac{1666}{11} = 151\frac{5}{11}$, Ans.

22. $19 \div \frac{14}{23} = \frac{19}{1} \times \frac{23}{14} = \frac{437}{14} = 31\frac{3}{14}$, Ans.

23. $167 \div \frac{15}{18} = \frac{167}{1} \times \frac{18}{15} = \frac{3006}{15} = 200\frac{2}{5}$, Ans.

24. $49 \div \frac{16}{29} = \frac{49}{1} \times \frac{29}{16} = \frac{1421}{16} = 88\frac{13}{16}$, Ans.

25. $15 \div \frac{1}{15} = \frac{15}{1} \times \frac{15}{1} = 225$, Ans.

26. $\frac{36}{51} \div \frac{3}{17} = \frac{12}{3} = 4$, Ans.

27. $\frac{31}{97} \div \frac{31}{301} = \frac{\cancel{31}}{97} \times \frac{301}{\cancel{31}} = \frac{301}{97} = 3\frac{10}{97}$, Ans.

28. $\frac{5}{6} \div \frac{8}{9} = \frac{5}{6} \times \frac{9}{8} = \frac{45}{48} = \frac{15}{16}$, Ans.

29. $\frac{11}{15} \div \frac{7}{11} = \frac{11}{15} \times \frac{11}{7} = \frac{121}{105} = 1\frac{16}{105}$, Ans.

30. $\frac{8}{25} \div \frac{14}{17} = \frac{\overset{4}{\cancel{8}}}{25} \times \frac{17}{\underset{7}{\cancel{14}}} = \frac{68}{175}$, Ans.

31. $\frac{16}{21} \div \frac{1}{15} = \frac{16}{21} \times \frac{15}{1} = \frac{240}{21} = 11\frac{3}{7}$, Ans.

32. $\frac{9}{25} \div 7\frac{3}{4} = \frac{9}{25} \times \frac{4}{31} = \frac{36}{775}$, Ans.

33. $\frac{8}{11} \div \frac{164}{9} = \frac{8}{11} \times \frac{9}{148} = \frac{72}{1628} = \frac{18}{407}$, Ans.

34. $11\frac{3}{7} \div \frac{4}{7} = \frac{\overset{20}{\cancel{80}}}{\cancel{7}} \times \frac{\cancel{7}}{\underset{1}{\cancel{4}}} = 20$, Ans.

35. $21\frac{4}{5} \div 18\frac{4}{7} = \frac{109}{5} \times \frac{7}{130} = \frac{763}{650} = 1\frac{113}{650}$, Ans.

36. $17\frac{3}{11} \div 28\frac{11}{26} = \frac{190}{11} \times \frac{26}{739} = \frac{4940}{8129}$, Ans.

37. $161\frac{3}{17} \div 14\frac{3}{5} = \frac{2740}{17} \times \frac{5}{73} = \frac{13700}{1241} = 11\frac{49}{1241}$, Ans.

38. $\frac{7}{11} \times \frac{4}{5} = \frac{28}{55}$; $\frac{3}{5} \times \frac{8}{11} = \frac{24}{55}$; $\frac{\overset{7}{\cancel{28}}}{\cancel{55}} \times \frac{\cancel{55}}{\underset{6}{\cancel{24}}} = \frac{7}{6} = 1\frac{1}{6}$, Ans.

39. $\frac{5}{9} \times 7\frac{3}{11} = \frac{400}{99}$; $\frac{4}{11} \times 17\frac{3}{7} = \frac{488}{77}$; $\frac{\overset{100}{\cancel{400}}}{\underset{9}{\cancel{99}}} \times \frac{\overset{7}{\cancel{77}}}{\underset{122}{\cancel{488}}} = \frac{700}{1098}$
[$= \frac{350}{549}$, Ans.

40. $\frac{6}{17} \times 15 = \frac{90}{17}$; $\frac{7}{15} \times 22 = \frac{154}{15}$; $\frac{90}{17} \times \frac{15}{154} = \frac{1350}{2618} =$
[$= \frac{675}{1309}$, Ans.

41. $\frac{7}{7} - \frac{5}{7} = \frac{2}{7}$; $\frac{2}{3} \times \frac{2}{7} = \frac{4}{21}$; $\frac{2}{7} - \frac{4}{21} = \frac{2}{21}$; $\frac{2}{21} \times 3675 =$
[350; ÷ 7 = \$50, Ans.

42. $1 - \frac{1}{3} = \frac{2}{3}$; $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$; $\frac{1}{3} + \frac{4}{9} = \frac{7}{9}$; $1 - \frac{7}{9} = \frac{2}{9}$; $\frac{2}{9} \times$
[$\frac{1}{7} = \frac{2}{63}$; $\frac{2}{63} \times 5000 = \$158\frac{46}{63}$, Ans.

(Art. **240**, p. **176**.)

2. $9\overline{)29\frac{3}{8}}$
3, rem. $2\frac{3}{8}$; $2\frac{3}{8} = \frac{19}{8}$; $\frac{19}{8 \times 9} = \frac{19}{72}$; $3 + \frac{19}{72} = 3\frac{19}{72}$, Ans.

3. $7\overline{)14\frac{1}{2}}$
2, rem. $\frac{1}{2}$; $\frac{1}{2 \times 7} = \frac{1}{14}$; $2 + \frac{1}{14} = 2\frac{1}{14}$, Ans.

4. $8\overline{)13\frac{3}{8}}$
1, rem. $5\frac{3}{8}$; $5\frac{3}{8} = \frac{43}{8}$; $\frac{43}{8 \times 8} = \frac{43}{64}$; $1 + \frac{43}{64} = 1\frac{43}{64}$, Ans.

5. $6)\underline{14\frac{3}{5}}$
2, rem. $2\frac{3}{5}$; $2\frac{3}{5}=\frac{13}{5}$; $\frac{13}{5\times 6}=\frac{13}{30}$; $2+\frac{13}{30}=2\frac{13}{30}$, Ans.

6. $9)\underline{37\frac{3}{7}}$
4, rem. $1\frac{3}{7}$; $1\frac{3}{7}=\frac{10}{7}$; $\frac{10}{7\times 9}=\frac{10}{63}$; $4+\frac{10}{63}=4\frac{10}{63}$, Ans.

7. $11)\underline{96\frac{2}{3}}$
8, rem. $8\frac{2}{3}$; $8\frac{2}{3}=\frac{26}{3}$; $\frac{26}{3\times 11}=\frac{26}{33}$; $8+\frac{26}{33}=\$8\frac{26}{33}$,
[Ans.

8. $8)\underline{167\frac{7}{11}}$
20, rem. $7\frac{7}{11}$; $7\frac{7}{11}=\frac{84}{11}$; $\frac{84}{11\times 8}=\frac{84}{88}=\frac{21}{22}$; $20+\frac{21}{22}$
[$=20\frac{21}{22}$cwt., Ans.

9. $\frac{7}{8}\times\frac{1723}{1}=\frac{12061}{8}=1507\frac{5}{8}$; $12)\underline{1507\frac{5}{8}}$
125, rem. $7\frac{5}{8}$; $7\frac{5}{8}=\frac{61}{8}$;
[$\frac{61}{8\times 12}=\frac{61}{96}$; $\$125+\frac{61}{96}=\$125\frac{61}{96}$, Ans.

10. $19)\underline{107\frac{3}{8}}$
5, rem. $12\frac{3}{8}$; $12\frac{3}{8}=\frac{99}{8}$; $\frac{99}{8\times 19}=\frac{99}{152}$; $5+\frac{99}{152}=$
$5\frac{99}{152}$; $\frac{99\times 100}{152}=\frac{9900}{152}=.65\frac{5}{38}$; $\$5+.65\frac{5}{38}=$
[$\$5.65\frac{5}{38}$, Ans.

11. $\frac{2}{7}$ of $\$1=\frac{2\times 100}{7}=\frac{200}{7}$; $\frac{200}{7}=.28\frac{4}{7}$; $\$3+.28\frac{4}{7}=3.28\frac{4}{7}$;
$15)\underline{3.28\frac{4}{7}}$
$\$0.21$, rem. $13\frac{4}{7}$; $13\frac{4}{7}=\frac{95}{7}$; $\frac{95}{7\times 15}=\frac{95}{105}=\frac{19}{21}$;
[$\$0.21+\frac{19}{21}=\$0.21\frac{19}{21}$, Ans.

12. $12)\underline{\$3.75\frac{3}{4}}$
$\$\ .31$, rem. $3\frac{3}{4}$; $3\frac{3}{4}=\frac{15}{4}$; $\frac{15}{4\times 12}=\frac{15}{48}=\frac{5}{16}$;
[$\$0.31+\frac{5}{16}=\$0.31\frac{5}{16}$, Ans.

13. $19)\underline{375\frac{11}{16}}$
19, rem. $14\frac{11}{16}$; $14\frac{11}{16}=\frac{235}{16}$; $\frac{235}{16\times 19}=\frac{235}{304}$;
[$19+\frac{235}{304}=19\frac{235}{304}$ acres, Ans.

REDUCTION OF COMPLEX FRACTIONS.

(ART. **242**, p. **178.**)

4. $\frac{4\frac{3}{7}}{\frac{2}{3}}=\frac{\frac{31}{7}}{\frac{2}{3}}=\frac{31}{7}\times\frac{3}{2}=\frac{93}{14}=6\frac{9}{14}$, Ans.

5. $\frac{\frac{3}{4}}{5\frac{2}{3}} = \frac{\frac{3}{4}}{\frac{17}{3}} = \frac{3}{4} \times \frac{3}{17} = \frac{9}{68}$, Ans.

6. $\frac{7}{4\frac{2}{3}} = \frac{\frac{7}{1}}{\frac{14}{3}} = \frac{7}{1} \times \frac{3}{14} = \frac{21}{14} = 1\frac{1}{2}$, Ans.

7. $\frac{7\frac{4}{11}}{8} = \frac{\frac{81}{11}}{\frac{8}{1}} = \frac{81}{11} \times \frac{1}{8} = \frac{81}{88}$, Ans.

8. $\frac{6\frac{2}{9}}{8\frac{2}{3}} = \frac{\frac{56}{9}}{\frac{26}{3}} = \frac{56}{9} \times \frac{3}{26} = \frac{168}{234} = \frac{28}{39}$, Ans.

9. $\frac{\frac{2}{7}}{\frac{3}{5}} = \frac{2}{7} \times \frac{5}{3} = \frac{10}{21}$, Ans.

10. $\frac{8}{\frac{1}{3}} = \frac{\frac{8}{1}}{\frac{1}{3}} = \frac{8}{1} \times \frac{3}{1} = 24$, Ans.

11. $\frac{\frac{4}{7}}{2} = \frac{\frac{4}{7}}{\frac{2}{1}} = \frac{4}{7} \times \frac{1}{2} = \frac{4}{14} = \frac{2}{7}$, Ans.

12. $\frac{5\frac{1}{3}}{\frac{3}{7}} = \frac{\frac{16}{3}}{\frac{3}{7}} = \frac{16}{3} \times \frac{7}{3} = \frac{112}{9} = 12\frac{4}{9}$, Ans.

13. $\frac{\frac{1}{4}}{6\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{13}{2}} = \frac{1}{4} \times \frac{2}{13} = \frac{2}{52} = \frac{1}{26}$, Ans.

14. $\frac{3}{2\frac{1}{2}} = \frac{\frac{3}{1}}{\frac{5}{2}} = \frac{3}{1} \times \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$, Ans.

15. $\frac{3\frac{1}{4}}{9} = \frac{\frac{13}{4}}{\frac{9}{1}} = \frac{13}{4} \times \frac{1}{9} = \frac{13}{36}$, Ans.

16. $\frac{11\frac{2}{3}}{12\frac{3}{5}} = \frac{\frac{35}{3}}{\frac{63}{5}} = \frac{35}{3} \times \frac{5}{63} = \frac{175}{189} = \frac{25}{27}$, Ans.

17. $\frac{\frac{7\frac{7}{9}}{11\frac{4}{5}}}{7} = \frac{\frac{\frac{70}{9}}{\frac{59}{5}}}{\frac{7}{1}} = \frac{70}{9} \times \frac{5}{59} \times \frac{1}{7} = \frac{350}{3717} = \frac{50}{531}$, Ans.

18. $\frac{\frac{7}{9}}{\frac{\frac{3}{5}}{\frac{5}{1}}} = \frac{7}{9} \times \frac{5}{3} \times \frac{5}{1} = \frac{175}{27} = 6\frac{13}{27}$, Ans.

(Art. **243**, p. **179**.)

1. $28\frac{3\frac{6}{7}}{4\frac{2}{7}} = 28\frac{\frac{27}{7}}{\frac{30}{7}};\ \frac{\frac{27}{7}}{\frac{30}{7}} = \frac{\overset{9}{\cancel{27}}}{\cancel{7}} \times \frac{\cancel{7}}{\underset{10}{\cancel{30}}} = \frac{9}{10};\ 28 + \frac{9}{10} = 28\tfrac{9}{10} =$

$\frac{289}{10};\ \frac{1}{3} \times \frac{2}{7} \times \frac{289}{10} = \frac{578}{210} = 2\tfrac{158}{210};\ 3\frac{39\frac{1}{2}}{105} = 3\frac{\frac{79}{2}}{\frac{105}{1}};$

$\frac{\frac{79}{2}}{\frac{105}{1}} = \frac{79}{2} \times \frac{1}{105} = \frac{79}{210};\ 3 + \frac{79}{210} = 3\tfrac{79}{210};\ 2\tfrac{158}{210} +$

$[3\tfrac{79}{210} = 6\tfrac{9}{70}$, Ans.

2. $\frac{1}{9} = \frac{1}{9};\ 2\tfrac{5}{8} = \frac{21}{8};\ \frac{45}{94\frac{7}{11}} = \frac{\frac{45}{1}}{\frac{1041}{11}} = \frac{45}{1} \times \frac{11}{1041} = \frac{495}{1041};$

$\frac{47\frac{5}{9}}{314\frac{3}{5}} = \frac{\frac{428}{9}}{\frac{1573}{5}} = \frac{428}{9} \times \frac{5}{1573} = \frac{2140}{14157};\ \frac{1}{9} + \frac{21}{8} +$

$\frac{495}{1041} + \frac{2140}{14157}.$ $\quad 3\underline{)\cancel{9}\ 8\ 1041\ 14157}$
$\qquad\qquad\qquad\quad\ 8\ \ 347\ \ 4719$

$3 \times 8 \times 347 \times 4719 = 39299832.$

$$\begin{array}{r|l}
9 & \underline{39299832} \\
8 & \underline{4366648} \times \quad 1 = \quad 4366648 \\
1041 & \underline{4912479} \times \ \ 21 = 103162059 \\
14157 & \underline{\ \ 37752} \times \ 495 = \ \ 18687240 \\
 & \ \ \ 2776 \times 2140 = \ \ \ 5940640 \\
\hline
 & \qquad\qquad \frac{132156587}{39299832} = 3\tfrac{14257091}{39299832}, \text{ Ans.}
\end{array}$$

3. $\frac{49\frac{5}{8}}{97} = \frac{\frac{397}{8}}{\frac{97}{1}} = \frac{397}{8} \times \frac{1}{97} = \frac{397}{776};\ \frac{34\frac{3}{5}}{145\frac{3}{11}} = \frac{\frac{173}{5}}{\frac{1598}{11}} = \frac{173}{5} \times$

$[\frac{11}{1598} = \frac{1903}{7990};\ \frac{397}{776} - \frac{1903}{7990} = \frac{847651}{3100120}$, Ans.

4. $\frac{27}{37\frac{4}{5}}=\frac{\frac{27}{1}}{\frac{189}{5}}=\frac{\overset{1}{\cancel{27}}}{1}\times\frac{5}{\underset{7}{\cancel{189}}}=\frac{5}{7}$;

$\frac{87\frac{2}{9}}{98\frac{1}{8}}=\frac{\frac{785}{9}}{\frac{785}{8}}=\frac{\cancel{785}}{9}\times\frac{8}{\cancel{785}}=\frac{8}{9}$;

$\frac{\frac{7}{8}}{2\frac{1}{3}}=\frac{\frac{7}{8}}{\frac{7}{3}}=\frac{\cancel{7}}{8}\times\frac{3}{\cancel{7}}=\frac{3}{8}$;

$\frac{81\frac{5}{11}}{128}=\frac{\frac{896}{11}}{\frac{128}{1}}=\frac{\overset{7}{\cancel{896}}}{11}\times\frac{1}{\cancel{128}}=\frac{7}{11}$;

$\frac{5}{\cancel{7}}\times\frac{\cancel{8}}{\underset{3}{\cancel{9}}}\times\frac{\cancel{3}}{\cancel{8}}\times\frac{\cancel{7}}{11}=\frac{5}{33}$, Ans.

5. $\frac{2}{3}\times7\frac{3}{4}=\frac{2}{3}\times\frac{31}{4}=\frac{62}{12}=\frac{31}{6}$; $\frac{4}{5}\times11\frac{4}{11}=\frac{4}{5}\times\frac{125}{11}=$ [$\frac{500}{55}=\frac{100}{11}$, $\frac{31}{6}\times\frac{11}{100}=\frac{341}{600}$, Ans.

6. $\frac{4}{9}\times\frac{91}{1}=\frac{364}{9}$; $\frac{9}{10}\times\frac{87}{1}=\frac{783}{10}$; $\frac{364}{9}\times\frac{10}{783}=\frac{3640}{7047}$, [Ans.

MISCELLANEOUS EXAMPLES IN MULTIPLICATION AND DIVISION OF FRACTIONS.

(PAGE 179.)

1. $2\frac{3}{7}=\frac{17}{7}$; $7\frac{4}{17}=\frac{123}{17}$; $\frac{123}{\cancel{17}}\times\frac{\cancel{17}}{7}=\frac{123}{7}=17\frac{4}{7}$, Ans.

2. $8\frac{1}{5}=\frac{41}{5}$; $4.68\frac{4}{7}=\frac{3280}{7}$; $\frac{3280}{7}\times\frac{5}{41}=\frac{16400}{287}$; $\frac{16400}{287}$ [$=\$0.57\frac{1}{7}$, Ans.

3. $96\frac{3}{8}=\frac{771}{8}$; $1-\frac{5}{8}=\frac{8}{8}-\frac{5}{8}=\frac{3}{8}$; $\frac{771}{8}\times\frac{3}{8}=\frac{2313}{64}$;

$\frac{2313}{64}\div2=\frac{2313}{64}\times\frac{1}{2}-\frac{2313}{128}=18\frac{9}{128}$yd., Ans.

4. $18\frac{1}{3} \times \frac{3}{4} = \frac{55}{3} \times \frac{3}{4} = \frac{165}{12}$; $6\frac{1}{2} \times \frac{2}{3} = \frac{13}{2} \times \frac{2}{3} = \frac{26}{6}$; $\frac{165}{12}$
[$\div \frac{26}{6} = \frac{165}{\underset{2}{\cancel{12}}} \times \frac{\cancel{6}}{26} = \frac{165}{52} = 3\frac{9}{52}$, Ans.

5. $1\frac{\frac{1}{2}}{3} = 1 + \frac{\frac{1}{2}}{3}$; $\frac{\frac{1}{2}}{3} = \frac{\frac{1}{2}}{\frac{3}{1}} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$; $1 + \frac{1}{6} = 1\frac{1}{6} = \frac{7}{6}$; $1 \div$
[$\frac{7}{6} = \frac{1}{1} \times \frac{6}{7} = \frac{6}{7}$, Ans.

6. $1 + \frac{11}{12} = \frac{12}{12} + \frac{11}{12} = \frac{23}{12}$; $250 \times 12 = 3000$; $3000 \div$
[$23 = \$130\frac{10}{23}$, horse.
$250 \times 11 = 2750$; $2750 \div 23 = \$119\frac{13}{23}$, chaise.
$\frac{7}{11} \times 130\frac{10}{23} = \frac{7}{11} \times \frac{3000}{23} = \frac{21000}{253} = \$83\frac{1}{253}$, harness.

7. $\$200 \times \frac{3}{4} = \$150$; $\$150 - \$144 = \$6$; $\$6 \times 4 =$
[$24.00, Ans.

8. $\frac{1}{5} \times \frac{3}{4} \times 3\frac{1}{8} = \frac{1}{5} \times \frac{3}{4} \times \frac{25}{8} = \frac{75}{160} = \frac{15}{32}$; $\frac{15}{32} \times \frac{1}{2} = \frac{15}{64}$;
$\frac{15}{32} \times \frac{15}{64} = \frac{225}{2048}$; $\frac{225}{2048} \div \frac{1}{2} = \frac{225}{2048} \times \frac{2}{1} = \frac{450}{2048} =$
[$\frac{225}{1024}$, Ans.

9. $13\frac{7}{8} \times 25\frac{3}{4} = \frac{111}{8} \times \frac{103}{4} = \frac{11433}{32}$; $\frac{11433}{32} \div 2\frac{3}{5} = \frac{11433}{32}$
[$\div \frac{13}{5}$; $\frac{11433}{32} \times \frac{5}{13} = \frac{57165}{416} = 137\frac{173}{416}$ bushels, Ans.

10. $3\frac{1}{2} \times 9\frac{7}{8} = \frac{7}{2} \times \frac{79}{8} = \frac{553}{16}$; $\frac{553}{1} \div \frac{553}{16} = \frac{\cancel{553}}{1} \times \frac{16}{\cancel{553}} =$
[16 days, Ans.

11. $\$0.90 \div 4 = \$0.22\frac{1}{2}$; that is, he will receive $\frac{1}{4}$ more.

12. $9\frac{4}{5} \times 16\frac{3}{4} = \frac{49}{5} \times \frac{67}{4} = \frac{3283}{20} = 164\frac{3}{20}$ miles per day.
$164\frac{3}{20} \times 4 = 656\frac{3}{5}$ miles in 4 days.
$164\frac{3}{20} \times 10 = 1641\frac{1}{2}$ miles in 10 days.
24h. — $16\frac{3}{4}$h. = $7\frac{1}{4}$h.; $7\frac{1}{4} \times 2 = 14\frac{1}{2}$ miles to deduct for drift in the night.
$164\frac{3}{20} - 14\frac{1}{2} = 149\frac{13}{20}$ miles gained per day.

$149\frac{13}{20} \times 10 = 1496\frac{1}{2}$ miles gained in 10 days.
$656\frac{3}{5}$m. + $1496\frac{1}{2}$m. = $2153\frac{1}{10}$ miles' distance sailed in 14 days.
$2317\frac{1}{4}$m. − $2153\frac{1}{10}$m. = $164\frac{3}{20}$ miles remaining, which distance will be sailed the last day; therefore the boat will arrive at her port in 15 days, Ans.

A PROPOSED NUMERATOR OR DENOMINATOR.

(Art. **244**, p. **181.**)

3. $34 \div 17 = 2$; $\frac{17 \times 2}{20 \times 2} = \frac{34}{40}$, Ans.

4. $3\frac{6}{7} = \frac{27}{7}$; $9 \div 27 = \frac{1}{3}$; $\frac{27 \times \frac{1}{3}}{7 \times \frac{1}{3}} = \frac{9}{2\frac{1}{3}}$, Ans.

5. $5 \div 4 = 1\frac{1}{4}$; $\frac{4 \times 1\frac{1}{4}}{1 \times 1\frac{1}{4}} = \frac{5}{1\frac{1}{4}}$, Ans.

6. $12 \div 16 = \frac{3}{4}$; $\frac{15 \times \frac{3}{4}}{16 \times \frac{3}{4}} = \frac{11\frac{1}{4}}{12}$, Ans.

7. $15 \div 5 = 3$; $\frac{3 \times 3}{5 \times 3} = \frac{9}{15}$, Ans.

8. $2 \div 3 = \frac{2}{3}$; $\frac{7 \times \frac{2}{3}}{3 \times \frac{2}{3}} = \frac{4\frac{2}{3}}{2}$, Ans.

9. $35 \div 20 = 1\frac{3}{4}$; $\frac{19 \times 1\frac{3}{4}}{20 \times 1\frac{3}{4}} = \frac{33\frac{1}{4}}{35}$, Ans.

10. $\frac{19}{57} = \frac{1}{3}$; $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$; $\frac{67}{201} = \frac{1}{3}$; $\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$, Ans.

COMMON NUMERATOR.

(Art. **246**, p. **182.**)

2. 24, the least common multiple of the numerators, = common numerator.

$$\begin{array}{lll} \frac{24}{8} \text{ of } 9 = 27, & \text{new denominator.} & \frac{8}{9} = \frac{24}{27} \\ \frac{24}{4} \text{ of } 7 = 42, & \text{"} \qquad \text{"} & \frac{4}{7} = \frac{24}{42} \\ \frac{24}{3} \text{ of } 4 = 32, & \text{"} \qquad \text{"} & \frac{3}{4} = \frac{24}{32} \\ \frac{24}{6} \text{ of } 7 = 28, & \text{"} \qquad \text{"} & \frac{6}{7} = \frac{24}{28} \end{array} \left.\vphantom{\begin{array}{l}1\\1\\1\\1\end{array}}\right\} \text{Ans.}$$

3. $\frac{5}{8}$, $2\frac{1}{2}$, $1\frac{3}{7} = \frac{5}{8}, \frac{5}{2}, \frac{10}{7}$.

10, the least common multiple of the numerators, = common numerator.

$$\begin{array}{lll} \frac{10}{5} \text{ of } 8 = 16, & \text{new denominator.} & \\ \frac{10}{5} \text{ of } 2 = 4, & \text{“} \qquad \text{“} & \\ \frac{10}{10} \text{ of } 7 = 7, & \text{“} \qquad \text{“} & \end{array} \qquad \left.\begin{array}{r} \frac{5}{8} = \frac{10}{16} \\ \frac{5}{2} = \frac{10}{4} \\ \frac{10}{7} = \frac{10}{7} \end{array}\right\} \text{Ans.}$$

4. $4\frac{4}{15}$, $6\frac{2}{5}$, $6\frac{2}{3} = \frac{64}{15}, \frac{32}{5}, \frac{20}{3}$.

320, the least common multiple of numerators, = common numerator.

$$\begin{array}{lll} \frac{320}{64} \text{ of } 15 = 75, & \text{new denominator.} & \\ \frac{320}{32} \text{ of } 5 = 50, & \text{“} \qquad \text{“} & \\ \frac{320}{20} \text{ of } 3 = 48, & \text{“} \qquad \text{“} & \end{array} \qquad \left.\begin{array}{r} \frac{64}{15} = \frac{320}{75} \\ \frac{32}{5} = \frac{320}{50} \\ \frac{20}{3} = \frac{320}{48} \end{array}\right\} \text{Ans.}$$

From the above process, it is evident they will meet in 320 days. A will have gone round the island 75 times, and have travelled 75 × 50 = 3750 miles. B will have gone round the island 50 times, and have travelled 50 × 50 = 2500 miles. C will have gone round the island 48 times, and have travelled 48 × 50 = 2400 miles.

GREATEST COMMON DIVISOR OF FRACTIONS.

(Art. **248**, p. **183**.)

2. $\left.\begin{array}{l} \text{Greatest common divisor of the numerators} = \\ \text{Least common denominator of the fractions} = \end{array} \dfrac{4}{315} \right\}$ Ans.

3. $12\frac{3}{8}$, $9\frac{3}{4}$, $8\frac{1}{4} = \frac{99}{8}, \frac{39}{4}, \frac{33}{4}$.

$\left.\begin{array}{l} \text{Greatest common divisor of the numerators} = \\ \text{Least common denominator of the fractions} = \end{array} \dfrac{3}{8} \right\}$ Ans.

Note. — $\frac{3}{8}$ will divide each of the fractions, $12\frac{3}{8}$, $9\frac{3}{4}$, and $8\frac{1}{4}$, without a remainder.

4. $\left.\begin{array}{l} \text{Greatest common divisor of the numerators} = \\ \text{Least common denominator of the fractions} = \end{array} \dfrac{1}{60} \right\}$ Ans.

5. $3\frac{4}{5}$, $5\frac{7}{10}$, $2\frac{8}{15} = \frac{19}{5}$, $\frac{57}{10}$, $\frac{38}{15}$.

$$\left.\begin{array}{l}\text{Greatest common divisor of the numerators} = \\ \text{Least common denominator of the fractions} =\end{array}\frac{19}{30}\right\}\text{Ans.}$$

6. $33\frac{3}{4}$, $67\frac{1}{2}$, $70\frac{7}{8} = \frac{135}{4}$, $\frac{135}{2}$, $\frac{567}{8}$.

$$\left.\begin{array}{l}\text{Greatest common divisor of the numerators} = \\ \text{Least common denominator of the fractions} =\end{array}\frac{27}{8} = 3\tfrac{3}{8}\right\}\text{Ans.}$$

$33\frac{3}{4} \div 3\frac{3}{8} = 10$ bags; $67\frac{1}{2} \div 3\frac{3}{8} = 20$ bags; $70\frac{7}{8} \div 3\frac{3}{8} =$ 21 bags. $10 + 20 + 21 = 51$ bags, Ans.

7. $73\frac{7}{11}$, $88\frac{4}{11}$, $139\frac{10}{11} = \frac{810}{11}$, $\frac{972}{11}$, $\frac{1539}{11}$.

$$\begin{array}{l}\text{Greatest common divisor of numerators} = \\ \text{Least common denominator of the fractions} =\end{array}\frac{81}{11}$$

$\frac{810}{11} + \frac{972}{11} + \frac{1539}{11} = \frac{3321}{11}$; $\frac{3321}{11} \div \frac{81}{11} = 41$ lots.

$81 \div 11 = 7\frac{4}{11}$ acres, the size of each lot.

LEAST COMMON MULTIPLE OF FRACTIONS.

(Art. **250, p. 184.**)

2. $\frac{4}{5}$, $\frac{8}{9}$, and $\frac{6}{7}$.

$$\begin{array}{l}\text{Least common mult. of the numerators} = \\ \text{Greatest com. div. of the denominators} =\end{array}\frac{24}{1} = 24\text{, Ans.}$$

3. $3\frac{15}{16}$, $7\frac{7}{8}$, $5\frac{1}{4} = \frac{63}{16}$, $\frac{63}{8}$, $\frac{21}{4}$.

$$\begin{array}{l}\text{Least common mult. of the numerators} = \\ \text{Greatest com. div. of the denominators} =\end{array}\frac{63}{4} = 15\tfrac{3}{4}\text{, Ans.}$$

4. $\frac{3}{8}$, $\frac{6}{7}$, $\frac{9}{10}$.

$$\begin{array}{l}\text{Least common mult. of the numerators} = \\ \text{Greatest com. div. of the denominators} =\end{array}\frac{18}{1} = 18\text{, Ans.}$$

Note. — By this operation we find that 18 is the least whole number that can be divided by the fractions $\frac{3}{8}$, $\frac{6}{7}$, and $\frac{9}{10}$, without a remainder.

5. $2\frac{1}{4}$, $4\frac{1}{2}$, $9\frac{3}{8} = \frac{9}{4}$, $\frac{9}{2}$, $\frac{75}{8}$.

$$\begin{array}{l}\text{Least common mult. of the numerators} = \\ \text{Greatest com. div. of the denominators} =\end{array}\frac{225}{2} = \$112\tfrac{1}{2}.$$

$\frac{225}{2} \div \frac{9}{4} = 50$ sheep; $\frac{225}{2} \div \frac{9}{2} = 25$ calves; $\frac{225}{2} \div \frac{75}{8}$ $= 12$ yearlings.

6. $3\frac{1}{2}, 4\frac{2}{3}, 5\frac{1}{4} = \frac{7}{2}, \frac{14}{3}, \frac{21}{4}$.
$80 \div \frac{7}{2} = 22\frac{6}{7}$ hours, the time A will go round the island. $80 \div \frac{14}{3} = 17\frac{1}{7}$ hours, the time B will go round the island. $80 \div \frac{21}{4} = 15\frac{5}{21}$ hours, the time C will go round the island.

$22\frac{6}{7}, 17\frac{1}{7}, 15\frac{5}{21} = \frac{160}{7}, \frac{120}{7}, \frac{320}{21}$.

$$\frac{\text{Least common mult. of } 160, 120, 320 = 960}{\text{Greatest common divisor of } 7, 7, 21 = 7} = 137\tfrac{1}{7} \text{ hours.}$$

In $137\frac{1}{7} \div 8 = 17\frac{1}{7}$ days, they will all meet at the point from which they started.

$\frac{960}{7} \div \frac{160}{7} = 6$ times, A will go round the island.
$\frac{960}{7} \div \frac{120}{7} = 8$ times, B will go round.
$\frac{960}{7} \div \frac{320}{21} = 9$ times, C will go round.
$80 \times 6 = 480$ miles, A travels. $80 \times 8 = 640$ miles, B travels. $80 \times 9 = 720$ miles, C travels.

7. $3\frac{1}{2}, 4\frac{3}{8}, 5\frac{1}{4} = \frac{7}{2}, \frac{35}{8}, \frac{21}{4}$.

$$\frac{\text{Least common multiple of } 7, 35, 21 = 105}{\text{Greatest common divisor of } 2, 8, 4 = 2} = 52\tfrac{1}{2}.$$

As $52\frac{1}{2}$ can be divided by each of the fractions $3\frac{1}{2}, 4\frac{3}{8}, 5\frac{1}{4}$, it is certain that twice $52\frac{1}{2} = 105$ may be divided by them; and no less whole number than this is a multiple of $52\frac{1}{2}$.

REDUCTION OF DENOMINATE FRACTIONS.

(ART. **253**, p. **185**.)

2. $\frac{1}{1200} \times \frac{20}{1} \times \frac{12}{4} \times \frac{4}{1} = \frac{960}{1200} = \frac{4}{5}$, Ans.
3. $\frac{1}{960} \times \frac{12}{1} \times \frac{20}{1} \times \frac{24}{1} = \frac{576}{960} = \frac{3}{5}$, Ans.
4. $\frac{5}{2592} \times \frac{12}{1} \times \frac{8}{1} \times \frac{3}{1} = \frac{1440}{2592} = \frac{5}{9}$, Ans.
5. $\frac{3}{8800} \times \frac{4}{1} \times \frac{25}{1} \times \frac{16}{1} = \frac{4800}{8800} = \frac{6}{11}$, Ans.
6. $\frac{3}{8000} \times \frac{20}{1} \times \frac{4}{1} \times \frac{25}{1} = \frac{6000}{8000} = \frac{3}{4}$, Ans.
7. $\frac{1}{225} \times \frac{5}{1} \times \frac{4}{1} \times \frac{2\frac{1}{4}}{1} = \frac{45}{225} = \frac{1}{5}$, Ans.
8. $\frac{1}{110880} \times \frac{8}{1} \times \frac{40}{1} \times \frac{16\frac{1}{2}}{1} \times \frac{12}{1} = \frac{63360}{110880} = \frac{4}{7}$, Ans.

9. $\frac{1}{38016} \times \frac{3}{1} \times \frac{8}{1} \times \frac{40}{1} \times \frac{16\frac{1}{2}}{1} \times \frac{12}{1} = \frac{570240}{1140480} = \frac{1}{2}$, Ans.

10. $\frac{3}{25090560} \times \frac{4}{1} \times \frac{40}{1} \times \frac{272\frac{1}{4}}{1} \times \frac{144}{1} = \frac{18817920}{25090560} = \frac{3}{4}$, Ans.

11. $\frac{1}{1152} \times \frac{4}{1} \times \frac{63}{1} \times \frac{4}{1} = \frac{1008}{1152} = \frac{7}{8}$, Ans.

12. $\frac{3}{320} \times \frac{4}{1} \times \frac{8}{1} \times \frac{2}{1} = \frac{192}{320} = \frac{3}{5}$, Ans.

13. $\frac{1}{4207680} \times \frac{365\frac{1}{4}}{1} \times \frac{24}{1} \times \frac{60}{1} = \frac{525960}{4207680} = \frac{1}{8}$, Ans.

14. $\frac{3}{32000} \times \frac{4}{1} \times \frac{25}{1} \times \frac{16}{1} = \frac{4800}{3200} = \frac{3}{2}$, Ans.

(ART. **254**, p. **186.**)

2. $\frac{\cancel{4}}{5} \times \frac{1}{\cancel{4}} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{1200}$, Ans.

3. $\frac{3}{5} \times \frac{1}{24} \times \frac{1}{20} \times \frac{1}{12} = \frac{3}{28800} = \frac{1}{9600}$, Ans.

4. $\frac{5}{9} \times \frac{1}{3} \times \frac{1}{8} \times \frac{1}{12} = \frac{5}{2592}$, Ans.

5. $\frac{6}{11} \times \frac{1}{16} \times \frac{1}{25} \times \frac{1}{4} = \frac{6}{17600} = \frac{3}{8800}$, Ans.

6. $\frac{3}{4} \times \frac{1}{25} \times \frac{1}{4} \times \frac{1}{20} = \frac{3}{8000}$, Ans.

7. $\frac{1}{5} \times \frac{1}{2\frac{1}{4}} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{225}$, Ans.

8. $\frac{\cancel{4}}{7} \times \frac{1}{12} \times \frac{1}{16\frac{1}{2}} \times \frac{1}{40} \times \frac{1}{\underset{2}{\cancel{8}}} = \frac{1}{110880}$, Ans.

9. $\frac{1}{2} \times \frac{1}{12} \times \frac{1}{16\frac{1}{2}} \times \frac{1}{40} \times \frac{1}{8} \times \frac{1}{3} = \frac{1}{38016}$, Ans.

10. $\frac{3}{4} \times \frac{1}{144} \times \frac{1}{272\frac{1}{4}} \times \frac{1}{40} \times \frac{1}{4} = \frac{3}{25090560}$, Ans.

11. $\frac{\cancel{7}}{8} \times \frac{1}{4} \times \frac{1}{\underset{9}{\cancel{63}}} \times \frac{1}{4} = \frac{1}{1152}$, Ans.

12. $\frac{3}{5} \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{4} = \frac{3}{320}$, Ans.

13. $\frac{1}{8} \times \frac{1}{60} \times \frac{1}{24} \times \frac{1}{365\frac{1}{4}} = \frac{1}{4207680}$, Ans.

14. $\frac{3}{2} \times \frac{1}{16} \times \frac{1}{25} \times \frac{1}{4} = \frac{3}{3200}$, Ans.

(Art. 255, p. 187.)

(2.)

	s.	d.
	1	0
		7
24)	7	0
Ans.		$3\frac{1}{2}$

(3.)

	s.	d.	qr.
	28	0	0
			7
9)	196	0	0
Ans.	21	9	$1\frac{1}{3}$

(4.)

	cwt.	qr.	lb.	oz.	dr.
	1	0	0	0	0
					7
11)	7	0	0	0	0
Ans.		2	13	10	$2\frac{10}{11}$

(5.)

	lb.	oz.	dr.
	1	0	0
			4
9)	4	0	0
Ans.		7	$1\frac{7}{9}$

(6.)

	lb.	oz.	pwt.	gr.
	1	0	0	0
				8
9)	8	0	0	0
Ans.		10	13	8

(7.)

	℔	℥	ʒ	℈	gr.
	1	0	0	0	0
					4
13)	4	0	0	0	0
Ans.	0	3	5	1	$12\frac{4}{13}$

(8.)

	E.E.	qr.	na.	in.
	1	0	0	0
				5
9)	5	0	0	0
Ans.		2	3	$0\frac{1}{4}$

(9.)

	m.	fur.	rd.	ft.	in.
	1	0	0	0	0
					11
13)	11	0	0	0	0
Ans.		6	30	12	$8\frac{4}{13}$

(10.)

	fur.	rd.	ft.	in.
	1	0	0	0
				8
9)	8	0	0	0
Ans.		35	9	2

(11.)

	A.	R.	p.	yd.	ft.	in.
	1	0	0	0	0	0
						7
13)	7	0	0	0	0	0
Ans.		2	6	4	5	$127\frac{5}{13}$

(12.)

	p.	ft.	in.
	1	0	0
			9
17)	9	0	0
Ans.		144	$19\frac{1}{17}$

(13.)

	cord.	ft.	in.
	1	0	0
			1
13)	1	0	0
Ans.		9	$1462\frac{2}{13}$

(14.)

$$\begin{array}{rrrrrr} & \text{hhd.} & \text{gal.} & \text{qt.} & \text{pt.} & \text{gi.} \\ & 1 & 0 & 0 & 0 & 0 \\ & & & & & 2 \\ \hline 19) & 2 & 0 & 0 & 0 & 0 \\ \hline \text{Ans.} & 6 & 2 & 1 & 0\frac{4}{19} & \end{array}$$

(15.)

$$\begin{array}{rrr} & \text{hhd.} & \text{gal.} \\ & 1 & 0 \\ & & 7 \\ \hline 9) & 7 & 0 \\ \hline \text{Ans.} & 42 & \end{array}$$

(16.)

$$\begin{array}{rrrrrr} & \text{y.} & \text{da.} & \text{h.} & \text{m.} & \text{sec.} \\ & 1 & 0 & 0 & 0 & 0 \\ & & & & & 11 \\ \hline 23) & 11 & 0 & 0 & 0 & 0 \\ \hline \text{Ans.} & 174 & 16 & 26 & 5\frac{5}{23} & \end{array}$$

17. $\$7\frac{3\frac{3}{11}}{4\frac{2}{5}} = \$7\frac{\frac{36}{11}}{\frac{22}{5}};\ \frac{36}{11} \times \frac{5}{22} = \frac{180}{242};\ \$7\frac{180}{242}$, Ans.

(Art. **256**, p. **188.**)

2. $\dfrac{3 \times 4 + 2 = 14}{12 \times 4 = 48} = \frac{7}{24}$, Ans.

3. $$\begin{array}{rl} \left.\begin{array}{r} 21 \times 12 + 9 = 261;\ 261 \times 4 + 1 = 1045; \\ 1045 \times 3 + 1 \end{array}\right\} = & \underline{3136} \\ 28 \times 12 \times 4 \times 3 = & 4032 \\ & [= \frac{7}{9},\ \text{Ans.} \end{array}$$

4. $$\begin{array}{rl} \left.\begin{array}{r} 2 \times 25 + 13 = 63;\ 63 \times 16 + 10 = 1018; \\ 1018 \times 16 + 2 = 16290;\ 16290 \times 11 + 10 \end{array}\right\} = & \underline{179200} \\ 4 \times 25 \times 16 \times 16 \times 11 = & 281600 \\ & [= \frac{7}{11},\ \text{Ans.} \end{array}$$

5. $$\begin{array}{rl} 7 \times 16 + 1 = 113;\ 113 \times 9 + 7 = & 1024 \\ \hline 16 \times 16 \times 9 = & 2304 \end{array} = \frac{4}{9},\ \text{Ans.}$$

6. $$\begin{array}{rl} 10 \times 20 + 13 = 213;\ 213 \times 24 + 8 = & 5120 \\ \hline 12 \times 20 \times 24 = & 5760 \end{array} = \frac{8}{9},\ \text{Ans.}$$

7. $\dfrac{268\frac{4}{5}}{231} = \dfrac{1344}{1155}$, Ans.

8. $$\begin{array}{rl} 2 \times 4 = 8 \times 2\frac{1}{4} + 1 = 19 \times 13 + 5 = & 252 \\ \hline 4 \times 4 \times 2\frac{1}{4} \times 13 = & 468 \end{array} = \frac{7}{13},\ \text{Ans.}$$

9. $$\begin{array}{rl} 2 \times 4 + 3 = 11 \times 2\frac{1}{4} = 24\frac{3}{4} \times 4 + 1 = & 100 \\ \hline 5 \times 4 \times 2\frac{1}{4} \times 4 = & 180 \end{array} = \frac{5}{9},\ \text{Ans.}$$

10. $$\begin{array}{rl} \left.\begin{array}{r} 6 \times 40 + 30 = 270 \times 16\frac{1}{2} + 12 = 4467 \times 12 + 8 \\ = 53612 \times 3 = 160836 \times 13 + 12 \end{array}\right\} = & \underline{2090880} \\ 8 \times 40 \times 16\frac{1}{2} \times 12 \times 3 \times 13 = & 2471040 \\ & [= \frac{11}{13},\ \text{Ans.} \end{array}$$

11. $$\begin{array}{r} 35\times16\frac{1}{2}+9=586\frac{1}{2}\times12+2=\underline{7040} \\ 40\times16\frac{1}{2}\times12=7920 \end{array}=\frac{8}{9}, \text{ Ans.}$$

12. $$\left.\begin{array}{r} 2\times40+6=86\times30\frac{1}{4}+4=2605\frac{1}{2}\times9+5 \\ =23454\frac{1}{2}\times144+127=3377575\times13+5 \end{array}\right\}=\underline{43908480}$$
$$4\times40\times30\frac{1}{4}\times9\times144\times13=81544320$$
$$[=\frac{7}{13}, \text{ Ans.}$$

13. $$\begin{array}{r} 144\times144+19=20755\times17+1=\underline{352836} \\ 272\frac{1}{4}\times144\times17=666468 \end{array}=\frac{9}{17}, \text{ Ans.}$$

14. $$\begin{array}{r} 9\times1728+1462=17014\times13+2=\underline{221184} \\ 128\times1728\times13=2875392 \end{array}=\frac{1}{13}, \text{ Ans.}$$

15. $$\begin{array}{r} 6\times4+2=26\times2+1=53\times4=212\times19+4=\underline{4032} \\ 63\times4\times2\times4\times19=38304 \end{array}$$
$$[=\frac{2}{19}, \text{ Ans.}$$

16. $$\begin{array}{ll} \text{1lb. avoirdupois} & =\underline{5760}\text{ grains} \\ \text{1lb. troy} & =7000\text{ grains} \end{array}=\frac{144}{175}, \text{ Ans.}$$

17. $$\begin{array}{r} 174\times24+16=4192\times60+26=251546\times60+5=15092765\times23+5=\underline{347133600} \\ 365\frac{1}{4}\times24\times60\times60\times23=725824800 \end{array}$$
$$[=\frac{11}{23}, \text{ Ans.}$$

ADDITION OF DENOMINATE FRACTIONS.

(ART. **257**, p. **190**.)

(2.)

	cwt.	qr.	lb.
$\frac{7}{11}$ of a ton =	12	2	$22\frac{8}{11}$
$\frac{17}{22}$ of a cwt. =		3	$2\frac{3}{11}$
Ans.	13	2	0

(3.)

	qr.	na.	in.
$\frac{3}{5}$ of a yd. =	2	1	$1\frac{7}{20}$
$\frac{1}{7}$ of an E. E. =		2	$1\frac{13}{14}$
$\frac{6}{7}$ of a qr. =		3	$0\frac{27}{28}$
Ans.	3	3	$1\frac{139}{140}$

(4.)

	fur.	rd.	ft.	in.
$\frac{7}{11}$ of a mile =	5	3	10	6
$\frac{4}{13}$ of a furlong =		12	5	$0\frac{12}{13}$
$\frac{9}{22}$ of a yard =			1	$2\frac{8}{11}$
	5	15	16	$9\frac{93}{143}$ =
Ans.	5	16	0	$3\frac{93}{143}$.

(5.)

	A.	R.	p.	ft.	in.
$\frac{5}{7}$ of an A. =	0	2	34	77	$113\frac{1}{7}$
$\frac{2}{3}$ of an A. =	0	2	26	181	72
$\frac{13}{14}$ of an A. =	0	3	28	155	$82\frac{2}{7}$
	2	1	9	$141\frac{3}{4}$	$123\frac{3}{7}$ =
Ans.	2	·1	9	142	$87\frac{3}{7}$

(6.)

	m.	fur.	rd.	ft.	in.
$18\frac{3}{7}$ miles =	18	3	17	2	$4\frac{2}{7}$
$23\frac{11}{14}$ miles =	23	6	11	7	$0\frac{6}{7}$
$19\frac{1}{21}$ miles =	19	0	15	3	$11\frac{1}{7}$
Ans.	61	2	3	13	$4\frac{2}{7}$

(7.)

	gal.	qt.	pt.	gi.
$\frac{11}{12}$ of a gal. =	0	3	1	$1\frac{1}{3}$
$\frac{1}{12}$ of a hhd. =	5	1	0	0
Ans.	6	0	1	$1\frac{1}{3}$

(8.)

	d.	h.	m.
$\frac{5}{16}$ of a week =	2	4	30
$\frac{1}{5}$ of a day =		4	48
Ans.	2	9	18

(9.)

	ft.	in.
$\frac{3}{4}$ of a square foot =	0	108
$\frac{1}{2}$ of a foot square =		36
Ans.	1	0

(10.)

rd.	ft.	in.
11	16	5
		6
11	16	11 =
12	0	5

SUBTRACTION OF DENOMINATE FRACTIONS.

(Art. **258**, p. **190**.)

(2.)

	qr.	na.	in.
$\frac{7}{8}$ E. E. =	4	1	$1\frac{1}{8}$
$\frac{2}{7}$ of a yd. =	1	0	$1\frac{2}{7}$
Ans.	3	0	$2\frac{5}{56}$

(3.)

	fur.	rd.	ft.	in.
$\frac{2}{9}$ of a mile =	1	31	1	10
$\frac{7}{11}$ of a fur. =		25	7	6
	1	5	$10\frac{1}{2}$	4 =
Ans.	1	5	10	10

(4.)

	m.	fur.	rd.	ft.	in.
$\frac{5}{7}$ deg. =	49	3	9	8	$7\frac{5}{7}$
$\frac{3}{5}$ mile =	0	4	32	0	0
Ans.	48	6	17	8	$7\frac{5}{7}$

(5.)

	R.	p.	yd.	ft.	in.
$\frac{4}{11}$ A. =	1	18	5	4	72
$\frac{4}{9}$ rod =	0	0	13	4	0
	1	17	$22\frac{1}{4}$	0	72
			$\frac{1}{4}$=	2	36
Ans.	1	17	22	2	108

(6.)

	ft.	in.
$\frac{9}{10}$ cord =	115	$345\frac{3}{5}$
$\frac{2}{11}$ cord =	23	$471\frac{3}{11}$
Ans.	91	$1602\frac{18}{55}$

(7.)

	gal.	qt.	pt.	gi.
$\frac{7}{13}$hhd. =	33	3	1	$1\frac{7}{13}$
$\frac{4}{5}$ of $\frac{7}{13}$hhd. = $\frac{28}{65}$ =	27	0	1	$0\frac{28}{65}$
Ans.	6	3	0	$1\frac{7}{65}$

(8.)

	m.	fur.	rd.	ft.	in.
72m. =	72	0	0	0	0
$\frac{4}{7}$ of 72m. =	41	1	5	11	$9\frac{3}{7}$
	30	6	34	$4\frac{1}{2}$	$2\frac{4}{7}$
				$\frac{1}{2}$=	6
Ans.	30	6	34	4	$8\frac{4}{7}$

(9.)

	d.	h.	m.	sec.
$\frac{2}{7}$ year =	104	8	34	$17\frac{1}{7}$
$\frac{4}{9}$ week =	3	2	40	0
Ans.	101	5	54	$17\frac{1}{7}$

(10.)

	R.	p.	yd.	ft.	in.
$\frac{4}{11}$A. =	1	18	5	4	72
$\frac{1}{2}$ft. =					72
Ans.	1	18	5	4	0

MISCELLANEOUS EXAMPLES IN FRACTIONS.

(PAGE 190.)

1. $17\frac{3}{11} = \frac{190}{11}$; $4\frac{7}{9} = \frac{43}{9}$; $\frac{190}{11} \times \frac{43}{9} = \frac{8170}{99} = 82\frac{52}{99}$ miles = 82m. 4fur. 8rd. 1ft. 4in., Ans.

2. $29\frac{7}{13} = \frac{384}{13}$; $\frac{384}{13} \times \frac{384}{13} = \frac{147456}{169} = 872\frac{88}{169}$ poles = 5A. 1R. $32\frac{88}{169}$p.; $\frac{88}{169}$p. = 141ft. $109\frac{155}{169}$in., Ans.

3. $17\frac{3}{4} = \frac{71}{4}$; $7\frac{1}{11} = \frac{78}{11}$; $4\frac{3}{5} = \frac{23}{5}$; $\frac{71}{4} \times \frac{78}{11} \times \frac{23}{5} = \frac{127374}{220}$ = $578\frac{107}{110}$ feet; $578\frac{107}{110} \div 128$ = 4 cords $66\frac{107}{110}$ cubic feet, Ans.

4. $19\frac{7}{8} = \frac{159}{8}$; $6\frac{3}{4} = \frac{27}{4}$; $\frac{159}{8} \times \frac{27}{4} = \frac{4293}{32}$ = \$134.15$\frac{5}{8}$, *Ans.*

5. $376\frac{11}{18} = \frac{6779}{18}$; $75\frac{3}{8} = \frac{603}{8}$; $\frac{6779}{18} \times \frac{603}{8} = \frac{4087737}{144} =$ \$28387.06$\frac{1}{4}$, Ans.

6. $17\frac{19}{112} = \frac{1923}{112}$; $4.75 = \frac{475}{1}$; $\frac{1923}{112} \times \frac{475}{1} = \frac{913425}{112} =$ \$81.55$\frac{65}{112}$, Ans.

7. $1670\frac{7}{13} = \frac{21717}{13}$; $12\frac{3}{4} = \frac{51}{4}$; $\frac{21717}{13} \times \frac{51}{4} = \frac{1107567}{52} =$ \$212.99$\frac{19}{52}$, Ans.

8. $28\frac{4}{11} = \frac{312}{11}$; $11\frac{3}{4} = \frac{47}{4}$; $\frac{312}{11} \times \frac{47}{4} = \frac{14664}{44} =$ \$333.27$\frac{3}{11}$, Ans.

9. $37\frac{11}{63} = \frac{2342}{63}$; $17.62\frac{1}{2} = \frac{3525}{2}$; $\frac{2342}{63} \times \frac{3525}{2} = \frac{8255550}{126}$ = \$655.20$\frac{5}{21}$, Ans.

10. $\frac{7}{8} \times \frac{575}{1} = \frac{4025}{8} =$ \$5.03$\frac{1}{8}$, Ans.

11. $139\frac{4}{7} = \frac{977}{7}$; $38\frac{4}{5} = \frac{194}{5}$; $\frac{977}{7} \times \frac{194}{5} = \frac{189538}{35} =$ $5415\frac{13}{35}$ poles = 33A. 3R. 15$\frac{13}{35}$p., Ans.

12. $11\frac{3}{5} = \frac{58}{5}$; $\frac{58}{5} \times \frac{15}{1} = \frac{870}{5}$; $\frac{870}{5} \times \frac{1}{9} = \frac{870}{45} = 19\frac{1}{3}$ feet, Ans.

13. $18\frac{3}{8} = \frac{147}{8}$; $48.15\frac{3}{4} = \frac{19263}{4}$; $\frac{19263}{4} \times \frac{8}{147} = \frac{154104}{588}$ = \$2.62$\frac{4}{49}$, Ans.

14. $98\frac{7}{8} = \frac{791}{8}$; $\frac{791}{8} \times \frac{1}{63} = \frac{791}{504} =$ \$1.56$\frac{119}{126}$, Ans.

15. $8\frac{2}{9} = \frac{74}{9}$; $\frac{74}{9} \times \frac{1}{5} = \frac{74}{45} =$ \$1.64$\frac{4}{9}$, Ans.

16. $11 \times 63 = 693$; $693 + 17\frac{1}{2} = 710\frac{1}{2} = \frac{1421}{2}$; $19\frac{3}{4} = \frac{79}{4}$; $\frac{1421}{2} \times \frac{79}{4} = \frac{112259}{8} =$ \$140.32$\frac{3}{8}$, Ans.

17. $63 \times 4 \times 2 = 504$pts.; $1\frac{3}{4} = \frac{7}{4}$; $\frac{504}{1} \times \frac{4}{7} = \frac{2016}{7} = 288$ bottles, Ans.

18. $18\frac{7}{12} = \frac{223}{12}$; $10\frac{5}{12} = \frac{125}{12}$; $7\frac{11}{12} = \frac{95}{12}$; $\frac{223}{12} \times \frac{125}{12} \times \frac{95}{12} = \frac{2648125}{1728} = 1532\frac{829}{1728}$; $1532\frac{829}{1728} \div 128 = 11$ cords, $124\frac{829}{1728}$ cubic feet, Ans.

19. $6\frac{7}{8} = \frac{55}{8}$; $65\frac{3}{4} = \frac{263}{4}$; $\frac{55}{8} \times \frac{263}{4} = \frac{14465}{32} =$ \$4.52$\frac{1}{32}$, Ans.

20. $8\frac{3}{4} = \frac{35}{4}$; $5\frac{7}{12} = \frac{67}{12}$; $3 = \frac{3}{1}$; $\frac{35}{4} \times \frac{67}{12} \times \frac{3}{1} = \frac{7035}{48} =$ $146\frac{9}{16}$ feet, Ans.

21. $46\frac{5}{12} = \frac{557}{12}$; $17\frac{1}{2} = \frac{35}{2}$; $\frac{557}{12} \times \frac{35}{2} = \frac{19495}{24} = 812\frac{7}{24}$ feet, Ans.

22. $2\frac{5}{12} + 3\frac{1}{12} = 5\frac{6}{12}$; $5\frac{6}{12} \times \frac{2}{1} = 11$; $11 \times 5\frac{1}{2} = 60\frac{1}{2}$; $2\frac{5}{12} = \frac{29}{12}$; $3\frac{1}{12} = \frac{37}{12}$; $\frac{29}{12} \times \frac{37}{12} \times \frac{2}{1} = \frac{2146}{144} = 14\frac{65}{72}$; $60\frac{1}{2} + 14\frac{65}{72} = 75\frac{29}{72}$; $75\frac{29}{72} \times \frac{7}{1} = 527\frac{59}{72}$ feet, Ans.
$5\frac{6}{12} = \frac{66}{12}$; $2\frac{5}{12} = \frac{29}{12}$; $3\frac{1}{12} = \frac{37}{12}$; $\frac{66}{12} \times \frac{29}{12} \times \frac{37}{12} \times \frac{7}{1} = \frac{495726}{1728} = 286\frac{253}{288}$ cubic feet, second Ans.

23. $12 + 11\frac{1}{2} = 23\frac{1}{2}$; $23\frac{1}{2} \times 2 = 47$; $47 \times 7\frac{1}{2} = 352\frac{1}{2}$; $12 \times 11\frac{1}{2} = 138$; $352\frac{1}{2} + 138 = 490\frac{1}{2}$; $490\frac{1}{2} \times .02\frac{3}{4} = \$13.48\frac{7}{8}$, Ans.

24. $14\frac{1}{2} = \frac{29}{2}$; $10\frac{1}{4} = \frac{41}{4}$; $16\frac{1}{2} = \frac{33}{2}$; $\frac{29}{2} \times \frac{33}{2} = \frac{957}{4} = 239\frac{1}{4}$; $\frac{41}{4} \times \frac{33}{2} = \frac{1353}{8} = 169\frac{1}{8}$; $239\frac{1}{4} + 169\frac{1}{8} = 408\frac{3}{8}$; $408\frac{3}{8} \times 2 = 816\frac{3}{4}$; $816\frac{3}{4} + 12 = 828\frac{3}{4}$; $828\frac{3}{4} \times 3 \times 4\frac{1}{2} \times 2 = \$223.76\frac{1}{4}$, Ans.

25. $14\frac{7}{12} = \frac{175}{12}$; $5\frac{11}{12} = \frac{71}{12}$; $4\frac{1}{4} = \frac{17}{4}$; $2150\frac{2}{5} = \frac{10752}{5}$; $\frac{175}{12} \times \frac{71}{12} \times \frac{17}{4} \times \frac{1728}{1} \times \frac{5}{10752} = \frac{1824984000}{6193152} = 294\frac{2429}{3584}$ bushels, Ans.

26. $10 \times 8 \times 6 = 480$; $8 \times 8 \times 8 = 512$; $512 - 480 = 32$ feet, Ans.

27. $7\frac{1}{2} \times 6 \times 5\frac{1}{2} = \frac{15}{2} \times \frac{6}{1} \times \frac{11}{2} = \frac{990}{4} = 247\frac{1}{2}$ cubic feet. $9\frac{1}{2} \times 4\frac{1}{2} \times 5\frac{1}{2} = \frac{19}{2} \times \frac{9}{2} \times \frac{11}{2} = \frac{1881}{8} = 235\frac{1}{8}$; $247\frac{1}{2} - 235\frac{1}{8} = 12\frac{3}{8}$; $1728 \times 12\frac{3}{8} = 21384$; $21384 \div 231 = 92\frac{4}{7}$ gallons, Ans.

The first cistern will contain $92\frac{4}{7}$ most gallons.

28. 31rd. $13\frac{3}{10}$ft., 41rd. $1\frac{9}{10}$ft., 38rd. $0\frac{1}{5}$ft., 45rd. $12\frac{7}{10}$ft. =
$524\frac{4}{5}$ft., $678\frac{2}{5}$ft., $627\frac{1}{5}$ft., $755\frac{1}{5}$ ft. =
$\frac{2624}{5}$ $\frac{3392}{5}$ $\frac{3136}{5}$ $\frac{3776}{5}$

The greatest common divisor of 2624, 3392, 3136, 3776, is $= 64$
The greatest common multiple of 5, 5, 5, 5, is $= \overline{5}$
Therefore $\frac{64}{5}$ will divide each of the fractions without a remainder; thus $\frac{2624}{5} \div \frac{64}{5} = 41$; $\frac{3392}{5} \div \frac{64}{5} = 53$; $\frac{3136}{5} \div \frac{64}{5} = 49$; $\frac{3776}{5} \div \frac{64}{5} = 59$. The number of rails will therefore be $41 + 53 + 49 + 59 = 202$; $202 \times 4 = 808$ rails.

$64 \div 5 = 12\frac{4}{5}$; $12\frac{4}{5} + \frac{7}{10} = 13\frac{1}{2}$ feet, length of the rails, Ans.

29. $\frac{5}{8}, \frac{3}{4}, \frac{6}{7} = \frac{35}{56}, \frac{42}{56}, \frac{48}{56}$.

$$\left.\begin{array}{l}\text{Least common multiple of the numerators} = 1680 \\ \text{Greatest common divisor of the denominators} = 56\end{array}\right\} \frac{1680}{56} = 30\text{yds., Ans.}$$

We therefore find that 30 is the least whole number that can be divided by $\frac{5}{8}$, $\frac{3}{4}$, or $\frac{6}{7}$, without a remainder.

30. $100 \div 30 = 3\frac{1}{3}$ times.

31. 31A. 3R. 6p. = 5086p.; 39A. 2R. $37\frac{1}{2}$p. = $6357\frac{1}{2}$p.

$$\frac{5086 \times 2 = 10172}{6357\frac{1}{2} \times 2 = 12715} = \frac{4}{5}, \text{ Ans.}$$

32. $68 \times \$7\frac{11}{12} = \$538.33\frac{1}{3}$, Ans.

33. $8\frac{2}{3} \times \$42\frac{3}{5} = \$369.20$, Ans.

34. $\frac{3-3}{3-3} = 0$, Ans.

35. $20 \times 15 = 300$ft., contents of the upper part of the room.

$20 + 15 = 35$; $35 \times 2 = 70$ft., length round the room.

$70 \times 8\frac{1}{2} = 595$ feet, contents of the upright ceiling of the room.

$2 \times 7 \times 3 = 42$ft., contents of the doors.

$4 \times 5\frac{1}{2} \times 3\frac{1}{3} = 73\frac{1}{3}$ft., contents of the windows.

$70 - 6 = 64$; $64 \times \frac{2}{3} = 42\frac{2}{3}$ft., contents of the mop-boards.

NOTE. — We deduct 6 feet for the two doors.

$595 + 300 = 895$; $42 + 73\frac{1}{3} + 42\frac{2}{3} = 158$.

$895 - 158 = 737$ft.; $737 \div 9 = 81\frac{8}{9}$ square yards.

$81\frac{8}{9} \times 6\frac{1}{4} = \frac{737}{9} \times \frac{25}{4} = \frac{18425}{36} = \$5.11\frac{29}{36}$, for plastering.

$595 - 158 = 437$ft.; $437 \div 9 = 48\frac{5}{9}$ square yards.

$48\frac{5}{9} \times .09 = \$4.37$, for papering.

$48\frac{5}{9} \times 3 = 145\frac{2}{3} = \frac{437}{3}$; $2\frac{3}{5} = \frac{13}{5}$; $\frac{437}{3} \div \frac{13}{5} = \frac{437}{3} \times$ [$\frac{5}{13} = \frac{2185}{39} = \$2.80\frac{5}{39}$, for paper, Ans.

QUESTIONS TO BE PERFORMED BY ANALYSIS.

(Page **194.**)

8. $30\frac{4}{15} = \frac{454}{15}$; $\frac{1728}{1} \times \frac{15}{454} = \frac{25920}{454}$; $7\frac{17}{30} = \frac{227}{30}$; $\frac{25920}{454} \times \frac{227}{30} = \frac{5883840}{13620} = \$432$, Ans.

4. $7\frac{17}{30} = \frac{227}{30}$; $\frac{432}{1} \times \frac{30}{227} = \frac{12960}{227}$; $\frac{1728}{1} \times \frac{227}{12960} = \frac{392256}{12960} = 30\frac{4}{15}$ tons, Ans.

5. $7\frac{17}{30} = \frac{227}{30}$; $\frac{432}{1} \times \frac{30}{227} = \frac{12960}{227}$; $30\frac{4}{15} = \frac{454}{15}$; $\frac{454}{15} \times \frac{12960}{227} = \frac{5883840}{3405} = \$1728$, Ans.

6. $30\frac{4}{15} = \frac{454}{15}$; $\frac{1728}{1} \times \frac{15}{454} = \frac{25920}{454}$; $\frac{432}{1} \times \frac{454}{25920} = 7\frac{17}{30}$ tons, Ans.

7. $7\frac{7}{10} = \frac{77}{10}$; $6\frac{4}{9} = \frac{58}{9}$; $\frac{58}{9} \times \frac{77}{10} = \frac{4466}{90} = 49\frac{28}{45}$ bushels, Ans.

8. $\frac{7}{11}$ of $\frac{17}{1} = \frac{119}{11} = 10\frac{9}{11}$; $10\frac{9}{11} + 15 = 25\frac{9}{11} = \frac{284}{11}$; $\frac{1728}{1} \times \frac{11}{284} = \frac{19008}{284} = \$66\frac{66}{71}$, each girl's share; $\frac{7}{11}$ of $66\frac{66}{71} = \frac{7}{11} \times \frac{19008}{284} = \frac{133056}{3124} = \$42\frac{42}{71}$, each boy's share.

9. $\frac{1449}{1} \times \frac{9}{7} = \frac{13041}{7} = 18.63$; $4\frac{3}{7} = \frac{31}{7}$; $\frac{1863}{1} \times \frac{31}{7} = \frac{57753}{7} = \$82.50\frac{3}{7}$, Ans.

10. $82.50\frac{3}{7} = \frac{57753}{7}$; $4\frac{3}{7} = \frac{31}{7}$; $\frac{57753}{7} \times \frac{7}{31} = \frac{404271}{217} = 18.63$; $\frac{1449}{1863} = \frac{7}{9}$, Ans.

11. $\frac{1449}{1} \times \frac{9}{7} = \frac{13041}{7} = 18.63$; $82.50\frac{3}{7} \div 18.63 = 4\frac{3}{7}$, Ans.

12. $82.50\frac{3}{7} = \frac{57753}{7}$; $4\frac{3}{7} = \frac{31}{7}$; $\frac{57753}{7} \times \frac{7}{31} = \frac{404271}{217} = 18.63$; $\frac{7}{9}$ of $\frac{1863}{1} = \frac{13041}{9} = \$14.49$, Ans.

13. $14\frac{7}{8} = \frac{119}{8}$; $\frac{500}{1} \times \frac{8}{119} = \frac{4000}{119}$; $9\frac{11}{12} = \frac{119}{12}$; $\frac{4000}{119} \times \frac{119}{12} = \frac{476000}{1428} = \$333.33\frac{1}{3}$, Ans.

14. $9\frac{11}{12} = \frac{119}{12}$; $333.33\frac{1}{3} = \frac{100000}{3}$; $\frac{100000}{3} \times \frac{12}{119} = \frac{1200000}{357}$; $14\frac{7}{8} = \frac{119}{8}$; $\frac{1200000}{357} \times \frac{119}{8} = \frac{142800000}{2856} = \$500$, [Ans.

15. $333.33\frac{1}{3} = \frac{100000}{3}$; $9\frac{11}{12} = \frac{119}{12}$; $\frac{100000}{3} \times \frac{12}{119} = \frac{1200000}{357}$;

$$[\frac{5\cancel{0}\cancel{0}\cancel{0}\cancel{0}}{1} \times \frac{357}{120\cancel{0}\cancel{0}\cancel{0}\cancel{0}} = \frac{1785}{120} = 14\frac{7}{8} \text{ tons, Ans.}$$

16. $14\frac{7}{8} = \frac{119}{8}$; $\frac{50000}{1} \times \frac{8}{119} = \frac{400000}{119}$; $333.33\frac{1}{3} = \frac{100000}{3}$; $\frac{100000}{3} \times \frac{119}{400000} = \frac{11900000}{1200000} = 9\frac{11}{12}$ tons, Ans.

17. $97\frac{7}{8} = \frac{783}{8}$; $\frac{3132}{1} \times \frac{8}{783} = \frac{25056}{783} = .32$; $763\frac{5}{8} \times .32 = \$244.36$, Ans.

18. $763\frac{5}{8} = \frac{6109}{8}$; $\frac{24436}{1} \times \frac{8}{6109} = \frac{195488}{6109} = .32$; $97\frac{7}{8} \times .32 = \$31.32$, Ans.

19. $763\frac{5}{8} = \frac{6109}{8}$; $\frac{24436}{1} \times \frac{8}{6109} = \frac{195488}{6109} = .32$; $31.32 \div .32 = 97\frac{7}{8}$gal., Ans.

20. $1975 \div 40 = 49\frac{3}{8}$; $49\frac{3}{8} \times 144 = 7110$lb., Ans.

21. $15.75 \div 17 = 92\frac{11}{17}$; $92\frac{11}{17} = \frac{1575}{17}$; $9\frac{1}{4} = \frac{37}{4}$; $\frac{1575}{17} \times \frac{37}{4} = \frac{58275}{68} = \$8.56\frac{67}{68}$, Ans.

22. $50\frac{4}{7} = \frac{354}{7}$; $\frac{1}{9} \times \frac{354}{7} = \frac{354}{63}$; $87\frac{3}{5} = \frac{438}{5}$; $\frac{354}{63} \times \frac{438}{5} = \frac{155052}{315} = \$492\frac{8}{35}$, Ans.

23. $78 \times 13 = 1014$; $13 + 7 = 20$; $1014 \div 20 = 50\frac{7}{10}$ days, Ans.

24. $10 \times 9 = 90$; $90 \div 15 = 6$ days, Ans.

25. $15 \times 6 = 90$; $90 \div 10 = 9$ days, Ans.

26. $10 \times 9 = 90$; $90 \div 6 = 15$ hours, Ans.

27. $17\frac{3}{11} = \frac{190}{11}$; $5\frac{3}{7} = \frac{38}{7}$; $\frac{38}{7} \times \frac{11}{190} = \frac{418}{1330}$; $97\frac{5}{9} = \frac{878}{9}$; $\frac{418}{1330} \times \frac{878}{9} = \frac{367004}{11970} = \$30\frac{208}{315}$, Ans.

28. $9\frac{3}{5} = \frac{48}{5}$; $19\frac{7}{9} = \frac{178}{9}$; $\frac{178}{9} \times \frac{48}{5} = \frac{8544}{45} = 189\frac{13}{15}$; $189\frac{13}{15} + 7 = 196\frac{13}{15}$; $9\frac{3}{5}$ tons $= 192$cwt.; $196\frac{13}{15} \div 192 = \$1\frac{73}{2880}$, Ans.

29. $9\frac{3}{5}$ tons $= 192$ cwt.; $192 \times 1\frac{3}{4} = 336$; $336 - 7 = \$329$, Ans.

30. $47\frac{3}{11} = \frac{520}{11}$; $2\frac{2}{3} = \frac{8}{3}$; $\frac{520}{11} \times \frac{8}{3} = \frac{4160}{33} = 126\frac{2}{33}$; $0.75 = \frac{3}{4}$; $126\frac{2}{33} \div \frac{3}{4} = 168\frac{8}{99}$ bushels, Ans.

31. $57\frac{9}{11} = \frac{636}{11}$; $\frac{636}{11} \times \frac{1}{15} = \frac{636}{165}$; $19\frac{7}{8} = \frac{159}{8}$; $\frac{636}{165} \times \frac{159}{8} = \frac{101124}{1320} = \$76\frac{67}{110}$, Ans.

32. $19\frac{7}{8} = \frac{159}{8}$; $76\frac{67}{110} = \frac{101124}{1320}$; $\frac{101124}{1320} \times \frac{8}{159} = \frac{808992}{209880} = \frac{636}{165}$; $57\frac{9}{11} = \frac{636}{11}$; $\frac{636}{11} \times \frac{165}{636} = 15$ cords, Ans.

33. $7\frac{3}{10} = \frac{73}{10}$; $47\frac{1}{3} = \frac{142}{3}$; $\frac{142}{3} \times \frac{73}{10} = \frac{10366}{30} = 345\frac{8}{15}$s. $= 17$£. 5s. $6\frac{2}{5}$d., Ans.

34. 172£. 15s. $0\frac{2}{5}$d. $= \frac{207302}{5}$d.; $47\frac{1}{3} = \frac{142}{3}$; $\frac{207302}{5} \times \frac{3}{142} = \frac{621906}{710} = 875\frac{328}{355}$d. $=$ 3£. 12s. $11\frac{328}{355}$d., Ans.

35. $43\frac{6}{7} = \frac{307}{7}$; $\frac{307}{7} \times \frac{7}{13} = \frac{2149}{91} = \$23.61\frac{7}{13}$, Ans.

36. $17\frac{3}{8} = \frac{139}{8}$; $2\frac{7}{11} = \frac{29}{11}$; $\frac{29}{11} \times \frac{8}{139} = \frac{232}{1529}$; $\frac{232}{1529} \times \frac{50}{1} = \frac{11600}{1529} = \$7.58\frac{1018}{1529}$, Ans.

37. $87\frac{3}{7} = \frac{612}{7}$; $\frac{612}{1} \times \frac{7}{612} = 7$; $14\frac{7}{10} \times 7 = \$102.90$, Ans.

38. $\frac{4375}{1} \times \frac{8}{7} = \frac{35000}{7} = 50$; $50 \times 10 = \$500$, Ans.

39. $500 \div 10 = 50$; $\frac{4375}{5000} = \frac{7}{8}$ acre, Ans.

40. $71.87 \div 9 = 7.98\frac{5}{9} = \frac{7187}{9}$; $\frac{4}{7} \times \frac{7187}{9} = \frac{28748}{63} = \$4.56\frac{20}{63}$, Ans.

41. $4.56\frac{20}{63} \times \frac{7}{4} = 7.98\frac{5}{9}$; $7.98\frac{5}{9} \times 9 = \$71.87$, Ans.

42. $111 \times 19 = 2109$; $2109 \div 47 = 44\frac{41}{47}$ days, Ans.

43. $44\frac{41}{47} \times 47 = 2109$; $2109 \div 19 = 111$ days, Ans.

44. $\frac{920}{1} \times \frac{11}{4} = \frac{10120}{4} = 25.30$; $25.30 \times 17 = \$430.10$, Ans.

45. $430.10 \div 17 = 25.30$; $\frac{4}{11} \times \frac{2530}{1} = \frac{10120}{11} = \$9.20$, Ans.

46. $\frac{715}{1} \times \frac{16}{7} = \frac{11440}{7} = 16.34\frac{2}{7}$; $16.34\frac{2}{7} \times 7 = \$114.40$, Ans.

47. $\frac{7}{16} \times \frac{11440}{7} = \frac{11440}{16} = \$7.15$, Ans.

48. $19\frac{3}{7} = \frac{136}{7}$; $87\frac{3}{7} = \frac{612}{7}$; $\frac{136}{\cancel{7}} \times \frac{\cancel{7}}{612} = \frac{136}{612} = \frac{34}{153}$;

$$[\frac{34}{153} \times \frac{60}{1} \times \frac{7}{1} = \frac{14280}{153} = 93\tfrac{1}{3}\text{bu.}, \text{ Ans.}$$

49. Smith will reap $\frac{1}{80}$ of the field in an hour; his wife will reap $\frac{1}{108}$ of the field in an hour. They will both reap $\frac{1}{80} + \frac{1}{108} = \frac{47}{2160}$ of the field, in an hour. Then they will reap the whole field in $\frac{2160}{47}$ hours $= 45\frac{45}{47}$; $45\frac{45}{47} \div 8 = 5\frac{35}{47}$ days, Ans.

DECIMAL FRACTIONS.

(ART. **266,** p. **199.**)

1. Fifty-six thousandths.
2. One thousand three ten thousandths.
3. Two thousand seven hundred eighty-six ten thousandths.
4. Sixteen thousand three hundred two hundred thousandths.
5. Nine hundred seventy-five thousandths.
6. One,and six hundred thirty one thousandths.
7. Forty-eight,and seven hundredths.

8. One, and three hundred fifteen thousandths.
9. Five, and six thousand one ten thousandths.
10. Eighty-seven, and six ten thousandths.
11. One, and seven millionths.
12. Five, and one hundred one thousand sixteen millionths.
13. One, and three hundred twenty-seven millionths.
14. One millionth.
15. Sixteen, and seven billionths.

16. .13
17. .006
18. .0019
19. .00406
20 .000001
21. .001031
22. 7.0017
23. 333.003
24. 1.000001
25. 325.7
26. 465.14
27. 93.07
28. 24.000009
29. 221.00009
30. 49000.049

31. 79002000.105
32. 69015.00015
33. 80000.0083
34. 9000019019.19
35. 27.927
36. 49000000000000.000000000001
37. 21.0001
38. 87000.000087
39. 99099.000009009
40. 17.0117
41. 33.33
42. 47000.0000029
43. 15.04007
44. 11000.11
45. 17.000000000000081
46. 9.000000000057
47. 69000.349

ADDITION OF DECIMALS.

(Art. **268**, p. **201**.)

2. 7564.0052656
3. 2071.449495
4. 7234.0968
5. 6913.5477

(6.)	(7.)	(8.)
$15.06	$137.50	23000010.
107.09	55.63	1000.00005
1.625	1.375	27.000019
93.765	.875	7.5
Ans. $217.54	Ans. $195.38	Ans. 23001044.500069

(9.)	(10.)	(11.)
59.059	25.000007	3.75
25000.0025	145.643	11.7
5.000005	175.89	16.125
205.05	17.00348	Ans. 31.575
Ans. 25269.111505	Ans. 363.536487	

(12.)	(13.)
73.29	209000.000046
87.047	98207.0015
3005.0106	15.08
28.03	.0049
29000.005	Ans. 307222.086446
Ans. 32193.3826	

SUBTRACTION OF DECIMALS.

(ART. **270,** p. **202.**)

6.	9.49989	7.	88.9429	8.	.001

(9.)	(10.)	(11.)
97.7	315.0027	29004005.
27.028	115.07	29000.
Ans. 70.672	Ans. 199.9327	349200.00024
		378200.00024
		Ans. 28625804.99976

(12.)	(13.)	(14.)
1000000.	$19.	$400.
.000001	1.375	316.875
Ans. 999999.999999	Ans. $17.625	Ans. $83.125

(15.)
19000000.
.000000019
Ans. 18999999.999999981

(16.) 1000.
93.45
124.
244.285
216.136
677.871
Ans. 322.129

MULTIPLICATION OF DECIMALS.

(Art. **271**, p. **203.**)

3.	.438496	10.	33.5175
4.	.0949416	11.	41448651.06
5.	.003721061	12.	.000019737
6.	26137.65	13.	$153.525
7.	152.2756	14.	$18.4375
8.	43910.073	15.	$46.95
9.	.00000081	16.	$149.5125
		17.	3616.175

CONTRACTIONS IN MULTIPLICATION OF DECIMALS.

1.	(Art. **272**, p. **204.**) 131634.	3.	1.
2.	347890.	4.	$7000.

(Art. **273**, p. **206.**)

(2.) 325.701428
3938127
227990
6514
326
261
10
3
Ans. 235.104

(3.)
56.7534916
8296735
28376746
1702605
397274
34052
5108
114
45
Ans. 305.15944

(4.) 843.7527
5714368
6750022
506252
25313
3375
84
59
4
Ans. 7285109.

DIVISION OF DECIMALS.

(Art. **274**, p. **207**.)

3. 321.2
4. .758
5. .561
6. 13.861
7. 749.084
8. 3124.3
9. .31243
10. 312430000.
11. .000031243
12. 312.43
13. 31243000000.
14. .31243
15. .31243
16. 32000.
17. .000032
18. .5403+
19. 140yd.
20. $50.
21. 64753000000.

CONTRACTIONS IN DIVISION OF DECIMALS.

(Art. **275**, p. **208**.)

1. 3.1675
2. 9.1605
3. .000070461
4. .00070461
5. .0070461
6. .070461
7. .70460
8. .0000070460.
9. .0001965
10. $3.50
11. $0.1025

2. (Art. **276**, p. **209**.) 4.95445
3. 426.1043
4. 8.7938+
5. 9876.54321

REDUCTION OF DECIMALS.

(Art. **277**, p. **210**.)

2. $\frac{875}{1000} = \frac{7}{8}$
3. $\frac{9375}{10000} = \frac{15}{16}$
4. $\frac{8125}{100000} = \frac{13}{160}$
5. $\frac{75}{100000} = \frac{3}{4000}$
6. $31\frac{75}{100} = 31\frac{3}{4}$
7. $96\frac{24}{1000} = 96\frac{3}{125}$
8. $163\frac{4}{100} = 163\frac{1}{25}$
9. $1001\frac{4375}{10000} = 1001\frac{7}{16}$
10. $1457\frac{222}{1000} = 1457\frac{111}{500}$
11. $19678\frac{36}{100} = 19678\frac{9}{25}$
12. $9163\frac{8755}{10000} = 9163\frac{1751}{2000}$

2. (Art. **278**, p. **211**.) .625
3. .5
4. .09375
5. .076932

6.	19.125	11.	$4.3125
7.	$315.875	12.	$60.1875
8.	1163.75	13.	.51
9.	.625	14.	2.9875
10.	.370625		

(ART. **279**, p. **213**.)

(2.)

$$\begin{array}{r|l} 20 & 9.00 \\ \hline & .45 \end{array}$$

.45, Ans.

(3.)

$$\begin{array}{r|l} 25 & 14. \\ 4 & 3.56 \\ 20 & 15.89 \\ \hline & .7945 \end{array}$$

.7945, Ans.

(4.)

$$\begin{array}{r|l} 16 & 12. \\ 16 & 8.75 \\ 25 & 21.546875 \\ 4 & 2.861875 \\ \hline & .71546875 \end{array}$$

.71546875, Ans.

(5.)

$$\begin{array}{r|l} 4 & 3.00 \\ 4 & 1.7500 \\ \hline & .4375 \end{array}$$

.4375, Ans.

(6.)

$$\begin{array}{r|l} 12 & 9.00 \\ 3 & 2.75 \\ 5\frac{1}{2} & 2.91666666 \\ 40 & 35.53030303 \\ 8 & 5.88825757 \\ \hline & .73603219+ \end{array}$$

.73603219+, Ans.

(7.)

$$\begin{array}{r|l} 2 & 1. \\ 4 & 2.5 \\ 63 & 3.625 \\ \hline & .0575396+ \end{array}$$

.0575396+, Ans.

(8.)

$$\begin{array}{r|l} 2 & 1.0 \\ 8 & 0.50 \\ 4 & 0.0625 \\ \hline & 0.015625 \end{array}$$

0.015625, Ans.

(9.)

$$\begin{array}{r|l} 40 & 16. \\ 4 & 2.4 \\ \hline & .6 \end{array}$$

.6, Ans.

(10.)

$$\begin{array}{r|l} 40 & 175 \\ \hline & 4.375 \end{array}$$

4.375, Ans.

(11.)

$$\begin{array}{r|l} 4 & 3.755 \\ \hline & \end{array}$$

Ans. .93875

(12.)

$$\begin{array}{r|l} 10 & 6.0 \\ 60 & 34.6 \\ 60 & 25.576 \\ \hline & \end{array}$$

Ans., .4262+

(13.)

$$\begin{array}{r|l} 25 & 20.0 \\ 4 & \ \ 2.8 \\ \hline & 243.7,\ \text{Ans.} \end{array}$$

(14.)

$$\begin{array}{r|l} 2 & 1.5 \\ 4 & 2.75 \\ \hline & 156.6875,\ \text{Ans.} \end{array}$$

(15.)

$$\begin{array}{r|l} 4 & 3. \\ 12 & 11.75 \\ 20 & 19.97916+ \\ \hline & .9989583+,\ \text{Ans.} \end{array} \qquad \begin{array}{r|l} 4 & 1.0 \\ 12 & 9.25 \\ 20 & 16.77083+ \\ \hline & .8385416+,\ \text{Ans.} \end{array} \qquad \begin{array}{r|l} 4 & 2. \\ 12 & 5.5 \\ 20 & 17.4583+ \\ \hline & .8729166+,\ \text{[Ans.} \end{array}$$

$$\begin{array}{r} .9989583+ \\ .8385416+ \\ .8729166+ \\ \hline 2.710416+ \end{array}$$

(Art. 280, p. 214.)

2. $.625 \times 12 = 7.5$d.; $.5 \times 4 = 2.0$; $7\frac{1}{2}$d., Ans.
3. $.6725 \times 4 = 2.69$; $.69 \times 25 = 17.25$; $25 \times 16 = 4$; 2qr. 17lb. 4oz., Ans.
4. $.9375 \times 4 = 3.75$; $.75 \times 4 = 3$; 3qr. 3na., Ans.
5. $.7895 \times 8 = 6.316$; $.316 \times 40 = 12.64$; $.64 \times 16\frac{1}{2} = 10.56$; $.56 \times 12 = 6.72$; $72 = \frac{18}{25}$; 6fur. 12rd. 10ft. $6\frac{18}{25}$in., Ans.
6. $.9378 \times 4 = 3.7512$; $.7512 \times 40 = 30.048$; $.048 \times 272\frac{1}{4} = 13.068$; $.068 \times 144 = 9.792$; $.792 = \frac{99}{125}$; 3R. 30p. 13ft. $9\frac{99}{125}$in., Ans.
7. $.5615 \times 63 = 35.3745$; $.3745 \times 4 = 1.498$; $.498 \times 2 = .996$; $.996 \times 4 = 3.984$; $.984 = \frac{123}{125}$; 35gal. 1qt. 0pt. $3\frac{123}{125}$gi., Ans.
8. $.367 \times 365\frac{1}{4} = 134\ 046\frac{3}{4}$; $.046\frac{3}{4} \times 24 = 1.122$; $.122 \times 60 = 7.32$; $.32 \times 60 = 19.2$; $.2 = \frac{1}{5}$; 134da. 1h. 7m. $19\frac{1}{5}$sec., Ans.

9. .6923828125 × 4 = 2.76953125; .76953125 × 25 = 19.23828125; .23828125 × 16 = 3.8125; .8125 × 16 = 13; 2qr. 19lb. 3oz. 13dr., Ans.

10. .015625 × 4 = .0625; .0625 × 8 = .5; .5 × 2 = 1; 1p., Ans.

11. .55 × 5 = 2.75; .75 × 4 = 3; 2qr. 3na., Ans.

12. .6 × 4 = 2.4; .4 × 40 = 16; 2R. 16p., Ans.

MISCELLANEOUS EXERCISES.

(1.)
25 | 18.
4 | 2.72
7.68
11.75
3840
5376
768
768
$90.24, Ans.

(2.)
25 | 14.
4 | 3.56
19.89
9.25
9945
3978
17901
$183.9825 Ans.

(3.)
40 | 15.000
4 | 2.37500
39.59375
87.375
19796875
27715625
11878125
27715625
31675000
3459.50390625 =
3459.503\frac{29}{32}$ Ans.

(4.)
40 | 15.000
8 | 3.375000
87.421875
578.75
437109375
611953125
699375000
611953125
437109375
50595.41015625 =
50595.41\frac{1}{64}$ Ans.

(5.)

12 | 9.00   12 | 3.50
| 18.75 | $2.29\frac{1}{6}$
$2.29\frac{1}{6}$

16875
3750
3750
3125

42.96875
.053

12890625
21484375

2.27734375 = \$$2.277\frac{11}{32}$, Ans.

(6.)

12 | 7.168
| $1.597\frac{1}{3}$
6.5

7985
9582
$21\frac{2}{3}$

$10.3826\frac{2}{3}$
12

4.5920

10ft. 4.592in., Ans.

(7.)

$29\frac{1}{2}$ = 29.5
4.316)29.500(6h. 50m. 6+sec., [Ans.
25896

3604
60

4.316)216240(50m.
21580

440
60

4.316)26400(6+sec.
25896

(8.)

4 | 2.0
4 | 1.500

5.375
5.625

26875
10750
32250
26875

30.234375 =
\$ $30.234\frac{3}{8}$, Ans.

(9.)

25 | 7.00
4 | 3.28
4.82
17
81.94
5.875
40970
57358
65552
40970
481.39750 =
$\$ 481.39\frac{3}{4}$, Ans.

(10.)

40 | 25.000
4 | 3.625
176.90625
75.375
88453125
123834375
53071875
88453125
123834375
13334.30859375 =
$\$13334.308\frac{19}{32}$, Ans.

(11.)

17.625
12.75
88125
123375
35250
17625
224.71875 =
$\$224.718\frac{3}{4}$, Ans.

(12.)

19ft. 3in. = 19.25
15ft. 9in. = 15.75
9625
13475
9625
1925
303.1875
144
7500
7500
1875
27.0000
303ft. 27in., Ans.

13. 14ft. 6in. = 14.5; 12ft. 6in. = 12.5; 8ft. 9in. = 8.75;
$14.5 + 12.5 = 27$; $27 \times 2 = 54$; $8.75 \times 54 = 472.5$;
$472.5 \div 9 = 52\frac{1}{2}$ yards, Ans.

14. 10ft. 7in. $=$ 127in.; 5ft. 10in. $=$ 70in.; $127 \times 70 \times 4 = 35560$; $35560 \div 144 = 246\frac{17}{18}$ft., Ans.

(15.)

$$\begin{array}{r|l} 4 & 3.00 \\ 4 & 2.75 \\ \hline & 4.6875 \end{array} \qquad \begin{array}{r|l} 4 & 2.0 \\ \hline \multicolumn{2}{c}{112.5} \end{array}$$

$112.5 \div 4.6875 = 24$, Ans.

(16.)

$$\begin{array}{r|l} 2 & 1.0 \\ 4 & 2.500 \\ \hline & 1.625 \end{array}$$

$1.82 \div 1.625 = 1.12$

$\$1.12 \times 63 = \$70.56$, Ans.

17. $\$15.06 \div 125.5 = \$.12$, Ans.

18. $\begin{array}{l} 4 \mid 3 \\ \hline 17.75 \times \end{array} 35.75 = \$634.562\frac{1}{2}$, Ans.

19. $\$87.25 \times 7\frac{47}{63} = \$675.84\frac{8}{63}$, Ans.

20. 34ft. 9in. $= 34.75$; 1ft. 3in. $= 1.25$; 1ft. 6in. $= 1.5$; $34.75 \times 1.25 \times 1.5 = 65.15625$ft., Ans.

(21.)

$$\begin{array}{r|l} 4 & 1.00 \\ & 18.25 \end{array}$$

$36.50 \div 18.25 = \$2.00$, Ans.

(22.)

$\$477.72 \div 9 = \$53.08$, Ans.

(23.)

$$\begin{array}{r} 5.375 \\ 1.78 \\ 1.1875 \\ 1.125 \\ 1.275 \\ 2.625 \\ \hline 13.3675 = \end{array}$$

$\$13.367\frac{1}{2}$, Ans.

(24.)

$2\frac{3}{8} = 2.375$; $3\frac{7}{8} = 3.875$; $1\frac{1}{16} = 1.0625$;

$2.375 + 3.875 + 1.0625 = 7.3125$

$$\begin{array}{r} 7.3125 \\ 17.625 \\ \hline 365625 \\ 146250 \\ 438750 \\ 511875 \\ 73125 \\ \hline 128.8828125 = \end{array}$$

$\$128.882\frac{13}{16}$, Ans.

(25.)

$15.25 \times 8.4 \times 10 = 1281$;
$1281 \times 1728 = 2213568$;
$2213568 \div 231 = 9682\frac{6}{11}$gal.
$9682\frac{6}{11} \div 63 =$ 152hhd. $6\frac{6}{11}$gal., Ans.

(26.)

25 | 7.00
4 | 2.28
3.57
13.625
1785
714
2142
1071
357
48.64125 =
$48.641\frac{1}{4}$, Ans.

(27.)

40 | 35.000
4 | 3.87500
37.96875
125.75
18984375
26578125
18984375
7593750
3796875
4774.5703125 =
$4774.570\frac{5}{16}$, Ans.

(28.)

25 | 21.00
4 | 2.84
17.71
11.25
8855
3542
1771
1771
$199.23\frac{3}{4}$, Ans.

(30.) 97.625
$7\frac{49}{63}$
683375
$75930\frac{35}{63}$
$759.305\frac{35}{63} =$
$759.305\frac{35}{63}$, Ans.

(29.)

9.375
3.37
65625
28125
28125
31.59375
7.75
15796875
22115625
22115625
244.8315625 =
$244.851\frac{9}{16}$, Ans.

(31.)

$$\begin{array}{r|l}
4 & 3.00 \\
4 & 3.7500 \\ \hline
 & 7.9375 \\
 & 4.75 \\ \hline
 & 396875 \\
 & 555625 \\
 & 317500 \\ \hline
 & 37.703125 =
\end{array}$$

$\$37.703\frac{1}{8}$, Ans.

(32.)

$$\begin{array}{r|l}
25 & 3.50 \\
4 & 1.14 \\
20 & 15.285 \\ \hline
 & 27.76425 \\
 & 183.62 \\ \hline
 & 5552850 \\
 & 16658550 \\
 & 8329275 \\
 & 22211400 \\
 & 2776425 \\ \hline
 & 5098.0715850 =
\end{array}$$

$\$5098.071\frac{117}{200}$, Ans.

(33.)

$$\begin{array}{r|l}
40 & 15.000 \\
8 & 3.375000 \\ \hline
 & 17.421875 \\
 & 1725.875 \\ \hline
 & 87109375 \\
 & 121953125 \\
 & 139375000 \\
 & 87109375 \\
 & 34843750 \\
 & 121953125 \\
 & 17421875 \\ \hline
 & 30067.978515625 =
\end{array}$$

$\$30067.978\frac{33}{64}$, Ans.

(34.)

$$\begin{array}{r|l}
40 & 15. \\
4 & 3.375 \\ \hline
\text{A.} & 17.84375
\end{array}$$

$\$624.53125 \div 17.84375 =$
$\$35$, Ans.

(35.)

$$\begin{array}{r|l}
25 & 12.5 \\
4 & 2.5 \\
20 & 15.625 \\ \hline
 & 19.78125
\end{array}$$

$\$494.53125 \div 19.78125 =$
$\$24.999\frac{172}{3297}$, Ans.

(36.)

$$\begin{array}{r}
40)1004.75 \\ \hline
\text{A. } 25.11875 \\
4 \\ \hline
\text{R. } .47500 \\
40 \\ \hline
\text{p. } 19.00000
\end{array}$$

Ans. 25A. 0R. 19p.

37. $20.5 \times 12.75 \times 7.6 = 1986.45$ cubic feet;
$1986.45 \div 128 = 15$ cords $66\frac{9}{20}$ feet.

38. $31 \times .08\frac{1}{4} = \$2.55\frac{3}{4}$; $\$2.75 \times 7\frac{1}{2} = \$20.62\frac{1}{2}$; $\$20.62\frac{1}{2} + \$2.55\frac{3}{4} = \$23.18\frac{1}{4}$; $23.18\frac{1}{4} \div .62\frac{1}{2} = 37\frac{23}{250}$yd., [Ans.

39. $3.50 \div 40 = 8\frac{3}{4}$ bushels.

$\frac{9}{20}, \frac{3}{10} = \frac{9}{20}, \frac{6}{20}$; $\frac{9}{20} + \frac{6}{20} = \frac{15}{20}$; $\frac{20}{20} - \frac{15}{20} = \frac{5}{20}$bu.; $8\frac{3}{4} \div 5 = 1\frac{3}{4}$ bushels.

$1\frac{3}{4}$bu. $\times 6 = 10\frac{1}{2}$bu.; $1\frac{3}{4}$bu. $\times 9 = 15\frac{3}{4}$bu.

Ans. $8\frac{3}{4}$ bushels of oats; $10\frac{1}{2}$ bushels corn; $15\frac{3}{4}$ bushels wheat.

Proof: $\$0.75 \times 10\frac{1}{2} = \$7.87{,}5$; $\$2 \times 15\frac{3}{4} = \$31.50$; $\$3.50 + \$7.87{,}5 + \$31.50 = \$42.87{,}5$.

40. $250.35 \div 2 = 125.175$; $125.175 \div 8 = 15.646875$; $15.646875 + 1 = 16.646875$ times, Ans.

41. $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$; $\frac{12}{12} - \frac{7}{12} = \frac{5}{12}$; $\frac{\cancel{5}}{12} \times \frac{1}{\cancel{5}} = \frac{1}{12}$, part for Mary.

$\frac{7}{12} + \frac{1}{12} = \frac{8}{12}$; $\frac{12}{12} - \frac{8}{12} = \frac{4}{12} = \frac{1}{3}$, part for wife.

$\frac{1}{4} - \frac{1}{12} = \frac{1}{6} = \$2243.26$, part Mary received less than James.

Therefore $\$2243.26 \times 6 = \$13459.56$, amount.

$\$13459.56 \times \frac{1}{4} = \$3364.89$, James' part.

$\$13459.56 \times \frac{1}{3} = \$4486.52$, William's part.

$\$13459.56 \times \frac{1}{12} = \$1121.63$, Mary's part.

$\$13459.56 \times \frac{1}{3} = \$4486.52$, wife's part.

INFINITE OR CIRCULATING DECIMALS.

(ART. **296**, p. **220.**)

3. $.\dot{6} = \frac{6}{9} = \frac{2}{3}$, Ans.

4. $\dot{1}.6\dot{2} = 1.\dot{6}2\dot{1} = 1\frac{621}{999} = 1\frac{23}{37}$, Ans.

5. $.5\dot{3} = \frac{5\frac{3}{9}}{10} = \frac{48}{90} = \frac{8}{15}$, Ans.

6. $.\dot{7}6923\dot{0} = \frac{769230}{999999} = \frac{10}{13}$, Ans.

7. $.5\dot{9}2\dot{5} = \frac{5\frac{925}{999}}{10} \times \frac{5920}{9990} = \frac{592}{999} = \frac{16}{27}$, Ans.

8. $3\dot{1}.6\dot{2} = 31.\dot{6}2\dot{1} = 31\frac{621}{999} = 31\frac{23}{37}$, Ans.

9. $.008\dot{4}97133\dot{3} = \frac{8\frac{497133}{999999}}{1000} = \frac{8497125}{999999000} = \frac{83}{9768}$, Ans.

(Art. **297**, p. **222.**)

3. $\frac{1}{11}=.\dot{0}\dot{9}$, Ans.
4. $\frac{39}{1320}=\frac{13}{440}=13\div 440=.029\dot{5}\dot{4}$, Ans.
5. $\frac{107}{253}=107\div 253=.\dot{4}22924901185770750988\dot{1}$, Ans.
6. $13\frac{17}{45}$; $17\div 45=.3\dot{7}$; $13.3\dot{7}$, Ans.
7. $\frac{166}{19536}=\frac{83}{9768}$; $83\div 9768=.008\dot{4}9713\dot{3}$, Ans.
8. $2\div 29=.06\dot{8}27586206896551724137931034\dot{8}$, Ans.

TRANSFORMATION OF REPETENDS.

(Art. **302**, p. **223.**)

(2.)

$3.\dot{6}7\dot{1}=3.\dot{6}7167167167\dot{1}$
$1.\dot{0}07\dot{1}=1.\dot{0}0710071007\dot{1}$
$8.\dot{5}\dot{2}=8.\dot{5}2525252525\dot{2}$
$7.\dot{6}1632\dot{5}=7.\dot{6}1632561632\dot{5}$

(3.)

$1.\dot{5}\dot{2}=1.52\dot{5}25\dot{2}$
$8.\dot{7}15\dot{6}=8.71\dot{5}67\dot{1}$
$3.56\dot{7}=3.56\dot{7}77\dot{7}$
$1.3\dot{7}\dot{8}=1.37\dot{8}78\dot{7}$

(4.)

$.00\dot{0}\dot{7}=.000\dot{7}07070\dot{0}$
$.141\dot{4}1\dot{4}=.141\dot{4}1441\dot{4}$
$887.\dot{1}=887.111\dot{1}1111\dot{1}$

(5.)

$.3\dot{1}2\dot{3}=.3\dot{1}2312\dot{3}$
$3.\dot{2}\dot{7}=3.2\dot{7}2727\dot{2}$
$5.0\dot{2}=5.0\dot{2}2222\dot{2}$

(6.)

$17.08\dot{8}\dot{4}=17.08\dot{8}4848484848484848484848484848\dot{4}$
$1563.0\dot{9}2\dot{9}=1563.09\dot{2}9929929929929929929929929929\dot{9}$
$15.\dot{1}234\dot{5}=15.12\dot{3}451234512345123451234512345 1\dot{2}$

ADDITION OF CIRCULATING DECIMALS.

(Art. **303**, p. **224.**)

(2.)

$3.5\dot{5}5555\dot{5}$
$7.6\dot{5}1651\dot{6}$
$1.7\dot{6}5765\dot{7}$
$6.1\dot{7}3737\dot{3}$
$51.\dot{7}77777\dot{7}$
$3.7\dot{0}0000\dot{0}$
$27.6\dot{3}1631\dot{6}$
$1.0\dot{0}3003\dot{0}$

Ans. $103.2\dot{5}9122\dot{7}$

(3.)

$\frac{1}{3}=\dot{3}=.\dot{3}3333\dot{3}$
$\frac{1}{7}=.\dot{1}4285\dot{7}=.\dot{1}4285\dot{7}$
$\frac{1}{9}=\dot{1}=.\dot{1}1111\dot{1}$

Ans. $.\dot{5}8730\dot{1}$

(4.)

$2\dot{7}.5\dot{6} = 27.5\dot{6}75675675675\dot{5}$
$5.\dot{6}3\dot{2} = 5.6\dot{3}263263263\dot{2}6$
$6.\dot{7} = 6.7\dot{7}7777777777\dot{7}$
$16.3\dot{5}\dot{6} = 16.3\dot{5}6565656565\dot{6}$
$.7\dot{1} = .7\dot{1}1111111111\dot{1}$
$6.\dot{1}23\dot{4} = 6.1\dot{2}341234123\dot{4}\dot{1}$

Ans. $63.1\dot{6}9067086888\dot{8}$

(5.)

$.165002 = .165002$
$31.64 = 31.64$
$1.\dot{6} = 1.666666\dot{6}$
$.3463\dot{4} = .346344\dot{4}$
$13. \quad 13.$

Ans. $46.818013\dot{1}$

(6.)

$.\dot{8}\dot{7} = .\dot{8}7878\dot{7}$
$.\dot{8} = .\dot{8}8888\dot{8}$
$.\dot{8}7\dot{6} = .\dot{8}7687\dot{6}$

Ans. $2.\dot{6}4455\dot{3}$

(7.)

$.\dot{3} = .3333\dot{3}3333\dot{3}$
$.45 = .45$
$.\dot{4}\dot{5} = .4545\dot{4}5454\dot{5}$
$.\dot{3}5\dot{1} = .3513\dot{5}1351\dot{3}$
$.6468 = .6468$
$.646\dot{8} = .6468\dot{8}8888\dot{8}$
$.64\dot{6}\dot{8} = .6468\dot{6}8686\dot{8}$
$.6\dot{4}6\dot{8} = .6468\dot{4}6846\dot{8}$

Ans. $4.1766\dot{3}4561\dot{8}$

(8.)

$1.25 = 1.250\dot{0}0000\dot{0}$
$3.\dot{4} = 3.444\dot{4}4444\dot{4}$
$.6\dot{3}\dot{7} = .637\dot{3}7373\dot{7}$
$7.88\dot{5} = 7.885\dot{5}5555\dot{5}$
$7.875 = 7.875\dot{0}0000\dot{0}$
$7.\dot{8}7\dot{5} = 7.875\dot{8}7587\dot{5}$
$11.\dot{1} = 11.111\dot{1}1111\dot{1}$

Ans. $40.079\dot{3}6072\dot{2}$

(9.)

$131.613 = 131.613$
$15.00\dot{1} = 15.001\dot{1}$
$67.13\dot{4} = 67.134\dot{4}$
$1000.6\dot{3} = 1000.633\dot{3}$

Ans. $1214.381\dot{8}$

(10.)

$5.\dot{1}634\dot{5} = 5.\dot{1}634516345163451634\dot{5}$

$8.\dot{6}38\dot{1} = 8.\dot{6}381638163816381638\dot{1}$

$3.\dot{7}\dot{5} = 3.\dot{7}575757575757575757\dot{5}$

Ans. $17.\dot{5}591912084737409030\dot{2}$

SUBTRACTION OF CIRCULATING DECIMALS.

(Art. **304**, p. **225.**)

(2.)

$7.\dot{1} = 7.1\dot{1}$

$5.0\dot{2} = 5.0\dot{2}$

Ans. $2.0\dot{8}$

(3.)

$31\dot{5}.8\dot{7} = 315.\dot{8}7587587587\dot{5}$

$78.\dot{0}37\dot{8} = 78.\dot{0}3780378037\dot{8}$

Ans. $237.\dot{8}3807209549\dot{7}$

(4.)

$\frac{2}{9} = .\dot{2} = .\dot{2}2222\dot{2}$

$\frac{1}{7} = .\dot{1}4285\dot{7} = .\dot{1}4285\dot{7}$

Ans. $.\dot{0}7936\dot{5}$

(5.)

$16.13\dot{4}\dot{7} = 16.13\dot{4}\dot{7}$

$11.08\dot{8}\dot{4} = 11.08\dot{8}\dot{4}$

Ans. $5.04\dot{6}\dot{2}$

(6.)

$18.16\dot{7}\dot{8} = 18.16\dot{7}\dot{8}$

$3.\dot{2}\dot{7} = 3.27\dot{2}\dot{7}$

Ans. $14.89\dot{5}\dot{1}$

(7.)

$3.\dot{1}2\dot{3} = 3.\dot{1}2312\dot{3}$

$.\dot{7}\dot{1} = .\dot{7}1717\dot{1}$

Ans. $2.\dot{4}0595\dot{1}$

(8.)

$\frac{3}{7} = .\dot{4}2857\dot{1} = .\dot{4}2857\dot{1}$

$\frac{2}{11} = \dot{1}\dot{8} = .\dot{1}8181\dot{8}$

Ans. $.\dot{2}4675\dot{3}$

(9.)

$\frac{4}{9} = .\dot{4} = .\dot{4}4444\dot{4}$

$\frac{2}{7} = .\dot{2}8571\dot{4} = .\dot{2}8571\dot{4}$

Ans. $.\dot{1}5873\dot{0}$

(10.)

$\frac{9}{17} = .\dot{5}29411764705882\dot{3}$

$\frac{6}{17} = .\dot{3}52941176470588\dot{2}$

Ans. $.\dot{1}76470588235294\dot{1}$

(11.)

$$\begin{array}{rr} 5.\dot{1}234\dot{5} = & 5.1\dot{2}3451234512345123451234512345\dot{1} \\ 2.3\dot{5}23456\dot{6} = & 2.3\dot{5}23456523456523456523456523456\dot{6} \\ \hline \text{Ans.} & 2.7\dot{7}11055821666927777988888599999\dot{4} \end{array}$$

MULTIPLICATION OF CIRCULATING DECIMALS.

(Art. **305**, p. **226**.)

(2.)

$$\begin{array}{rr} & 87.32\dot{5}8\dot{6} \\ & 437 \\ \hline & 61128106 \\ & 26197759_{7} \\ & 34930346_{34} \\ \hline \text{Ans.} & 381.6140\dot{3}3\dot{8} \end{array}$$

3. $582.\dot{3}4\dot{7} = 582\frac{347}{999} = \frac{581765}{999}$; $.0\dot{3} = \frac{3}{90} = \frac{1}{30}$; $\frac{581765}{999} \times \frac{1}{30} = \frac{581765}{59970} = \frac{116353}{5994}$; $\frac{116353}{5994} = 19.41\dot{1}578244\dot{9}$, Ans.

4. $3.1\dot{4}\dot{5} = 3.1\frac{45}{99} = 3.1\frac{5}{11} = \frac{346}{110}$; $4.\dot{2}9\dot{7} = 4\frac{297}{999} = 4\frac{11}{37} = \frac{159}{37}$; $\frac{346}{110} \times \frac{159}{37} = \frac{55014}{4070} = 13.5\dot{1}69533\dot{3}$, Ans.

(5.)

$$\begin{array}{rr} & .\dot{2}8571\dot{4} \\ & 21 \\ \hline & 285714 \\ & 571428_{5} \\ \hline \text{Ans.} & 8.\dot{0}0000\dot{0} \end{array}$$

$$\begin{array}{rr} (7.) & .2\dot{8}493150\dot{6} \\ & 365 \\ \hline & 1424657534 \\ & 1709589041_{0} \\ & 854794520_{54} \\ \hline \text{Ans.} & 104.\dot{0}0000000\dot{0} \end{array}$$

(6.)

$$\begin{array}{r} .461607\dot{1}4285\dot{7} \\ 20 \\ \hline 9.2321428\dot{8}5714\dot{2} \\ 4 \\ \hline 0.928571\dot{4}2857\dot{1} \\ 25 \\ \hline 4642857\dot{1}42855 \\ 18571428571428 \\ \hline 23.2142857\dot{7}1428\dot{3} \end{array}$$

Ans. 9cwt. 0qr. 23+lb.

DIVISION OF CIRCULATING DECIMALS.

(ART. **306**, p. **226**.)

2. $345.\dot{8} = 345\frac{8}{9}$; $.\dot{6} = \frac{6}{9} = \frac{2}{3}$; $345\frac{8}{9} \div \frac{2}{3} = 518\frac{5}{6} = 518.8\dot{3}$, Ans.
3. $234.\dot{6} = 234\frac{2}{3}$; $.\dot{7} = \frac{7}{9}$; $234\frac{2}{3} \div \frac{7}{9} = 301\frac{5}{7} = 301.7142\dot{8}\dot{5}$, Ans.
4. $13.5\dot{1}6953\dot{3} = 13\frac{5}{10} + \frac{169533}{9999990} = \frac{1351693980}{99999900}$; $3.1\dot{4}\dot{5} = 3\frac{1}{10} + \frac{45}{990} = \frac{31140}{9900}$; $\frac{1351693980}{99999900} \div \frac{31140}{9900} = 4.\dot{2}9\dot{7}$, Ans.
5. $381.6140\dot{3}3\dot{8} = 381\frac{6140}{10000} + \frac{338}{9990000} = \frac{38123241980000}{99900000000000}$; $4.37 = 4\frac{37}{100}$; $\frac{38123241980000}{99900000000000} \div 4\frac{37}{100} = 87.325\dot{8}\dot{6}$, Ans.
6. $.\dot{4}2857\dot{1} = \frac{428571}{999999}$; $.625 = \frac{5}{8}$; $\frac{428571}{999999} \div \frac{5}{8} = .\dot{6}85714\dot{2}$, Ans.
7. $2.\dot{3}7\dot{0} = 2\frac{370}{999} = \frac{2368}{999}$; $4.\dot{9}2307\dot{6} = 4\frac{923076}{999999} = \frac{4923072}{999999}$; $\frac{2368}{999} \div \frac{4923572}{999999} = .\dot{4}8\dot{1}$, Ans.
8. $\frac{9}{99} \div \frac{230769}{999999} = .\dot{3}\dot{9}$, Ans.
9. $316.31015 \div \frac{3}{9} = 948.93045$, Ans.
10. $100006 \div \frac{6}{9} = 150009$, Ans.
11. $.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$; $.2\dot{5} = \frac{2}{10} + \frac{5}{90} = \frac{23}{90}$; $\frac{4}{11} \div \frac{23}{90} = 1.\dot{4}22924901185770750988\dot{1}$, Ans.

CONTINUED FRACTIONS.

(ART. **309**, p. **229**.)

(3.) $\frac{261}{82} = 82)261(3$

$$\begin{array}{l}
82)261(3\\
\quad\underline{246}\\
\quad\quad 15)82(5\\
\quad\quad\quad\underline{75}\\
\quad\quad\quad\quad 7)15(2\\
\quad\quad\quad\quad\quad\underline{14}\\
\quad\quad\quad\quad\quad\quad 1)7(7\\
\quad\quad\quad\quad\quad\quad\quad\underline{7}
\end{array}$$

First approx. val. $= \frac{3}{1} = 3$.

$$\begin{array}{l} 3 \times 5 + 1 = 16 \\ \overline{1} \times 5 + 0 = \overline{5} \end{array} = 3\tfrac{1}{5}.$$

$$\begin{array}{l} 16 \times 2 + 3 = 35 \\ \overline{5} \times 2 + 1 = \overline{11} \end{array} = 3\tfrac{2}{11}.$$

$$\begin{array}{l} 35 \times 7 + 16 = 261 \\ \overline{11} \times 7 + \overline{5} = \overline{82} \end{array} = 3\tfrac{15}{82}.$$

Ans. $3, 3\frac{1}{5}, 3\frac{2}{11}, 3\frac{15}{82}$, real val.

(4.) $\frac{29}{77} =$

$$\begin{array}{l}
29)77(2 \\
\quad \underline{58} \\
\quad 19)29(1 \\
\qquad \underline{19} \\
\qquad 10)19(1 \\
\qquad\quad \underline{10} \\
\qquad\quad 9)10(1 \\
\qquad\qquad \underline{9} \\
\qquad\qquad 1)9(9 \\
\qquad\qquad\quad \underline{9}
\end{array}$$

First approx. val. $= \frac{1}{2}$.

$$\begin{array}{l} 1 \times 1 + 0 = 1 \\ \overline{2} \times 1 + 1 = \overline{3}, \end{array}$$

$$\begin{array}{l} 1 \times 1 + 1 = 2 \\ \overline{3} \times 1 + 2 = \overline{5}, \end{array}$$

$$\begin{array}{l} 2 \times 1 + 1 = 3 \\ \overline{5} \times 1 + 3 = \overline{8}, \end{array}$$

$$\begin{array}{l} 3 \times 9 + 2 = 29 \\ \overline{8} \times 9 + 5 = \overline{77}, \text{ original val.} \end{array}$$

Ans. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{29}{77}$.

(5.) $\frac{1327}{1631} =$

$$\begin{array}{l}
1327)1631(1 \\
\quad \underline{1327} \\
\quad 304)1327(4 \\
\qquad \underline{1216} \\
\qquad 111)304(2 \\
\qquad\quad \underline{222} \\
\qquad\quad 82)111(1 \\
\qquad\qquad \underline{82} \\
\qquad\qquad 29)82(2 \\
\qquad\qquad\quad \underline{58} \\
\qquad\qquad\quad 24)29(1 \\
\qquad\qquad\qquad \underline{24} \\
\qquad\qquad\qquad 5)24(4 \\
\qquad\qquad\qquad\quad \underline{20} \\
\qquad\qquad\qquad\quad 4)5(1 \\
\qquad\qquad\qquad\qquad \underline{4} \\
\qquad\qquad\qquad\qquad 1)4(4 \\
\qquad\qquad\qquad\qquad\quad \underline{4}
\end{array}$$

First approx. val. $= \frac{1}{1}$.

$$\begin{array}{l} 1 \times 4 + 0 = 4 \\ \overline{1} \times 4 + 1 = \overline{5}\cdot \end{array}$$

$$\begin{array}{l} 4 \times 2 + 1 = 9 \\ \overline{5} \times 2 + 1 = \overline{11}\cdot \end{array}$$

$$\begin{array}{l} 9 \times 1 + 4 = 13 \\ \overline{11} \times 1 + 5 = \overline{16}\cdot \end{array}$$

$$\begin{array}{l} 13 \times 2 + 9 = 35 \\ \overline{16} \times 2 + 11 = \overline{43}\cdot \end{array}$$

$$\begin{array}{l} 35 \times 1 + 13 = 48 \\ \overline{43} \times 1 + 16 = \overline{59}\cdot \end{array}$$

$$\begin{array}{l} 48 \times 4 + 35 = 131 \\ \overline{59} \times 4 + 43 = \overline{161}\cdot \end{array}$$

$$\begin{array}{l} 131 \times 1 + 48 = 179 \\ \overline{161} \times 1 + 59 = \overline{220}\cdot \end{array}$$

$$\begin{array}{l} 179 \times 4 + 131 = 847 \\ \overline{220} \times 4 + 161 = \overline{1041}\cdot \end{array}$$

Ans. $\frac{1}{1}$, $\frac{4}{5}$, $\frac{9}{11}$, $\frac{13}{16}$, $\frac{35}{43}$, $\frac{48}{59}$, [$\frac{131}{161}$, $\frac{179}{220}$, $\frac{847}{1041}$.

(6.)

$\frac{829}{347} = 347)829(2$
694
135)347(2
270
77)135(1
77
58)77(1
58
19)58(3
57
1)19(19
19

$\frac{2}{1} = 2$, first approx. val.

$$\begin{array}{l} 2 \times 2 + 1 = 5 \\ \overline{1} \times 2 + 0 = \overline{2} \end{array} = 2\tfrac{1}{2}.$$

$$\begin{array}{l} 5 \times 1 + 2 = 7 \\ \overline{2} \times 1 + 1 = \overline{3} \end{array} = 2\tfrac{1}{3}.$$

$$\begin{array}{l} 7 \times 1 + 5 = 12 \\ \overline{3} \times 1 + 2 = \overline{5} \end{array} = 2\tfrac{2}{5}.$$

$$\begin{array}{l} 12 \times 3 + 7 = 43 \\ \overline{5} \times 3 + 3 = \overline{18} \end{array} = 2\tfrac{7}{18}.$$

$$\begin{array}{l} 43 \times 19 + 12 = 819 \\ \overline{18} \times 19 + 5 = \overline{347} \end{array} = [2\tfrac{125}{347}.$$

Ans. $2,\ 2\frac{1}{2},\ 2\frac{1}{3},\ 2\frac{2}{5},\ 2\frac{7}{18},\ 2\frac{125}{347}$.

(7.) $\dfrac{1}{13\frac{568}{1000}} = \frac{1000}{13568}$.

1000)13568(13
13000
568)1000(1
568
432)568(1
432
136)432(3
408
24)136(5
120
16)24(1
16
8)16(2
16

$\frac{1}{13}$ = first approx. val.

$$\begin{array}{l} 1 \times 1 + 0 = 1 \\ \overline{13} \times 1 + 1 = \overline{14} \end{array}.$$

$$\begin{array}{l} 1 \times 1 + 1 = 2 \\ \overline{14} \times 1 + 13 = \overline{27} \end{array}.$$

$$\begin{array}{l} 2 \times 3 + 1 = 7 \\ \overline{27} \times 3 + 14 = \overline{95} \end{array}.$$

$$\begin{array}{l} 7 \times 5 + 2 = 37 \\ \overline{95} \times 5 + 27 = \overline{502} \end{array}.$$

$$\begin{array}{l} 37 \times 1 + 7 = 44 \\ \overline{502} \times 1 + 95 = \overline{597} \end{array}.$$

$$\begin{array}{l} 44 \times 2 + 37 = 125 \\ \overline{579} \times 2 + 44 = \overline{1396} \end{array}$$

Ans. $\frac{1}{13},\ \frac{1}{14},\ \frac{2}{27},\ \frac{7}{95},\ \frac{37}{502},\ \frac{44}{597},\ \frac{125}{1396}$.

(8.) $1.27 = \frac{127}{100}$.

```
100)127(1
    100
    ---
     27)100(3
         81
         --
         19)27(1
            19
            --
             8)19(2
               16
               --
                3)8(2
                  6
                  -
                  2)3(1
                    2
                    -
                    1)2(2
                      2
                      -
```

First approx. val. $= \frac{1}{1}$.

$$\begin{array}{l} 1 \times 3 + 1 = 4 \\ \overline{1} \times 3 + 0 = \overline{3} \end{array} = 1\frac{1}{3}.$$

$$\begin{array}{l} 4 \times 1 + 1 = 5 \\ \overline{3} \times 1 + 1 = \overline{4} \end{array} = 1\frac{1}{4}.$$

$$\begin{array}{l} 5 \times 2 + 4 = 14 \\ \overline{4} \times 2 + 3 = \overline{11} \end{array} = 1\frac{3}{11}.$$

$$\begin{array}{l} 14 \times 2 + 5 = 33 \\ \overline{11} \times 2 + 4 = \overline{26} \end{array} = 1\frac{7}{26}.$$

$$\begin{array}{l} 33 \times 1 + 14 = 47 \\ \overline{26} \times 1 + 11 = \overline{37} \end{array} = 1\frac{10}{37}.$$

$$\begin{array}{l} 47 \times 2 + 33 = 127 \\ \overline{37} \times 2 + 26 = \overline{100} \end{array} = 1\frac{27}{100}.$$

Ans. $\frac{1}{1}, \frac{4}{3}, \frac{5}{4}, \frac{14}{11}, \frac{33}{26}, \frac{47}{37}, \frac{127}{100} = 1, 1\frac{1}{3}, 1\frac{1}{4}, 1\frac{3}{11}, 1\frac{7}{26}, 1\frac{10}{37}, 1\frac{27}{100}$.

RATIO.

REDUCTION AND COMPARISON OF RATIOS.

(ART. 328, p. 232.)

2. $63 : 72 = \frac{63}{72} = \frac{7}{8}$, Ans.
3. $66 : 24 = \frac{66}{24} = \frac{11}{4}$, Ans.
4. $4 \times 6 \times 3 : 8 \times 9 \times 2 = 72 : 144 = \frac{72}{144} = \frac{1}{2}$, Ans.
5. $19 \times 5 \times 2 \times 3 : 15 \times 12 \times 38 = 570 : 684 = \frac{570}{684} = \frac{95}{114}$, Ans.

(ART. 329, p. 233.)

3. $\frac{5}{6} : \frac{4}{7} = \frac{5}{6} \times \frac{7}{4} = \frac{35}{24} = 35 : 24$, Ans.
4. $13\frac{1}{2} : 27 = \frac{13\frac{1}{2}}{27} = \frac{27}{54} = \frac{1}{2} = 1 : 2$, Ans.
5. $6.25 : 3.125 = \frac{6.25}{3.125} = \frac{2}{1} = 2 : 1$, Ans.

6. $\left.\begin{array}{r}4 : 16\\ 25 : 10\end{array}\right\} = 4 \times 25 : 16 \times 10 = 100 : 160 = \frac{100}{160} =$
[$\frac{5}{8} = 5 : 8$, Ans.

7. $\left.\begin{array}{r}3 : 6\\ 9 : 27\\ 108 : 12\end{array}\right\} = 3 \times 9 \times 108 = 2916 : 6 \times 27 \times 12 =$
[$1924 = \frac{2916}{1924} = \frac{3}{2} = 3 : 2$, Ans.

8. $\left.\begin{array}{r}12\frac{1}{2} : 6\frac{1}{4}\\ 76.5 : 25.5\end{array}\right\} = 12.5 \times 76.5 = 956.25 : 6.25 \times 25.5 =$
[$159.375 = \frac{956.25}{159.375} = \frac{6}{1} = 6 : 1$, Ans.

(Art. **330**, p. **233.**)

3. $39 : 13 = \frac{39}{13} = 3$, Ans.

4. $2\frac{1}{2} : 9 = \dfrac{2\frac{1}{2}}{9} = \frac{5}{18}$, Ans.

5. $21 : 21 = \frac{21}{21} = 1$, Ans.

6. $\frac{1}{3} \times \frac{1}{6} \times \frac{200}{1} = \frac{200}{18} = \frac{100}{9} : \frac{1}{2} \times \frac{50}{1} = \frac{50}{2} = \frac{25}{1}$; $\frac{100}{9} \times \frac{1}{25} = \frac{4}{9}$, Ans.

7. $24 : 6 = \frac{6}{24} = \frac{1}{4}$, Ans.

8. $4 : 36 = \frac{4}{36} = \frac{1}{9}$, Ans.

9. 94A. 2R. 16p. = 15136p.; 11A. 3R. = 1880p.; $1880 : 15136 = \frac{1880}{15136} = \frac{235}{1892}$, Ans.

10. $17 : 9 = \frac{17}{9} = 1\frac{8}{9}$; $39 : 19 = \frac{39}{19} = 2\frac{1}{19}$; therefore the ratio of 39 to 19 is the larger.

11. $36 \times 4 \times 3 = 432 : 12 \times 16 \times 2 = 384 = \frac{432}{384} = \frac{9}{8}$; $60 \div (3 \times 5) = 4$; $(20 \times 2) \div 8 = 5$; $\frac{9}{8} - \frac{4}{5} = = \frac{13}{40}$, Ans.

12. $.0\dot{2} = \frac{2}{90}$; $2.\dot{5}0\dot{3} = 2\frac{503}{999} = \frac{2501}{999} = \frac{90}{2} \times \frac{2501}{999} = \frac{225090}{1998} = 112\frac{73}{111}$, Ans.

13. $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$; $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$; $\frac{10}{1} \times \frac{1}{12} = \frac{10}{12} = \frac{5}{6}$.
$4 : 5 = \frac{4}{5}$; $\frac{5}{6} - \frac{4}{5} = \frac{1}{30}$, Ans. That is, the first ratio is the greater by $\frac{1}{30}$.

14. $220 : 500 = \frac{220}{500} = \frac{11}{25}$, Ans.

15. $36 : 180 = \frac{1}{5}$, Ans.

16.) 4900)11283(2
9800
1483)4900(3
4449
451)1483(3
1353
130)451(3
390
61)130(2
122
8)61(7
56
5)8(1
5
3)5(1
3
2)3(1
2
1)2(2
2

First approx. ratio = $\frac{1}{2}$.

$1 \times 3 + 0 = 3$
$\overline{2} \times 3 + 1 = \overline{7}$

$3 \times 3 + 1 = 10$
$\overline{7} \times 3 + 2 = \overline{23}$

$10 \times 3 + 3 = 33$
$\overline{23} \times 3 + 7 = \overline{76}$

$33 \times 2 + 10 = 76$
$\overline{76} \times 2 + 23 = \overline{175}$

$76 \times 7 + 33 = 565$
$\overline{175} \times 7 + 76 = \overline{1301}$

$565 \times 1 + 76 = 641$
$\overline{1301} \times 1 + 135 = \overline{1436}$

$641 \times 1 + 555 = 1196$
$\overline{1436} \times 1 + 1301 = \overline{2737}$

$1196 \times 1 + 641 = 1837$
$\overline{2737} \times 1 + 1436 = \overline{4173}$

$1837 \times 2 + 1196 = 4870$
$\overline{4173} \times 2 + 2637 = \overline{11083}$

Ans. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{10}{23}$, $\frac{33}{76}$, $\frac{76}{175}$, $\frac{565}{1301}$, $\frac{641}{1436}$, $\frac{1196}{2737}$, $\frac{1837}{4173}$, $\frac{4870}{11083}$.

(17.) 1000000)3141592(3
3000000
141592)1000000(7
991144
8856)141592(15
8856
53032
44280
8752)8856(1
8752
104)8752(84
832
432
416
16)104(6
96
8)16(2
16

First approx. ratio $= \frac{3}{1}$.

$3 \times 7 + 1 = 22$
$\overline{1} \times 7 + 0 = \overline{7}$

$22 \times 15 + 3 = 333$
$\overline{7} \times 15 + 1 = \overline{106}$

$333 \times 1 + 22 = 355$
$\overline{106} \times 1 + 7 = \overline{113}$

$355 \times 84 + 333 = 30153$
$\overline{113} \times 84 + 106 = \overline{9492}$

$30153 \times 6 + 355 = 181273$
$\overline{9492} \times 6 + 113 = \overline{57065}$

$181273 \times 2 + 30153 = 392699$
$\overline{57065} \times 2 + 9492 = \overline{123622}$

Ans. $\frac{3}{1}, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{30153}{9492}, \frac{181273}{57065}, \frac{392699}{123622} =$
$3, 3\frac{1}{7}, 3\frac{15}{106}, 3\frac{16}{113}, 3\frac{1677}{9492}, 3\frac{10078}{57065}, 3\frac{21833}{123622}$.

ANALYSIS BY RATIO.

(Art. **331**, p. **235**.)

(2.) $80
17
$1360 = 17A.
(2R. : 1A. = $\frac{1}{2}$); $\frac{1}{2}$ of 80 = 40 = 2R.
(1R. : 2R. = $\frac{1}{2}$); $\frac{1}{2}$ of 40 = 20 = 1R.
(20r. : 1R. = $\frac{1}{2}$); $\frac{1}{2}$ of 20 = 10 = 20r.
(10r. : 20r. = $\frac{1}{2}$); $\frac{1}{2}$ of 10 = 5 = 10r.
(5r. : 10r. = $\frac{1}{2}$); $\frac{1}{2}$ of 5 = 2.50 = 5r.
$1437.50, Ans.

(3.) $2.50
16
$40.00 = 16cwt.
(2qr. : 1cwt. = ½); ½ of $2.50 = 1.25 = 2qr.
(1qr. : 2qr. = ½); ½ of $1.25 = .625 = 1qr.
(10lb. : 1cwt. = ⅒); ⅒ of $2.50 = .25 = 10lb.
$42.125, Ans.

(4.) $14.00
27
$378.00 = 27cwt.
(1qr. : 1cwt. = ¼); ¼ of $14 = 3.50 = 1qr.
(20lb. : 1cwt. = ⅕); ⅕ of $14 = 2.80 = 20lb.
$384.30, Ans.

(5.) $5.60
7
$39.20 = 7yd.
(2qr. : 1yd. = ½); ½ of $5.60 = 2.80 = 2qr.
(1qr. : 2qr. = ½); ½ of $2.80 = 1.40 = 1qr.
(2na. : 1qr. = ½); ½ of $1.40 = .70 = 2na.
$44.10, Ans.

(6.) $20
7
$140.00 = 7ton.
(10cwt. : 1 ton = ½); ½ of $20 = 10.00 = 10cwt.
(2cwt. : 10cwt. = ⅕); ⅕ of $10 = 2.00 = 2cwt.
(1cwt. : 2cwt. = ½); ½ of $ 2 = 1.00 = 1cwt.
(2qr. : 1cwt. = ½); ½ of $ 1 = .50 = 2qr.
$153.50, Ans.

(8.) $4.40
19
$83.60 = 19yd.
(2qr. : 1yd. = ½); ½ of $4.40 = 2.20 = 2qr.
(1qr. : 1yd. = ¼); ¼ of $4.40 = 1.10 = 1qr.
(2na. : 1qr. = ½); ½ of $1.10 = .55 = 1na.
$87.45, Ans.

(9.) $32
24
$768
(3R. : 1A. = ¾); ¾ of $32 = 24
(20p. : 1A. = ⅛); ⅛ of $32 = 4
$796, Ans.

(10.) 200
$0.30
$60.00 = 200lb.
(4oz. : 1lb. = ¼) = ¼ of $0.30 = .075 = 4oz.
$59.925, Ans.

(11.) 714
15s.
3570
714
10710s.
(6d. : 1s. = ½); ½ of 714 = 357
20)11067
553£. 7s., Ans.

(12.)

$450
2
$900 = 2y.
(6mo. : 1y. = ½); ½ of $450 = 225 = 6mo.
(3mo. : 6mo. = ½); ½ of $225 = 112.50 = 3mo.
(15da. : 3mo. = ⅙); ⅙ of $112.50 = 18.75 = 15da.
$1256.25, Ans.

(13.)

$80.50 × 25 = $2012.50 = 25A.
(2R. : 1A. = ½); ½ of $80.50 = 40.25 = 2R.
(20r. : 2R. = ¼); ¼ of $40.25 = 10.06+ = 20r.
(10r. : 20r. = ½); ½ of $10.06+ = 5.03+ = 10r.
(5r. : 10r. = ½); ½ of $ 5.03+ = 2.51+ = 5r.
$2070.35+, Ans.

(14.)

498
(2s. : 1£. = ⅒); ⅒ of 498 = 49£. 16s.
(6d. : 2s. = ¼); ¼ of 49£. 16s. = 12£. 9s.
62£. 5s., Ans.

(16.)

35gal. 2qt. 1pt. : 11gal. 3qt. 1pt. = 3;
$5.83¾ × 3 = $17.51¼, Ans.

(17.)

12yd. 1qr. 2na. : 24yd. 3qr. = ½;
$49.50 × ½ = $24.75, Ans.

(18.)

73bu. 3pk. : 14bu. 3pk. = 5;
17bu. 2pk. 4qt. × 5 = 88bu. 0pk. 4qt., Ans.

(19.)

$9.75 : $3.25 = ⅓;
1T. 2cwt. 2qr. 15lb. × ⅓ = 7cwt. 2qr. 5lb., Ans.

(20.)

4h. : 1h. 20m. = 3;
27m. 3fur. 20rd. × 3 = 82m. 2fur. 20rd., Ans.

SIMPLE PROPORTION.

(Art. **338**, p. **240**.)

3. 16A. : 197A. : : \$720 : \$8865, Ans.

$$\frac{197 \times \overset{45}{\cancel{720}}}{\cancel{16}} = \$8865, \text{ Ans.}$$

4. \$8865 : \$720 : : 137A. : 16A., Ans.

$$\frac{720 \times 137}{8865} = 16\text{A., Ans.}$$

5. 15hhd. : 84hhd. : : \$17595 : \$985.32, Ans.

$$\frac{84 \times 17595}{15} = \$985.32, \text{ Ans.}$$

6. \$12 : \$40 : : \$6 : \$12.

$$\frac{\overset{20}{\cancel{40}} \times \cancel{6}}{\underset{\cancel{2}}{\cancel{12}}} = \$20, \text{ Ans.}$$

7. 15 men : 10 men : : 45 days : 30 days, Ans.

$$\frac{10 \times \overset{3}{\cancel{45}}}{\underset{1}{\cancel{15}}} = 30 \text{ days, Ans.}$$

8. $7 + 9 = 16 : 8 + 4 = 12 : : 12 : 9$.

$$\frac{12 \times 12}{16} = 9, \text{ Ans.}$$

9. 3 men : 9 men : : 17 days : 51 days, Ans.
10. 17 days : 51 days : : 3 men : 9 men; $9 - 3 = 6$ men, Ans.
11. 5½ rods : 160 rods : : 1 rod : 29$\frac{1}{11}$ rods, Ans.
12. \$100 : \$850 : : \$6 : \$51, Ans.
13. \$6 : \$32 : : \$100 : \$533.33⅓, Ans.
14. 20gal. : 180gal. : : 167lb. : 1503lb., Ans.
15. 2ft. : 3ft. : : 75ft. : 112½ft., Ans.
16. \$4.75 : \$160 : : 36 miles : 1212$\frac{12}{19}$ miles, Ans.

17. 8 days : 12 days : : 100 men : 150 men, Ans.
18. $\frac{7}{12}$yd. : $\frac{4}{5}$yd. : : $\$\frac{7}{20}$: \$0.48, Ans.
19. 36A. 3R. : 21A. 3R. 20p. : : \$1260 : \$750, Ans.
20. 10pwt. 18gr. : 2000lb. : : \$10 : $\$446511\frac{27}{43}$, Ans.
21. $4\frac{1}{2}$yd. : $13\frac{1}{2}$yd. : : \$9.75 : \$29.25, Ans.
22. $2\frac{1}{2}$in. : 144in. : : 1in. : $57\frac{3}{5}$in., Ans.
23. $\frac{7}{16}$: $\frac{3}{32}$: : 51£. : 10£. 18s. $6\frac{6}{7}$d., Ans.
24. 13° 10′ 35″ : 360° : : 1 day : 27da. 7h. 43m.+, Ans.
25. 7lb. : 12lb. : : $\$\frac{3}{4}$: $\$1.28\frac{4}{7}$, Ans.
26. \$1.75 : \$213.50 : : 7lb. : 8cwt. 2qr. 4lb., Ans.
27. 7oz. : 7lb. 11oz. : : 30£. : 407£. 2s. $10\frac{2}{7}$d., Ans.
28. \$600 : \$500 : : 6 months : 5 months, Ans.
29. \$7.50 : \$8.00 : : 7oz. : $7\frac{7}{15}$oz., Ans.
30. 1 man : 1000 men : : $3\frac{3}{4} \times 1\frac{7}{8} = \frac{225}{32}$yd. : $7031\frac{1}{4}$yd.
    $1\frac{1}{4}$yd. : 1yd. : : $7031\frac{1}{4}$yd. : 5625yd., Ans.
31. 10h. : 14h. : : 9 days : $12\frac{3}{5}$ days, Ans.
32. 75 — 40 = 35gal. : 500gal. : : 1h. : 14h. 17m. $8\frac{4}{7}$sec., Ans.
33. \$0.56 : \$120.96 : : 1 glove : 216 gloves.
    216 ÷ 12 = 18doz., Ans.
34. 20m. : 1m. : : 1cist. : $\frac{1}{20}$cist.
    40m. : 1m. : : 1cist. : $\frac{1}{40}$cist.
    75m. : 1m. : : 1cist. : $\frac{1}{75}$.
    $\frac{1}{20} + \frac{1}{40} + \frac{1}{75} = \frac{53}{600}$.
    $\frac{53}{600}$cist. : 1cist. : : 1m. : 11m. $19\frac{13}{53}$sec., Ans.
35. 5da. : 1da. : : 1 field : $\frac{1}{5}$ field; 6da. : 1da. : : 1 field : $\frac{1}{6}$ field.
    $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$ field; $\frac{11}{30}$ field : 1 field : : 1da. : $2\frac{8}{11}$da., Ans.
36. 8ft. : 24ft. : : 6 men : 18 men.
    Then 6 days : 12 days : : 18 men : 36 men, Ans.
37. A can do $\frac{1}{20}$ of the labor in 1 day, A and C can do $\frac{1}{12}$ of it; therefore, C alone can do $\frac{1}{12} - \frac{1}{20} = \frac{1}{30}$ of it.
    Then $\frac{1}{30}$ work : 1 work : : 1da. : 30da., Ans.
38. 700 men : 1 man : : 184000lb. : $262\frac{6}{7}$lb.
    Then 5lb. : $262\frac{6}{7}$lb. : : 1 week : 52 weeks 4 days, Ans.
39. $\frac{3}{4} \times 3 \times 25 = 56\frac{1}{4}$lb.; $56\frac{1}{4}$lb. : 3150lb. : : 1 week : 56 weeks, Ans.

40. 8 × 8 = 64in.: 20 × 16 × 144 = 46080in.
64in. : 46080in. : : 1 tile : 720 tiles, Ans.

41. 10 × 9 × 4 = 360 cubic inches in each stone.
80 × 20 × $2\frac{1}{2}$ × 1728 = 6220800 cubic inches in the wall.
360in. : 6220800in. : : 1 stone : 17280 stones, Ans.

42. 1T. 7cwt. 3qr. 20lb. = 2795lb.; 13T. 5cwt. 2qr. = 26550lb.
2795lb. : 26550lb. : : $9.50 : $90.24+, Ans.

43. 61.3lb. : 1lb. : : $44.99 42 : $0.734, Ans.

44. 1hhd. : .15hhd. : : $2.39 : $0.3585, Ans.

45. .75 ton : 1 ton : : $15 : $20, Ans.

46. .5yd. : 6yd. : : 10yd. : 120yd., Ans.

47. 10h. : 12h. : : 15da. : 18da., Ans.

48. 9 months : 5 months : : 450 men : 250 men.
450 — 250 = 200 men, Ans.

49. As the hour and minute hand pass each other 11 times in 12 hours, and as they are together at 12 o'clock, it is evident they will next pass each other in $\frac{1}{11}$ of 12 hours = 1h. 5m. $27\frac{3}{11}$sec., Ans.
11h. : 12h. : : 1h. : 1h. 5m. $27\frac{3}{11}$sec., Ans.

BY ANALYSIS.

50. If A and B can perform a piece of labor in $5\frac{5}{11}$ days, it is evident that in 1 day they would do $\frac{1}{5\frac{5}{11}} = \frac{11}{60}$ of the work. If B and C can do the work in $6\frac{2}{3}$ days, in 1 day they would perform $\frac{1}{6\frac{1}{2}} = \frac{3}{20}$ of the work. If A and C can do the work in 6 days, in 1 day they would perform $\frac{1}{6}$ of it. It then appears that A, B, and C, by laboring each 2 days, will perform $\frac{11}{60} + \frac{3}{20} + \frac{1}{6} = \frac{30}{60} = \frac{1}{2}$ of the work; and therefore, by laboring 1 day each, they would do $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$ of it. And if $\frac{1}{4}$ of the labor be performed in 1 day, it is evident that the whole work will be performed in 4 days, Ans.

If A, B, and C, can do $\frac{1}{4}$ of the work in one day, and A and B can do $\frac{11}{60}$, it is evident that C can do $\frac{1}{4} - \frac{11}{60} =$

$\frac{4}{60} = \frac{1}{15}$ of it in 1 day, and therefore will be 15 days in performing the whole. B and C can do $\frac{3}{20}$ of it in a day; therefore A can do $\frac{1}{4} - \frac{3}{20} = \frac{1}{10}$ of it in a day, or he will be 10 days in doing the whole. A and C can do $\frac{1}{6}$ of the work in a day; therefore B can do $\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ in a day, or, in performing the whole labor, he will be 12 days, Ans.

FORM OF STATEMENT.

$5\frac{5}{11}$ days : 1 day : : 1 work : $\frac{11}{60}$ work = A and B.
$6\frac{2}{3}$ days : 1 day : : 1 work : $\frac{3}{20}$ work = B and C.
6 days : 1 day : : 1 work : $\frac{1}{6}$ work = A and C.
$\frac{1}{2}$ work = $\overline{\text{2 A, 2 B, 2 C} =}$
$\frac{1}{4}$ work = A, B, and C.

$\frac{1}{4} - \frac{11}{60} = \frac{1}{15}$ work : 1 work : : 1 day : 15 days = C.
$\frac{1}{4} - \frac{3}{20} = \frac{1}{10}$ work : 1 work : : 1 day : 10 days = A.
$\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ work : 1 work : : 1 day : 12 days = B.
$\frac{1}{4}$ work : 1 work : : 1 day : 4 days = A, B, and C.

(Art. **339**, p. **243.**)

(2.)

246A. 1R. 32p. = 39432p.; 3 + 4 + 5 = **12.**

$$\left.\begin{array}{l}12 : 3 : : 39432\text{p.} : \ 61\text{A. } 2\text{R. } 18\text{p.}\\ 12 : 4 : : 39432\text{p.} : \ 82\text{A. } 0\text{R. } 24\text{p.}\\ 12 : 5 : : 39432\text{p.} : 102\text{A. } 2\text{R. } 30\text{p.}\end{array}\right\}\text{Ans.}$$

(3.)

$$\begin{array}{l}4\frac{1}{4} = 4\frac{5}{20} = \frac{85}{20};\\ 6\frac{1}{2} = 6\frac{10}{20} = \frac{130}{20};\\ 6\frac{3}{5} = 6\frac{12}{20} = \frac{132}{20};\\ 7 \ = \underline{7} \quad = \underline{\frac{140}{20}};\\ \quad 24\frac{7}{20} = \frac{487}{20}.\end{array}$$

$$\left.\begin{array}{l}\frac{487}{20} : \frac{85}{20} : : 319 : 55\frac{330}{487}\\ \frac{487}{20} : \frac{130}{20} : : 319 : 85\frac{75}{487}\\ \frac{487}{20} : \frac{132}{20} : : 319 : 86\frac{226}{487}\\ \frac{487}{20} : \frac{140}{20} : : 319 : 91\frac{343}{487}\end{array}\right\}\text{Ans.}$$

4. Gold 9 + silver $\frac{1}{2}$ + copper $\frac{1}{2}$ = 10.

$$\left.\begin{array}{l}10:9::1\text{oz. }1\text{pwt. }12\text{gr.}:19\text{pwt. }8\tfrac{2}{5}\text{gr.}=\text{gold.}\\10:\tfrac{1}{2}::1\text{oz. }1\text{pwt. }12\text{gr.}:\ 1\text{pwt. }1\tfrac{4}{5}\text{gr.}=\text{silver.}\\10:\tfrac{1}{2}::1\text{oz. }1\text{pwt. }12\text{gr.}:\ 1\text{pwt. }1\tfrac{4}{5}\text{gr.}=\text{copper.}\end{array}\right\}\text{Ans.}$$

5. Silver 9 + copper 1 = 10; 192gr. × 20 = 3840gr.

10 : 9 : : 3840gr. : 7oz. 4pwt. silver.
10 : 1 : : 3840gr. : 16pwt. copper.

6. First, 1; second, $1\frac{1}{3} \times 1 = 1\frac{1}{3}$; third, $1\frac{2}{3} \times 1\frac{1}{3} = 2\frac{2}{9}$;
$1 = \frac{9}{9}$; $1\frac{1}{3} = \frac{12}{9}$; $2\frac{2}{9} = \frac{20}{9}$; $\frac{9}{9} + \frac{12}{9} + \frac{20}{9} = \frac{41}{9}$.

41 : 9 : : \$600 : \$131$\frac{29}{41}$ = first man receives.
41 : 12 : : \$600 : \$175$\frac{25}{41}$ = second man receives.
41 : 20 : : \$600 : \$292$\frac{28}{41}$ = third man receives.
Proof, \$131$\frac{29}{41}$ + \$175$\frac{25}{41}$ + \$292$\frac{28}{41}$ = \$600.

7. 98 + 86 + 64 = 248.

$$\left.\begin{array}{l}248\text{ tons}:98\text{ tons}::93\text{ tons}:36\tfrac{3}{4}\text{ A's tons.}\\248\text{ tons}:86\text{ tons}::93\text{ tons}:32\tfrac{1}{4}\text{ B's tons.}\\248\text{ tons}:64\text{ tons}::93\text{ tons}:24\ \ \text{C's tons.}\end{array}\right\}\text{Ans.}$$

8. 15 + 32 = 47 miles; 25 + 32 = 57 miles.

$$\left.\begin{array}{l}47\text{mi.}:15\text{mi.}::160\text{mi.}:51\tfrac{3}{47}\text{mi. A.}\\57\text{mi.}:25\text{mi.}::160\text{mi.}:70\tfrac{10}{57}\text{mi. B.}\end{array}\right\}\text{Ans.}$$

COMPOUND PROPORTION.

(Art. **340**, p. **245.**)

(2.)

$$\left.\begin{array}{r}30\text{ days}:20\text{ days}\\9\text{ hours}:12\text{ hours}\end{array}\right\}::117\text{ miles}:104\text{ miles, Ans.}$$

$$\frac{\overset{2}{\cancel{20}} \times \overset{4}{\cancel{12}} \times 117}{\underset{3}{\cancel{9}} \times \underset{3}{\cancel{30}}} = 104\text{ miles, Ans.}$$

(3.)

$$\left.\begin{array}{lcl} 24\text{ men} & : & 6\text{ men} \\ 8\text{ hours} & : & 9\text{ hours} \\ 20\text{ feet} & : & 200\text{ feet} \\ 6\text{ feet} & : & 8\text{ feet} \\ 4\text{ feet} & : & 6\text{ feet} \end{array}\right\} :: 16\text{ days} : 90\text{ days, Ans.}$$

OPERATION.

$$\frac{6\times 9\times 200\times 8\times 6\times 16}{24\times 8\times 20\times 6\times 4}\begin{array}{l} =829440 \\ =92160 \end{array} \quad \frac{829440}{92160} = 90\text{ days, Ans.}$$

(4.)

$$\left.\begin{array}{lcl} \$100 & : & \$500 \\ 12\text{ months} & : & 4\text{ months} \end{array}\right\} :: \$6 : \$10,\text{ Ans.}$$

(5.)

$$\left.\begin{array}{lcl} \$6 & : & \$10 \\ 4\text{ months} & : & 12\text{ months} \end{array}\right\} :: \$100 : \$500,\text{ Ans.}$$

(6.)

$$\left.\begin{array}{lcl} \$500 & : & \$100 \\ \$6 & : & \$10 \end{array}\right\} :: 12\text{ months} : 4\text{ months, Ans.}$$

(7.)

$$\left.\begin{array}{lcl} \$500 & : & \$100 \\ 4\text{ months} & : & 12\text{ months} \end{array}\right\} :: \$10 : \$6,\text{ Ans.}$$

(8.)

$$\left.\begin{array}{lcl} 9\text{ comp.} & : & 5\text{ comp.} \\ 10\text{ hours} & : & 11\text{ hours} \\ 25\text{ sheets} & : & 36\text{ sheets} \\ 24\text{ pages} & : & 16\text{ pages} \\ 44\text{ lines} & : & 50\text{ lines} \\ 40\text{ letters} & : & 45\text{ letters} \end{array}\right\} :: 16\text{ days} : 12\text{ days, Ans.}$$

(9.)

$$\left.\begin{array}{lcl} 60\text{ men} & : & 12\text{ men} \\ 30\text{ feet} & : & 300\text{ feet} \\ 6\text{ feet} & : & 8\text{ feet} \\ 3\text{ feet} & : & 6\text{ feet} \\ 8\text{ hours} & : & 12\text{ hours} \end{array}\right\} :: 15\text{ days} : 120\text{ days, Ans.}$$

CANCELLING.

$$\frac{12 \times \overset{10}{\cancel{300}} \times \cancel{8} \times \cancel{6} \times \cancel{12} \times \overset{1}{\cancel{15}}}{\underset{4}{\cancel{60}} \times \underset{1}{\cancel{30}} \times \cancel{6} \times \underset{1}{\cancel{3}} \times \cancel{8}} = 120 \text{ days, Ans.}$$

(10.)

$$\left.\begin{array}{l} 16 \text{ horses} : 32 \text{ horses} \\ 24 \text{ days} \;\; : 48 \text{ days} \end{array}\right\} :: 84 \text{ bushels} : 336 \text{ bushels, Ans.}$$

CANCELLING.

$$\frac{\overset{2}{\cancel{32}} \times \overset{2}{\cancel{48}} \times 84}{\underset{1}{\cancel{16}} \times \underset{1}{\cancel{24}}} = 336 \text{ bushels, Ans.}$$

(11.)

$$\left.\begin{array}{l} 575 \text{ pounds} : 775 \text{ pounds} \\ 150 \text{ miles} \;\; : \; 64 \text{ miles} \end{array}\right\} :: \$24.58 : \$14.135 + \text{Ans.}$$

(12.)

$7\frac{1}{4}$oz. = 7.25; 4s. 2d. = 50d.; 5s. 6d. = 66d.; 1s. 2d. = 14d.

$$\left.\begin{array}{l} 66\text{d.} \;\; : 50\text{d.} \\ 4\frac{3}{4}\text{d.} : 14\text{d.} \end{array}\right\} :: 7.25\text{oz.} : 16\tfrac{118}{627}\text{oz., Ans.}$$

CANCELLING.

$$\frac{\overset{2}{\cancel{50}} \times \overset{7}{\cancel{14}} \times 7.25}{\underset{33}{\cancel{66}} \times \underset{19}{\cancel{4.75}}} = 16\tfrac{118}{627}\text{oz., Ans.}$$

(13.)

$$\left.\begin{array}{rlrl} 24 & \text{men} & : 496 & \text{men} \\ 9 & \text{hours} : & 11 & \text{hours} \\ 7 & \text{hard.} : & 4 & \text{hard.} \\ 465 & \text{feet} & : 337\frac{1}{2} & \text{feet} \\ 3\frac{2}{3} & \text{feet} & : \; 5\frac{3}{5} & \text{feet} \\ 2\frac{1}{3} & \text{feet} & : \; 3\frac{1}{2} & \text{feet} \end{array}\right\} :: 5\tfrac{1}{2} \text{ days} : 132 \text{ days, Ans.}$$

CONJOINED PROPORTION.

(ART. **341**, p. **247.**)

(2.)

FORM OF STATEMENT.

100A. Bradford = 120A. Haverhill;
50A. Haverhill = 65A. Methuen;
150A. Methuen.

OPERATION.

$$\begin{array}{r} 100 \times 50 \times 150 = \\ 120 \times 65 = \end{array} \frac{750000}{7800} = 96\tfrac{2}{13}\text{A., Ans.}$$

(3.)

FORM OF STATEMENT.

10lb. cheese = 7lb. butter;
11lb. butter = 2bu. corn;
11bu. corn = 8bu. rye;
4bu. rye = 1 cord wood;
10 cords wood.

OPERATION.

$$\begin{array}{r} 10 \times 11 \times 11 \times 4 \times 10 = \\ 7 \times 2 \times 8 \times 1 = \end{array} \frac{48400}{112} = 432\tfrac{1}{7}\text{lb., Ans.}$$

(4.)

STATEMENT.

12 men = 25 women.
5 women = 6 boys.
75 boys.

OPERATION.

$$\frac{12 \times 5 \times 75}{25 \times 6} = 30 \text{ men, Ans.}$$

(5.)

STATEMENT.

6 gallons = 5 imperial gal.
10 imp. gal. = 6 Velts.
26 Velts = 16 Vedros.
63 gallons.

OPERATION.

$$\frac{5 \times 6 \times 16 \times 63}{6 \times 10 \times 26} = \frac{30240}{1560} = 19\tfrac{5}{13} \text{ Vedros, Ans.}$$

(6.)

STATEMENT.

7 Boston = 8 Buffalo.
10 Buffalo = 14 Chicago.
21 Chicago = 25 Davenport.
120 Davenport.

OPERATION.

$$\frac{7 \times 10 \times 21 \times 120}{8 \times 14 \times 25} = \frac{1764}{2800} = 630 \text{ bushels, Ans.}$$

(7.)

STATEMENT.

24s. Massachusetts = 32s. New York.
48s. New York = 45s. Pennsylvania.
15s. Pennsylvania = 10s. Canada.
100s. Massachusetts.

OPERATION.

$$\frac{32 \times 45 \times 10 \times 100}{24 \times 48 \times 15} = \frac{1440000}{1728} = 83\tfrac{1}{3}\text{s., Ans.}$$

MISCELLANEOUS EXAMPLES IN PROPORTION.

(PAGE 248.)

(1.)

BY ANALYSIS.

$27 \times 7 = 189 =$ miles A is ahead of B.

$36 - 27 = 9$ miles that B gains each day on A. If, therefore, 9 miles are gained in one day, it will require, to gain 189 miles, $189 \div 9 = 21$ days, Ans.

FORM OF STATEMENT.

$36 - 27 = 9$m. : 189m. : : 1 day : 21 days, Ans.

(2.)

BY ANALYSIS.

2s. 3d. = 27d., price obtained for the coffee. It is evident that 27d. is $\frac{135}{100}$ of the cost; therefore, $\frac{100}{135}$ of 27d. = 20d. was the cost, Ans.

FORM OF STATEMENT.

135d. : 100d. : : 27d. : 20d., Ans.

(3.)

FORM OF STATEMENT.

$2000 \times 12 \times 7 \times 14 = 2352000 \div 16 = 147000$lb. whole quantity.

$105 \times 200 = 21000$lb. wholly spoiled.

$147000 - 21000 = 126000$lb. left to subsist on.

$2000 \times 12 \times 7 = 168000$ rations.

$2016000 \div 168000 = 12$oz. for each man per day, Ans.

(4.)

FORM OF STATEMENT.

$2000 \times 12 \times 7 \times 12 = 2016000 \div 16 = 126000$lb. quantity subsisted on.

$126000 \div 6 = 21000$lb. spoiled.

$21000 \times 7 = 147000$lb., the whole quantity, Ans.

(5.)

FORM OF STATEMENT.

2000 × 12 × 7 × 14 = 2352000 ÷ 16 = 147000lb., whole weight.

2000 × 12 × 7 × 12 = 2016000 ÷ 16 = 126000lb. left to subsist on, Ans.

(6.)

FORM OF STATEMENT.

$3.00 × 60 = $180, price given for the Holland.
$4.00 × 60 = $240, price obtained for it.
$180 : $240 : : $240 : $320, Ans.

7. $1\frac{1}{2}$lb. : 20lb. : : $ 1 : $ $13.33\frac{1}{3}$.
15lb. : 62lb. : : $ 12 : $ 49.60.
$ $13.33\frac{1}{3}$ + $ 49.60 = $ $62.93\frac{1}{3}$, price of the tea.
20lb. + 62lb. = 82lb., whole quantity of the tea.
3lb. : 82lb. : : $ 4 : $ $109.33\frac{1}{3}$; $ $109.33\frac{1}{3}$ − $ $62.93\frac{1}{3}$ =
[$ 46.40, Ans.

8. 2 fur. 3rd. 3yd. = 1824 feet.
66ft. : 64ft. : : 1824ft. : $1768\frac{8}{11}$ft.
$1768\frac{8}{11}$ft. × 4 = $7074\frac{10}{11}$ feet, Ans.

9. 7 cows : 3 cows : : 5 oxen : $2\frac{1}{7}$ oxen.
2 + $2\frac{1}{7}$ = $4\frac{1}{7}$ oxen; $4\frac{1}{7}$ oxen : 5 oxen : : 87 days : 105 days, Ans.

10. 8mo. : 6 − 4 = 2mo. : : 360 men : 90 men.
360 − 90 = 270 men, Ans.

11. $10\frac{1}{4}$ = 10.25; $1\frac{7}{8}$ = 1.875; 100 − 5 = 95; 100 : 95 : : 1.875yd. : 1.78125yd.; 100 : 95 : : 1.78125yd. : 1.6921875yd.; 1.6921875 : 1 : : 10.25yd. : $6\frac{62}{1083}$yd., Ans.

12. 130A. 2R. 20p. = 20900p.; 20900 × 4 = 83600;
100A. 0R. 30p. = 16030p.; 16030 × 5 = 80150;
83600p. : 80150p. : : $6537.50 : $6267.71+, Ans.

13. 192 tons 17cwt. 16lb. = 385716lb.; 800 + 101 + 56 + 43 = 1000.
1000 parts : 800 parts : : 385716lb. : $308572\frac{4}{5}$lb., copper.

1000 parts : 101 parts : : 385716lb. : $38957\frac{79}{250}$lb. tin.
1000 parts : 56 parts : : 385716lb. : $21600\frac{12}{125}$lb. zinc.
1000 parts : 43 parts : : 385716lb. : $16585\frac{197}{250}$lb. lead.

14. 81 w. pine : 42 w. pine : : 1 oak : $\frac{42}{81}$ oak.
7 : 8 : : $\frac{42}{81}$: $\frac{48}{81}$; that is, oak is to pitch pine as 48 to 81; $\frac{42}{81} + \frac{48}{81} = \frac{90}{81}$; that is, $\frac{90}{81}$ cords of oak are equal to 2 cords of pine; therefore, $\frac{90}{81}$ oaks : 20 oak : : 2 pine : 36 pine.
$36 \div 2 = 18$ cords of each, Ans.

15. $63 \times \frac{4}{5} = 50\frac{2}{5}$ gallons.
85gal. : $50\frac{2}{5}$: : \$116.95 : \$$69.34\frac{38}{85}$, Ans.

16. 4 cows : 15 cows : : 3 horses : $11\frac{1}{4}$ horses.
1 sheep : 7 sheep : : $\frac{1}{3}$ cow : $\frac{7}{3}$ cows.
1 cow : $\frac{7}{3}$ cows : : $\frac{3}{4}$ horse : $1\frac{3}{4}$ horses.
$10 + 11\frac{1}{4} + 1\frac{3}{4} = 23$ horses.

$$\left.\begin{array}{r}\text{Then as 23 horses : 8 horses}\\ 3\frac{1}{2}\text{tons : }4\frac{9}{10}\text{ tons}\end{array}\right\} :: 30\text{ days} : 14\frac{98}{161}\text{ days, Ans.}$$

17. 25 : 14 : : 1 : $\frac{14}{25}$.
7 : 5 : : 1 : $\frac{5}{7}$.
5 : 2 : : 1 : $\frac{2}{5}$.
$\frac{14}{25} \times \frac{5}{7} \times \frac{2}{5} = \frac{140}{875} = \frac{4}{25}$; $2000 \div \frac{4}{25} = 12500$ pounds of turnips, are equal to 2000 pounds of potatoes; 14 : 25 : : 35 cts. for beets : $62\frac{1}{2}$ cts., the proportionate price of potatoes; that is, 80 cts. for potatoes is as much dearer than 35 cts. for beets, as 80 is more than $62\frac{1}{2} = 17\frac{1}{2}$ cts.; again, 5 : 2 : : 25 cts. for carrots : 10 cts., the proportionate price for turnips; but 20 cts. for turnips is 10 cts. dearer than the proportionate price; hence, 20 cts. for turnips is 10 cts. dearer than 25 cts. for carrots, Ans.

18. A travels $22\frac{1}{2}$ days at the rate of 18 miles per hour $= 405$ miles. He not only travels as far as B, but the distance that B would travel in twice 9 days $= 18$ days; therefore, B, to travel the whole distance which A has travelled, would require $22\frac{1}{2} + 18 = 40\frac{1}{2}$ days; therefore, $405 \div 40\frac{1}{2} = 10$ miles per day, Ans.

19. By the conditions of the question, 2 men bear the expenses of the ride for 20 miles, 3 men for 52 miles, 4 men for 42 miles, and 5 men for 30 miles; therefore, each of the "two men" will pay $\frac{1}{2}$ the hire for 20 miles, $+ \frac{1}{3}$ for 52 miles, $+ \frac{1}{4}$ for 42 miles, $+ \frac{1}{5}$ for 30 miles, $=$ the hire for $43\frac{5}{6}$ miles.

The "two men" will pay for $43\frac{5}{6} \times 2 =$ hire for $87\frac{2}{3}$ miles.

A will pay $\frac{1}{3}$ for 52, $+ \frac{1}{4}$ for 42, $+ \frac{1}{5}$ for 30 $=$ " $33\frac{5}{6}$ miles.

B will pay $\frac{1}{4}$ for 42, $+ \frac{1}{5}$ for 30 $=$ " $16\frac{1}{2}$ miles.

C will pay $\frac{1}{5}$ for 30 $=$ " 6 miles.

$\overline{144}$

$144 : 87\frac{2}{3} :: \$25 : \$7.609\frac{103}{108}$, share of each of the "two men."

$144 : 33\frac{5}{6} :: \$25 : \$5.873\frac{91}{108}$, A's share.

$144 : 16\frac{1}{2} :: \$25 : \$2.864\frac{7}{12}$, B's share.

$144 : 6 :: \$25 : \$1.041\frac{2}{3}$, C's share.

PERCENTAGE.

(Art. 346, p. 252.)

1.	.19	6.	$.77\frac{1}{6}$
2.	.27	7.	1.06
3.	.135	8.	1.07
4.	.0175	9.	3.05
5.	.074	10.	9.998

Art. 347, p. 252.)

2.	75bu.	8.	$ 990
3.	15cwt.	9.	48bbl.
4.	45 tons.	10.	60hhd.
5.	$ 375	11.	$ 0.25
6.	665chal.	12.	15.12lb.
7.	.8 miles.		

(13.)

$$\begin{array}{r} 900 \\ .08 \\ \hline 72.00 \end{array}$$

$$\begin{array}{r} 900 \\ 72 \\ \hline 828 \\ .50 \\ \hline 414.00 \end{array}\text{, Ans.}$$

(14.)

$$\begin{array}{r} \$\,1728 \\ .15 \\ \hline 8640 \\ 1728 \\ \hline \$\,259.20 \end{array}$$

$$\begin{array}{r} \$\,1728 \\ 259.20 \\ \hline \$\,1468.80 \end{array}\text{, Ans.}$$

(15.)

$$\begin{array}{r} \$\,25000 \\ .40 \\ \hline \$\,10000.00 \end{array} = \text{wife's share.}$$

$$\begin{array}{r} \$\,15000 \\ .30 \\ \hline \$\,4500.00 \end{array} = \text{son's share.}$$

$$\begin{array}{r} \$\,25000 \\ 10000 \\ \hline \$\,15000 \end{array}$$

$$\begin{array}{r} \$\,10000 \\ 4500 \\ \hline \$\,14500 \end{array}$$

$$\begin{array}{r} \$\,25000 \\ 14500 \\ \hline 10500 \\ 60 \\ \hline 3)\overline{10440} \\ \$\,3480 \end{array} = \text{each daughter's share.}$$

Ans. Wife, \$ 10000; son, \$ 4500; each daughter, \$ 3480.

(ART. **348,** p. **253.**)

2. 12½ per cent.
3. 10 per cent.
4. 5⅝ per cent.
5. 25 per cent.
6. $\$\,36.00 \times .25 = \$\,9.00$; *$\$\,9.00 \div 150 = .06$*, Ans.
7. 75 per cent.
8. 37½ per cent.
9. 50 per cent.
10. 20 per cent.
11. 20 per cent.

12. $100 \div 140 = .71\frac{3}{7}$, the per cent. that the grammar class is of the geography class. $100 - 71\frac{3}{7} = 28\frac{4}{7}$ per cent., Ans.

13. $4 \div 32 = .12\frac{1}{2}$, Ans.

14. $22 + 3 = 25$; $25 : 22 :: 100 : 88$ per cent. of copper. $25 : 3 :: 100 : 12$ per cent. of nickel.

15. 25 pear-trees are $\frac{25 \times 100}{250} = 10$ per cent. of the whole. 40 per cent. + 12 per cent. + 8 per cent. + 10 per cent. = 70 per cent.; $100 - 70 = 30$ per cent. peach-trees, Ans.

(Art. 349, p. 255.)

2.	250.	5.	$2\frac{3}{16}$.
3.	$203\frac{1}{8}$.	6.	$ 13.
4.	600.	7.	$ 140.

8. $17 + 6 = 15\frac{1}{3}$ per cent. of the whole number; $\frac{23 \times 100}{15\frac{1}{3}}$ [= 150, Ans.

9. $.33\frac{1}{3}$ of $.45 = .15$; $\frac{450 \times 100}{15} = 3000$, Ans.

10. 3m. 1fur. 1rd. = 1001rd.; $\frac{1001 \times 100}{12\frac{1}{2}} = 8008$rd. = 25m. [0fur. 8rd., Ans.

11. $\frac{19 \times 100}{17\frac{1}{7}} = 110\frac{5}{6}$; $110\frac{5}{6} - 19 = 91\frac{5}{6}$, Ans.

12. $\frac{\$36.89 \times 100}{13\frac{2}{9}} = \$270$; $\frac{\$36.89 \times 100}{16\frac{2}{3}} = \$221.34$; [$\$279.00 - \$221.34 = \$57.66$, Ans.

(Art. 350, p. 256.)

3.	120	5.	$ $20.96\frac{24}{31}$
4.	365	6.	$ $34.27\frac{119}{803}$

7. $\$24 \div 1.33\frac{1}{3} = \$18$, Ans.

8. 2m. 6fur. 24rd. = 904rd.; $904 \div .87\frac{1}{2} = 1033\frac{1}{7}$rd. = 3m. 1fur. $33\frac{1}{7}$rd., Ans.

9. $\$123.16 \div .84 = \$146.61$, Ans.

10. 6yd. $1\frac{1}{10}$qr. ÷ .25 = 25yd. $0\frac{4}{10}$qr. = quantity cut off; 25yd. $0\frac{4}{10}$qr. + 6yd. $1\frac{1}{10}$qr. = 31yd. 1qr. 2na., Ans.

11. 279 ÷ .90 = 310, taken; 310 — 279 = 31, lost, Ans.

12. 100 — $86\frac{1}{2}$ = $13\frac{1}{2}$ per cent., which A had left.
100 — 55 = 45 per cent., which B had left.
Hence, $.13\frac{1}{2} + .45 = .58\frac{1}{2}$, which they both have left of 2.00 =

$$\$\,36.85\tfrac{1}{2}\,;\ \frac{\$\,36.85\frac{1}{2} \times 1.00}{58\frac{1}{2}} = \$\,63\,;$$

100 : $13\frac{1}{2}$: : \$63 : \$ $8.50\frac{1}{2}$, what A had left.
100 : 45 : : \$63 : \$ 28.35, what B had left.

MISCELLANEOUS EXAMPLES.

(PAGE 257.)

1.	$117\frac{1}{25}$	7.	.07
2.	$174\frac{14}{75}$	8.	$81\frac{1}{4}$.
3.	351	9.	\$ 66.
4.	$2\frac{7}{10}$	10.	$5\frac{35}{93}$.
5.	$316\frac{2}{3}$	11.	\$ 22.
6.	40	12.	$117\frac{6}{67}$lb.

13. 6 × 30 × 12 = 2160; 2160 × .15 = 324, Ans.

14. 12 : 100 : : \$ 0.69 : \$ 5.75, Ans.

15. 5400 + 6000 = what was excavated in the first two weeks = 11400; 40500 — 11400 = 29100; 29100 × .25 = 7275 = what was excavated in the third week; 11400 + 7275 = 18675; 40500 ÷ 2 = 20250; 20250 — 18675 = 1575 cubic feet, Ans.

16. $.25 \times .50 = .12\frac{1}{2}$; $\frac{.12\frac{1}{2} \times 100}{75} = .16\frac{2}{3}$, Ans.

17. \$ 0.50 × .80 = \$ 0.40, the cost of a gallon; \$ 0.40 × .25 = \$ 0.10; \$ 0.40 + \$ 0.10 = \$ 0.50, Ans.

18. 20 × .04 = .8; 1 × .05 = .05; 20 — .8 = 19.2 = the length after shrinking; 1 — .05 = .95 = width after shrinking; 19.2 × .95 = $18\frac{6}{25}$; 20 — $18\frac{6}{25}$ = $1\frac{19}{25}$yd., Ans.

19. 15 per cent. of 1.00 $= .15$ $=$ daughter's;
10 per cent. of .85 $= .085 + \frac{1}{10}$ of an acre $=$ son's;
25 per cent. of $.765 - \frac{1}{10} = \underline{.19125 - \frac{1}{40}}$ of an acre $=$ wife's;
$.42625 + \frac{3}{40} =$ the shares of the wife, the son, and the daughter; and $100 - .42625$ $=$ the remainder $+ \frac{3}{40}$ of an acre which is not included in the .42625. Hence $1.00 - .42625 = .57375 = 39 \times 5$ $= 195 + \frac{3}{40}$ of an acre $= 195\frac{3}{40}$ acres.
$.57375 : 1.00 :: 195\frac{3}{40}$ acres : 340 acres, Ans.

20. $30{,}500{,}000 \times 1.34\frac{1}{2} = 41{,}022{,}500$, Ans.

21. Let the English $= 100$ per cent.;
$100 + 33\frac{1}{3} = 133\frac{1}{3}$ per cent. $=$ French;
$133\frac{1}{3} + 8\frac{1}{3}$ of $133\frac{1}{3} = 144\frac{4}{9}$ per cent. $=$ Turks;
$144\frac{4}{9} - 100 = 44\frac{4}{9}$ per cent. $= 1600$. Hence
$44\frac{4}{9} : 100 :: 1600 : 3600 =$ English;
$100 : 133\frac{1}{3} :: 3600 : 4800 =$ French;
$100 : 144\frac{4}{9} :: 3600 : \underline{5200} =$ Turks;
13600 the whole number.
$13600 : 3600 :: 100 : 26\frac{8}{17}$, percentage of English;
$13600 : 4800 :: 100 : 35\frac{5}{17}$, percentage of French;
$13600 : 5200 :: 100 : 38\frac{4}{17}$, percentage of Turks.

22. $\$7.25 \times .10 = \$0.725$; $\$7.25 - \$0.725 = \$6.525$; $\$7.25 + \$0.725 = \$7.975$; $\$7.975 - \$6.525 =$ $\$1.45$; $\$6.525 : \$1.45 :: 100 : 22\frac{58}{261} =$ the per cent. of the proceeds of the flour above the cost; $560 \times \$1.45$ $= \$812.00 =$ profits, Ans.

23. $87500 \div 1.25 = 70000$, Ans.

24. Let the cost of the horse $= 100$; but the horse cost $62\frac{1}{2}$ per cent. as much as the buggy; hence $62\frac{1}{2} : 100 :: 100 :$ $160 =$ the proportionate percentage of the buggy; $100 -$ $70 = 30$, the proportionate percentage of the harness; $100 + 160 + 30 = 290$;
$290 : 100 :: \$500 : \$172\frac{12}{29} =$ horse;
$290 : 160 :: \$500 : \$275\frac{25}{29} =$ buggy;
$290 : 30 :: \$500 : \$\ 51\frac{21}{29} =$ harness.

INTEREST.

(ART. **354**, p. **262.**)

2.	$40.50
3.	$476.144
4.	$161.33
5.	$888.546
6.	$108.587
7.	$44.442
8.	$540.000
9.	$1.30
10.	$1.275
11.	$0.6867
12.	$31.85
13.	$116.99
14.	$40.10
15.	$0.0758
16.	$0.9739
17.	$14.607
18.	$0.227
19.	$31.146
20.	$18.674
21.	$193.09
22.	$\$3.37\frac{1}{2}$
23.	$26.343
24.	$0.584
25.	$17.438

26.

y.	mo.	da.
1863	6	27
1852	0	11
11	6	16

$\$76.895 \times .692\frac{2}{3} = \$53$-
[.262, Ans.

27.

y.	mo.	da.
1859	9	19
1856	6	4
3	3	15

$\$98.25 \times .197\frac{1}{2} = \$19.$
[.404, Ans.

28.

y.	mo.	da.
1860	6	18
1836	1	19
24	4	29

$\$22.763 \times 1.464\frac{5}{6} = \$33$-
[.344, Ans.

29.

y.	mo.	da.
1859	9	12
1855	0	7
4	9	5

$\$175.07 \times .285\frac{5}{6} = \$50$-
[.04, Ans.

30.

y.	mo.	da.
1854	0	11
1852	11	6
1	1	5

$\$197.285 \times .065\frac{5}{6} = \$12$-
[.987, Ans.

31. $\$4377.15 \times .18 = \$787.887$; $\$787.887 + \$4377.15 = \$5165.037$ = the amount, Ans.

32. $\$444.60 \times .33 = \$146.718$, Ans.

(ART. 355, p. 265.)

(2.)

	$16.75
	.07
Interest for 1 year	=1.1725
Int. for 4 mo., $\frac{1}{3}$ of a year	= .3908
Int. for 2 mo., $\frac{1}{2}$ of 4 mo.	= .1954
Int. for 1 mo., $\frac{1}{2}$ of 2 mo.	= .0977
Int. for 15 da., $\frac{1}{2}$ of 1 mo.	= .0488
Int. for 2 da., $\frac{1}{15}$ of 1 mo.	= .0065
Int. for 7 mo. and 17 da.	= $0.7392, Ans.

(3.)

y.	mo.	d.
1852	11	7
1852	3	17
	7	20

	$11.105
	.07
Int. for 1 year	= .77735
Int for 6 mo., or $\frac{1}{2}$ of 1 yr.	= .38867
Int. for 1 mo., or $\frac{1}{6}$ of 6 mo.	= .06477
Int. for 15 da., or $\frac{1}{2}$ of 1 mo.	= .03238
Int. for 5 da., or $\frac{1}{3}$ of 15 da.	= .01079
Int. for 7 mo. and 20 da.	= $0.49661, Ans.

4.

y.	mo.	d.
1854	7	30
1853	0	2
1	7	28

$12.69 × .099$\frac{2}{3}$ = $1.2647; $1.2647 ÷ 6 = $0.2107, × 7 = $1.475, Ans.

5. $871.111

6. $504.64

7. $0.70

8.

y.	mo.	d.
1860	0	11
1852	1	7
7	11	4

$17869.75 × .475$\frac{2}{3}$ = $8500.0444; $8500.0444 ÷ 6 = $1416.67406, × 5 = $7083.3703, Ans.

9. $21.78

10. $0.149

11.

	y.	mo.	d.
	1863	11	15
	1861	10	11
	2	1	4

$35.61 × .125$\frac{2}{3}$ = $4.474, Ans.

12. $10.139

13.

	y.	mo.	d.
	1860	6	17
	1854	4	7
	6	2	10

$1728.19
.371$\frac{2}{3}$
6 × 4 = 24)642.31061$\frac{2}{3}$
$26.762, [Ans.

14. $397.16
.05$\frac{1}{2}$
198580
19858
Int. for 1 yr. = 21.8438
Int. for $\frac{1}{2}$ of 1 yr. = 10.9219
Int. for 1 da., or
$\frac{1}{180}$ of 6 mo. = .0608
Ans. $32.8265

15. $100.25
.014$\frac{5}{6}$
At 6 per cent. = 1.48704$\frac{1}{6}$
At 1 per cent. = .24784
4
At 4 per cent. = .99136
$100.25
Amount, $101.241

16. $51.17
.04
2)$2.0468
Int. for 6 mo. = 1.0234
Int. for 3 mo. = .5117
Int. for 15 da., or $\frac{1}{6}$ of
3 mo. = .08528
Int. for 1 da. = .00568
Int. for 13 da. = .07384
Ans. 1.6999

17. $42.20 × .062$\frac{2}{3}$ = $2.6445; 4$\frac{1}{2}$ × $2.6445 ÷ 6 = 1.983, Ans.

18. $0.975

19.

	y.	mo.	d.
	1852	6	3
	1849	10	27
	2	7	6

$96.84
.07$\frac{1}{2}$
67788
4842
Int. for 1 yr. = 7.2630
2
Int. for 2 yr. = 14.5260
Int. for 6 mo. = 3.6315
Int. for 1 mo. = .6052
Int. for 6 da., or
$\frac{1}{5}$ of 1 mo. = .1210
Ans. $18.8837

20. $225.925

	y.	mo.	d.
21.	1857	5	19
	1853	6	29
	3	10	20

$71.09,1

.233$\frac{1}{3}$

213273

213273

142182

23697

Interest at 6 per cent.* } $16.587900

2

Ans. $33.17,5800

* The interest will be twice this amount at 12 per cent.

22. $444.163

23. $7.864

24. $0.886

	y.	mo.	d.
25.	1857	1	18
	1829	4	16
	27	9	2

$36.72 × 1.665$\frac{1}{3}$ = $61.15104; $61.15104 × 7 ÷ 6 = $71.342, Ans.

26. $8.628

27. $515.60

28. $3.167

29. $9750 × .08 = $780.00, [Ans.

30. $9162×3×1$\frac{1}{2}$=$412.29, [Ans.

31. $95.833

32. $58.00

33. $246.295

34. $10000

.15$\frac{1}{3}$

$150000

3333$\frac{1}{3}$

Int. at 6 per cent. = 1533.33\frac{1}{3}$

7

6)10733.33$\frac{1}{3}$

Int. at 7 per cent. = $1788.88

$1788.88 — $1533.33 = $255.55, Ans.

(ART. **356**, p. **268**.)

2. $96.00

Int. for 60 da. = .96

Int. for 30 da. = .48

Int. for 3 da. = .048

6).528

.088

7

Int. at 7 per cent. = $0.616 [Ans.

3. $320.40

Int. for 60 da. = $\frac{1}{100}$ of the principal = 3.204

Int. for 3 da. = $\frac{1}{20}$ of 60 da. = .1602

6)3.3642

.5607

5

Int. at 5 per cent. = $2.8035 [Ans.

4.

$$\begin{array}{lr}
 & \$131.20 \\ \hline
\text{Int. of 60 da.} & =1.3120 \\
\text{Int. of 60 da.} & =1.3120 \\
\text{Int. of 3 da.} & =\ \ .0656 \\ \hline
 & \text{Ans. } \$2.6896
\end{array}$$

5.	$0.345
6.	$245.65
7.	$1.681
8.	$0.263
9.	177.029\frac{13}{18}$
10.	166.681\frac{2}{3}$
11.	$569.70
12.	$35.295
13.	$72.888
14.	$2.223

15.

$$\begin{array}{r}
\$144.50 \\
.024 \\ \hline
6)3.46800 \\ \hline
578 \\
5 \\ \hline
73)2.890 \\
39 \\ \hline
\$2.851=\$2.89
\end{array}$$

[diminished* by $\frac{1}{73}$ of itself, [Ans.

16.	$20.662
17.	$19.65
18.	$80.38
19.	$1005.50
20.	$7321.90

---

* See note 2, p. 267.

(ART. **357**, p. **269**.)

2. 179£. 12s. 11d. = 179.645£. × .095 = 17.0662£. = interest at 6 per cent.; 17.0662£. ÷ 6 = 2.8443; 2.8443 × 5 = 14.2215£. = 14£. 4s. 5$\frac{1}{4}$d., Ans.

(3.)

$$\begin{array}{lcll}
\text{Principal} & = & 25£. & \\ \cline{3-3}
\frac{1}{10}\text{ of the principal} & = & 2.5 & \text{int. for 20 mo.} \\
\frac{1}{20}\text{ of int. for 1yr. 8mo.} & = & .125 & \text{int. for 1 mo.} \\ \cline{3-3}
 & & 2.625 \div 6 = .4375;\ .4375 \times 5 = &
\end{array}$$

2.1875£. = 2£. 3s. 9d. = int. for 1yr. 9mo., Ans.

(4.)

$$\begin{array}{lcll}
\text{Principal} & = & 5440.5£. & \\ \cline{3-3}
\frac{1}{5}\text{ of the principal} & = & 1088.1 & \text{int. for 3yr. 4mo.} \\
\frac{1}{8}\text{ of int. for 3yr. 4mo.} & = & 136.0125 & \text{int. for 5mo.} \\
\frac{2}{5}\text{ of int. for 5 mo.} & = & \left\{\begin{array}{r}27.2025 \\ 27.2025\end{array}\right\} & \text{int. for 2mo.} \\ \cline{3-3}
 & & 1278.5175£. & \text{(Carried forward.)}
\end{array}$$

(Brought forward :) 1278.5175£. = the interest for 3yr. 11mo. = 1278£. 10s. 4d., Ans.

5. 943£. 1s. 8d. = 943.0833£.; 30da. in May + 30 in June + 31 in July + 31 in Aug. + 30 in Sept. + 21 in Oct. = 173da. ; 943.0833£. × .028$\frac{5}{6}$ = 27.19223£. ; 27.19223£. × 5$\frac{1}{4}$ ÷ 6 = 23.793£.; 23.793 — .3259 ($\frac{1}{73}$ of 23.793£.) = 23.467 = 23£. 9s. 4d., Ans.

(Art. 358, p. 270.)

2. 10.08 ÷ .07 = $144, Ans.
3. 1.00 × .08$\frac{1}{4}$ = .0825 × 1$\frac{1}{3}$ = .11; 13.20 ÷ .11 = $120.00, Ans.
4. 40.50 ÷ .06 = $675, Ans.
5. 1.00 × .02 × 3 = .06 ; 24.00 ÷ .06 = $400, Ans.
6. 1.00 × .0305 = 0.0305 ÷ 6 = .0050$\frac{5}{6}$; .0050$\frac{5}{6}$ × 7$\frac{1}{4}$ = .0368$\frac{13}{24}$; 206.38$\frac{1}{3}$ ÷ .0368$\frac{13}{24}$ = $5600, Ans.

(Art. 359, p. 270.)

2. $144.00 × .01 = $1.44; 10.08 ÷ 1.44 = 7 per cent., Ans.
3. $120 × .01 = 1.20; 1.20 × 1$\frac{1}{3}$ = 1.60; 133.20 — 120 = 13.20 ÷ 1.60 = 8$\frac{1}{4}$ per cent., Ans.
4. $1.00 × .01 = $0.01; $0.01 × 14$\frac{2}{7}$ = 0.14\frac{2}{7}$; 1.00 ÷ 14$\frac{2}{7}$ = 7 per cent., Ans.
5. $1.00 × .01 = $.01 × 33$\frac{1}{3}$ = $.33$\frac{1}{3}$; 3.00 ÷ .33$\frac{1}{3}$ = 9 per cent., Ans.
6. $4650 × .01 = $46.50; 232.50 ÷ 46.50 = 5 per cent., Ans.
7. 48da. = 1$\frac{3}{5}$mo. = $\frac{1\frac{3}{5}}{12}$ of 1yr.; $7500 × .01 = $75;

   $75 × $\frac{1\frac{3}{5}}{12}$ = $10; 60 ÷ 10 = 6 per cent., Ans.

8. $\$280 \times .01 = \$2.80 \times 6\frac{1}{2} = \$18.20$; $\$411.95 - \$280 = \$131.95$; $131.95 \div 18.20 = 7\frac{1}{4}$ per cent., Ans.

9. $\$480 \times .01 = \$4.80 \times 1\frac{7}{24} = \$6.20$; $\$529.60 - \$480 = \$49.60$; $49.60 \div 6.20 = 8$ per cent., Ans.

(Art. **360**, p. **271**.)

2. $\$120 \times .08\frac{1}{4} = \$9.90$; $13.20 \div 9.90 = 1\frac{1}{3}$yr., Ans.

3. $\$144 \times .07 = \$10.08$; $10.08 \div 10.08 = 1$yr., Ans.

4. $\$240 \times .06 = \$14.40$; $\$280 - \$240 = \$40$; $40 \div 14.40 = 2\frac{7}{9}$yr. = 2yr. 9mo. 10da., Ans.

5. $\$1.00 \times .05 = \$.05$; $1.00 \div .05 = 20$yr., Ans.

6. $\$1.00 \times .10 = \$.10$; $1.00 \div .10 = 10$yr., Ans.

7. $\$1500 \times .05 = \$75.00$; $\$2250 - \$1500 = \$750$; $750 \div 75 = 10$yr., Ans.

8. $\$480 \times .04\frac{1}{2} = \$21.60$; $\$561.60 - \$480 = \$81.60$; $81.60 \div 21.60 = 3\frac{7}{9}$yr. = 3yr. 9mo. 10da., Ans.

9. $\$1728 \times .12 = \$207.36$; $\$3853.44 - \$1728 = \$2125.44$; $2125.44 \div 207.36 = 10\frac{1}{4}$yr. = 10yr. 3mo., Ans.

10. $\$240 \times .06 = \$14.40$; $\$720 - \$240 = \$480$; $480 \div 14.40 = 33\frac{1}{3}$yr. = 33yr. 4mo., Ans.

11. $\$400 \times .06 = \$24.00$; $100 \div 24 = 4\frac{1}{6}$yr. = 4yr. 2mo.;

$$\begin{array}{rrr} 1857 & 4 & 16 \\ 4 & 2 & 0 \\ \hline 1861 & 6 & 16 \end{array} = \text{July 16, 1861, Ans.}$$

PROMISSORY NOTES.

(Page **275**.)

(2.)

Principal on interest from June 17, 1849, . . .	$769.870
Interest from June 17, 1849, to March 1, 1850, 8mo. 14da., ...	32.591
Amount (carried forward), . . .	$802.461

(Brought forward :)		$802.461
First payment, March 1, 1850,		75.500
New principal, bearing interest from March 1, 1850,		726.961
Interest from March 1, 1850, to June 11, 1851, 15mo. 10da.,		55.733
Amount,		782.694
Second payment, June 11, 1851,		165.000
New principal, bearing interest from June 11, 1851,		617.694
Interest from June 11, 1851, to Sept. 15, 1851, 3mo. 4da.,		9.677
Amount,		627.371
Third payment, Sept. 15, 1851,		161.000
New principal, bearing interest from Sept. 15, 1851,		466.371
Interest from Sept. 15, 1851, to Jan. 21, 1852, 4mo. 6da.,		9.793
Amount,		476.164
Fourth payment, January 21, 1852,		47.250
New principal, bearing interest from Jan. 21, 1852,		428.914
Interest from Jan. 21, 1852, to December 6, 1853, 22mo. 15da.,		48.252
Amount,		477.166
Fifth payment, less than interest, March 5, 1853,	12.17	
Sixth payment, more than interest, Dec. 6, 1853,	98.00	
		110.170
New principal, bearing interest from Dec. 6, 1853,		366.996
Interest from Dec. 6, 1853, to July 7, 1854, 7mo. 1d.,		12.906
Amount (carried forward),		$379.902

(Brought forward :)	$379.902
Seventh payment, July 7, 1854,	169.000
New principal, bearing interest from July 7, 1854,	210.902
Interest from July 7, 1854, to Sept. 25, 1855, 14mo. 18da.,	15.395
Balance due Sept. 25, 1855,	$226.297

(3.)

Principal on interest from April 30, 1851,	$300.000
Interest from April 30, 1851, to June 27, 1852, 13mo. 27da.,	20.850
Amount,	320.850
First payment, June 27, 1852,	150.000
New principal, bearing interest from June 27, 1852,	170.850
Interest from June 27, 1852, to December 9, 1852, 5mo. 12da.,	4.612
Amount,	175.462
Second payment, Dec. 9, 1852,	150.000
New principal, bearing interest from Dec. 9, 1852,	25.462
Interest from Dec. 9, 1852, to Oct. 9, 1853, 10mo.,	1.273
Balance due Oct. 9, 1853,	$26.735

(4.)

Principal on interest from Feb. 11, 1852,	$54.180
Interest from Feb. 11, 1852, to July 11, 1853, 17mo.,	4.605
Amount,	58.785
First payment, July 11, 1853,	12.250
Principal bearing interest from July 11, 1853,	46.535
Interest from July 11, 1853, to Aug. 21, 1855, 25mo. 10da.,	5.894
Amount (carried forward),	$52.429

(Brought forward:)		$52.429
Second payment, less than interest, Aug. 15, 1854,	2.100	
Third payment, less than interest, July 9, 1855,	3.120	
Fourth payment, more than interest, Aug. 21, 1855,	37.180	
		42.400
Principal bearing interest from Aug. 21, 1855,		10.029
Interest from Aug. 21, 1855, to Dec. 17, 1855, 3mo. 26da.,		.193
Balance due Dec. 17, 1855,		$10.222

(6.)

Principal carrying interest from Jan. 1, 1850,		$1000.00
Interest from Jan. 1, 1850, to Sept. 28, 1850, 8mo. 27da.,		51.91
Amount,		1051.91
First payment,		144.00
Balance for new principal,		907.91
Interest from Sept. 28, 1850, to July 17, 1851, 9mo. 19da.,		51.01
Amount,		958.92
Second payment, March 1, 1851, a sum less than interest,	20.00	
Third payment, July 17, 1851, a sum greater than interest,	360.00	
		380.00
Balance for new principal,		578.92
Interest from July 17, 1851, to Aug. 9, 1851, 22da.,		2.47
Amount (carried forward),		$581.39

(Brought forward:)	$581.39
Fourth payment, Aug. 9, 1851,	190.00
Balance for new principal,	391.39
Interest from Aug. 9, 1851, to Sept. 25, 1852, 1yr. 1mo. 16da.,	30.89
Amount,	422.28
Fifth payment, Sept. 25, 1852,	170.00
Balance for new principal,	252.28
Interest from Sept. 25, 1852, to Dec. 11, 1853, 1yr. 2mo. 16da.,	21.38
Amount,	273.66
Sixth payment, Dec. 11, 1853,	200.00
Balance for new principal,	73.66
Interest from Dec. 11, 1853, to July 4, 1855, 1yr. 6mo. 23da.,	8.06
Amount,	81.72
Seventh payment, July 4, 1855,	75.00
Balance for new principal,	6.72
Interest from July 4, 1855, to June 1, 1857, 1yr. 10mo. 27da.,	.89
Balance due at the time of payment,	$7.61

NOTE. — Interest on the above note is computed at 7 per cent.

(ART. 372, p. 278.)

(2.)

Principal,		$700.00
Interest for 9mo. 24da.,		34.30
		734.30
First payment,	160.00	
Interest for 8mo. 10da.,	6.66⅔	
(Carried forward:)	$166.66⅔	$734.30

(Brought forward:)	$166.66⅔	$734.30
Second payment, ...	200.	
Interest for 5mo. 4da., ...	5.13⅓	
Third payment, ...	120.	
Interest for 2mo. 17da., ...	1.54	
Fourth payment, ...	60.	
Interest for 1mo. 23da., ...	.53	
		553.87
Balance due Nov. 28, 1854, . .		$180.43

(3.)

Principal, ...		$500.00
Interest for 12mo., ...		35.00
		535.00
Payment, ...	200.00	
Interest for 9mo., ...	10.50	
		$210.50
Balance due April 1, 1858, . .		$324.50

COMPOUND INTEREST.

(Art. **376**, p. **280**.)

(2.)

$500	(Brought up:) 31.80
.06	530
30.00	561.80
500	.06
530	33.7080
.06	561.80
31.80 (Carried up.)	$595.5080, Ans.

(3.)

$970
.06
58.20
970
1028.20
.06
61.6920
1028.20
1089.8920
.049 Int. of $1 for 9mo. 24da.
98090280
43595680
53.404708
1089.8920
1143.2967
970 First principal subtracted.
$173.296, Ans.

(4.)

$300
.07
21.00
300
321
.07
22.47
321
343.47
.07
24.0429
343.47
367.5129
.07
25.725903
367.5129
393.2388
.035
19661940
11797164
13.763358
393.238
407.001
300
$107.001, Ans.

(5.)

$$\begin{array}{r}
\$316 \\
.06 \\ \hline
18.96 \\
316 \\ \hline
334.96 \\
.06 \\ \hline
20.0976 \\
334.96 \\ \hline
355.0576 \\
.06 \\ \hline
21.303456 \\
355.0576 \\ \hline
376.361 \text{ (Carried up.)}
\end{array}$$

$$\begin{array}{r}
\text{(Brought up.) } 376.361 \\
.023 \\ \hline
1129083 \\
752722 \\ \hline
8.656303 \\
376.361 \\ \hline
385.017 \\
316 \\ \hline
\$69.017\text{, Ans.}
\end{array}$$

(Art. **377**, p. **282.**)

2. $1200 × 2.104852 = 2525.8224; 2525.8224 — 1200 = $1325.822, Ans.

3. $300 × 1.790848 = 537.2544; 537.2544 × 1.0375 = 557.401; 557.401 — 300 = $257.401, Ans.

4. $5 × 29.457 = $147.285; $147.285 — $5 = $142.285, Ans.

5. $480 × 10.285718 = $4937.144, Ans.

6. $40 × .310796 = $12.431, Ans.

7. The amount of $1 for 50 years is $18.420154; if now this amount be taken for a principal and multiplied by the amount of $1 for 50 years, the result will be the amount of $1 for 100 years; therefore 18.420154 × 18.420154 = 339.302072, the amount of $1 for 100 years; and 339.302072 × $100 = $33930.2072; and $33930.2072 — $100 = $33830.2072, Ans.

8. $1000 × 6.84059 = $6840.59; $6840.59 × 1.02 = $6977.4018; $6977.4018—$1000 = $5977.4018 = interest at compound interest; $1000 × 2.00 = $2000 =

interest at simple interest; $5977.4018 — $2000 = $3977.4018, Ans.

9. For 21y., payable semi-annually, is the same as for 42y., payable annually; $50 × 3.460696 = $173.034, Ans.

(10.)

Principal bearing interest from March 26, 1854,		$100.000
Interest for 1y. 5mo., ...		8.650
Amount of the principal to Aug. 25, 1855, . . .		108.650
First payment, June 11, 1854, . . .	50.000	
Compound interest from June 11, 1854, to Aug. 25, 1855, 14mo. 14da., . . .	3.653	
Second payment, Sept. 25, 1854, . . .	50.000	
Compound interest from Sept. 25, 1854, to Aug. 25, 1855, 11mo., ...	2.750	
Amount of the endorsements, ...		106.403
Balance due Aug. 25, 1855, ...		$2.247

(11.)

Principal bearing interest from Jan. 1, 1850, . .		$1000.00
Compound interest on $1000 from Jan. 1, 1850, to April 1, 1855, 5y. 3mo.		427.09
Amount of principal to April 1, 1855, . . .		1427.09
First payment, June 10, 1850, ...	70.00	
Compound interest from June 10, 1850, to April 1, 1855, 4y. 9mo. 21d., . . .	26.94	
Second payment, Sept. 25, 1851, . . .	80.00	
Compound interest from Sept. 25, 1851, to April 1, 1855, 3y. 6mo. 6da., . . .	21.54	
Third payment, July 4, 1852, ...	100.00	
Compound interest from July 4, 1852, to April 1, 1855, 2y. 8mo. 27da., . . .	20.43	
Fourth payment, Nov. 11, 1853, . . .	30.00	
(Carried forward:)	$348.91	$1427.09

(Brought forward:)	$348.91	$1427.09
Compound interest from Nov. 11, 1853, to April 1, 1855, 1y. 4mo. 20da.,	2.97	
Fifth payment, June 5, 1854,	50.00	
Compound interest from June 5, 1854, to April 1, 1855, 9mo. 26da.,	2.87	
Amount of endorsements,		404.75
Balance due April 1, 1855,		$1022.34

Note. — The above note and endorsements are reckoned at 7 per cent.

(Art. **378**, p. **284**.)

2. 1.60578 × 1.035 = 1.553255 = the amount of $1 for 6y. 6mo.; $205.90 ÷ .5532 = $372.16, Ans.
3. 1.191 × 1.012 = 1.2052 = amount of $1 for 3y. 2mo. 12da.; $1026.54 ÷ .2052 = $5000, Ans.
4. 1.905 = interest of $1 for 31y. at 3½ per cent.; $857.25 ÷ 1.905 = $450, Ans.

(Art. **379**, p. **285**.)

2. 640.405 ÷ 400 = 1.60101, which for 12 years indicates by the table an interest of 4 per cent., Ans.
3. 2096.147 ÷ 2500 = .8384588; and in the table this is found to be the *interest* of $1 for 9 years at 7 per cent.
4. 200 ÷ 100 = 2.000000; by the table it is seen that at 6 per cent. this will be the amount of $1 for a time somewhere between 11 and 12 years. The amount for 11y. is 1.898299; if this, increased by $\frac{89}{100}$ of a year, be 2., then the rate is 6 per cent.; 1.898299 × 1.0534 = 2.00000 very nearly.
5. $31479.70 ÷ 10000 = 3.14797; 3.0256, the amount of $1 for 19y. at 6 per cent., is the approximation; 3.0256 × $1.04\frac{3}{8}$ = 3.14797 nearly; hence the rate is 6 per cent.

(Art. **380**, p. **285**.)

2. 640.405 ÷ 400 = 1.601012; which is the amount of $1 at 4 per cent. for 12 years, Ans.

3. 9021.78 ÷ 6000 = 1.50363, which is the amount of $1 at 7 per cent. for 6 years, Ans.

4. 200 ÷ 100 = 2.000000; by the table the amount at 5 per cent., which approximates to 2., is the amount for 14y., which is 1.979932; 2.0000 — 1.979932 = that interest which the amount for 14y. (1.979932) must gain before the $1 will be doubled; therefore, to get the time required for gaining this principal, divide (Art. 360) the given interest by the interest of 1.979932 for 1y.; that is, divide 2.000 — 1.979932 by 2.078928 — 1.979932; .020068 ÷ .098996 = .2026y. = 2mo. 13da. nearly; 14y. 2mo. 13da., Ans.

5. 200 ÷ 100 = 2.; by the table the nearest approximation to 2., at 6 per cent., is the amount for 11y., 1.898299; 2.012196 — 1.898299 = .113897; 2. — 1.898299 = .101701; .101701 ÷ .113897 = .89y. = 10mo. 20+da.; 11y. 10mo. 20+da., Ans.

6. 1781.665 ÷ 450 = 3.959, which by the table is found to be the amount of $1 at 3½ per cent. for 40y.; but, as the principal in the problem pays 3½ per cent. in periods of half a year, its 40 periods will be only 20y. The deposit was made when the son was 1 year old; and 20 + 1 = 21y., Ans.

DISCOUNT AND PRESENT WORTH.

(Art. **386**, p. **287**.)

2. $1.07 = amount of $1 for the given time; $802.50 ÷ 1.07 = $750; $802.50 — 750 = $52.50, Ans.

3. $1.12 = amount of $1 for the given time; $117.60 ÷ 1.12 = $105.00, Ans.

4. $1.205 = amount of $1 for 3y. 5mo.; $769.60 ÷ 1.205 = $638.672, Ans.

5. \$1.26875 = amount of \$1 for 3y. 7mo.; \$678.75 ÷ 1.26875 = \$534.975, Ans.

6. \$1.25 = amount of \$1 for the time;
\$600 ÷ 1.25 = \$480 present worth;
\$600 — \$480 = \$120 discount, Ans.

7. \$$1.056\frac{5}{6}$ = amount of \$1 for 11mo. 11da; \$79.87 ÷ $1.056\frac{5}{6}$ = \$75.57,4+.
\$$1.111\frac{1}{6}$ = amount of \$1 for 22mo. 7da.; \$87.75 ÷ $1.111\frac{1}{6}$ = \$78.971; \$78.971 + \$75.574 = \$154.545+, Ans.

8. \$1.118 = amount of \$1 for 23mo. 18da.; \$365.87 ÷ 1.118 = \$327.254+.
\$1.1525 = amount of \$1 for 30mo. 15da.; \$161.15 ÷ 1.1525 = \$139.826+.
\$$1.028\frac{2}{3}$ = amount of \$1 for 5mo. 22da.; \$112.50 ÷ $1.028\frac{2}{3}$ = \$109.364+.
\$1.258 = amount of \$1 for 51mo. 18da.; \$96.81 ÷ 1.258 = \$76.955+.
\$327.254 + \$139.826 + \$109.364 + \$76.955 + = \$653.40+, Ans.

9. \$67.25 ÷ 1.18 = \$56.99, Ans.

10. \$80.095 ÷ 1.191 = \$67.25, Ans.

11. \$110.364 ÷ 1.402552 = 78.687; \$110.364 — \$78.687 = \$31.677 discount, Ans.

(Art. 387, p. 288.)

1. \$960 × .05 = \$48; \$960 — \$48 = \$912, Ans.

2. \$5000 ÷ 1.07 = \$4672.89; \$5000 — \$4672.89 = \$327.11 true discount; \$5000 × .07 = \$350 nominal discount; \$350 — \$327.11 = \$22.89, Ans.

3. \$4440 × .03 = 133.20; \$4440 — \$133.20 = \$4306.80; \$4306.80 is the nominal present worth, and is the sum which must be hired at 6 per cent. to pay the debt; \$4306.80 × .03 = \$129.204; \$133.20 — \$129.204 = \$3.996, Ans.

BANKING.

(Art. **392**, p. **290.**)

2. $7800 × (.015 + .0005 for 3 days of grace) .0155 = $120.90, Ans.

3. $1200 × .01225 = $14.70, Ans.

4. $8000 × .0105 = $84 discount; $8000 — $84 = $7916 proceeds, Ans.

5. $760 × .0255 = $19.38; $760 — $19.38 = $740.62, Ans.

6. $7860 × .030$\frac{1}{2}$ = $239.73; $7860 — $239.73 = $7620.27, Ans.

7. The time from Dec. 3 to April 3 is 4mo.; 4mo. from 6mo. leave 2mo. or 60da.; 60da. + 3da. = 63da., the time the note is to run; $160.40 × .0105 = $1.68; $160.40 — $1.68 = $158.72, Ans.

8. 123da. from Apr. 16, 1856 = Aug. 17, 1856, the time when the note is legally due; from May 16 to Aug. 17 = 93da., the time for which the note was discounted; $890.50 × .0155 = $13.80275; $13.80275 ÷ 6, × 7 = $16.1032, Ans.

9. 90 + 3da. from May 1 = Aug. 2, the legal time of maturity; May 1 to June 11 = 41da.; 93 — 41 = 52da., for which the note was discounted; $1340 × .008$\frac{2}{3}$ = $11.61; $1340 — $11.61 = $1328.39 proceeds, Ans.

10. From June 19 to July 5 are 16da.; from June 19 to Aug. 19 are 61da.; 61 + 3 = 64da.; 64 — 16 = 48, the time the note had to run, and for which it was discounted; $9000 × .032 = $288; $9000 — $288 = $8712, Ans.

(Art. **393**, p. **292.**)

2. $1 × .0180$\frac{5}{6}$ = 0.0180\frac{5}{6}$; $1.00 — 0.0180\frac{5}{6}$ = $0.9819, the proceeds of $1; $680 ÷ .9819 = $692.523, Ans.

3. Proceeds of \$1 for 120 + 3da. = .9795; \$540.50 ÷ .9795 = \$551.81, Ans.

4. \$1938 ÷ .9695 = \$1998.968, Ans.

5. \$1631.60 ÷ .9795 = \$1665.74, Ans.

6. Present worth of \$1 for 33da. at 2 per cent. per month = .989; \$1500 ÷ .989 = \$1516.68, Ans.

(Art. **394**, p. **292.**)

2. The proceeds of \$1 for 30 + 3 da. = .9945; .06 ÷ .9945 = $6\frac{66}{1989}$ per cent., Ans.

3. .06 ÷ .9845 = $6\frac{186}{1969}$ per cent., Ans.

4. .02 ÷ .918 = $2\frac{82}{459}$ per cent. a month, Ans.

5. The proceeds of \$1 are .96442; .07 ÷ .96442 = $7\frac{12453}{48221}$ per cent., Ans.

6. .06 ÷ .94 = $6\frac{18}{47}$ per cent., Ans.

(Art. **395**, p. **293.**)

2. The amount of \$1 for 33da. = 1.0055; .06 ÷ 1.0055 = $5\frac{1945}{2011}$, per cent., Ans.

3. 2 per cent. a month = 24 per cent. per annum; .24 ÷ 1.042 = $23\frac{17}{521}$ per cent., Ans.

4. .06 ÷ 1.0155 = $5\frac{1845}{2031}$ per cent., Ans.

5. .08 ÷ 1.027333 = $7\frac{808669}{1027333}$ per cent., Ans.

6. .07 ÷ 1.03558 = $6\frac{39326}{51779}$ per cent., Ans.

7. .06 ÷ 1.06 = $5\frac{35}{53}$ per cent., Ans.

8. The amount of \$1 for 8yr. 4mo. at 5 per cent. = \$$1.41\frac{2}{3}$; .05 ÷ $1.41\frac{2}{3}$ = $.03\frac{9}{17}$, Ans.

MISCELLANEOUS EXAMPLES.

(Page **294.**)

1. 18 of gold; 20 : 18 : : 100 : 90 per cent. of gold;
 1 of copper; 20 : 1 : : 100 : 5 per cent. of copper;
 $\underline{\ 1\ }$ of silver; 20 : 1 : : 100 : 5 per cent. of silver;
 20;

2. $\frac{16}{16}$ = amount, $\frac{15}{16}$ = principal, $\frac{1}{16}$ = discount. Therefore, 32 × 16 = 512 ÷ 15 = $34\frac{2}{15}$qt., Ans.

3. $25000 × .04 = $1000, Ans.

4. $206\frac{1}{4}$ — 192 = $14\frac{1}{4}$; 192 : $14\frac{1}{4}$: : 1.00 : $.07\frac{27}{64}$, Ans.

5. 1.50 : 1.00 : : 1.00 : $.66\frac{2}{3}$; 1.00 — $.66\frac{2}{3}$ = $.33\frac{1}{3}$, Ans.

6. How long will it take $10 to double itself, or gain $10 interest? (Art. 360.) 10 ÷ .50 = 20yr.; 10 ÷ 60 = $16\frac{2}{3}$yr.; 20 — $16\frac{2}{3}$ = $3\frac{1}{3}$yr., Ans.

7. $500 ÷ 1.2155 = $411.351, Ans.

8. 1.00 : 1.10 : : $5 : $5.50, the price at which the cloth was actually sold, it being 10 per cent. less than the asking price; therefore, .90 : 1.00 : : $5.50 : $$6.11\frac{1}{9}$, Ans.

9. $7860 × $.030\frac{1}{2}$ = $239.73; $7860 — $239.73 = $7620.27, Ans.

10. $1500 ÷ 1.06 = $$1415.09\frac{23}{53}$; $1500 — $$1415.09\frac{23}{53}$ = $$84.90\frac{30}{53}$, the true discount; $1500 × .06 = $90; $90 — $$84.90\frac{30}{53}$ = $$5.09\frac{23}{53}$, Ans.

11. .24 ÷ .958 = $25\frac{25}{479}$ per cent., Ans.

12. Interest of $1 at $1\frac{1}{2}$ per cent. a month = $.0225; $36.40 ÷.0225 = $$1617.77\frac{7}{9}$, Ans.

13. .12 ÷ 1.011 = $11\frac{293}{337}$ per cent., Ans.

14. In the course of 693 days the bank can discount notes of 60 days 11 times, and notes of 30 days 21 times; then (Art. 393) $50000 ÷ .9945 = $50276.52 = the face of the note or notes discounted for 33 days; $50276.52 — $50000 = $276.52 = the sum gained in each period of 33da.; $276.52 × 21 = $5806.92 = the whole amount gained on 30 days' notes in 693 days; and $50000 ÷ .9895 = $50530.57; $50530.57 — $50000 = $530.57; $530.57 × 11 = $5836.27 = the whole sum gained on 60 days' notes in 693 days; $5836.27 — $5806.92 = $29.35 = the amount which is gained on notes of 60 days *more* than on notes of 30 days, Ans.

15. 450 × \$3.50 = \$1575; 450 × \$4.00 = \$1800; \$1800 ÷ .0305 = \$54.90; \$1800 — \$54.90 = \$1745.10; \$1745.10 — \$1575 = \$170.10, Ans.

16. $6\frac{1}{2} - 6 = \frac{1}{2}$ per cent. $6\frac{1}{2} : \frac{1}{2} :: \$1.00 : \$0.076\frac{2}{13}$, the interest of \$1 for the required time at 6 per cent.; $\$0.06 : \$0.076\frac{2}{13}$: : 12 months : 15m. $11\frac{7}{13}$d., Ans. [See solution, p. 252.]

STOCKS.

(ART. **400**, p. **296**.)

2. \$24360 × 1.35 = \$32886, Ans.
3. \$100 × 15 × 1.13 = \$1695, Ans.
4. \$100 × 10 × .85 = \$850, Ans.
5. \$100 × 30 = \$3000; $\$3000 \times 1.08\frac{3}{4} = \$3262.50$, amount; \$3262.50 — \$3000 = \$262.50, premium, Ans.
6. \$50 × 25 × .97 = \$1212.50, Ans.
7. \$250 × 22 = \$5500; \$5500 × .95 = \$5225, amount paid; \$5500 — \$5225 = \$275, Ans.
8. \$50000 × 1.17 = \$58500, Ans.
9. \$19500 × .93 = \$18135; \$19500 × 1.03 = \$20085; \$20085 — \$18135 = \$1950, Ans.

(ART. **401**, p. **297**.)

2. \$6210 ÷ 1.035 = \$6000, Ans.
3. \$1155 ÷ 11 = \$105; \$105 ÷ 1.05 = \$100, Ans.
4. \$1230 ÷ 41 = \$30; \$30 ÷ .60 = \$50, Ans.
5. \$2052 ÷ 1.08 = \$1900; \$2052 — \$1900 = \$152, Ans.
6. \$16245 ÷ .95 = \$17100; \$17100 — \$16245 = \$855, Ans.
7. 862.50 ÷ 57.50 = 15 shares; \$862.50 ÷ 1.15 = \$750; \$862.50 — \$750 = \$112.50, Ans.
8. \$1000 × .88 = \$880; 7920 ÷ 880 = 9 bonds; \$1000 — \$880 = \$120; \$120 × 9 = \$1080, Ans.

(ART. **402**, p. **298.**)

2. 1.00 — .25 = .75; .06 ÷ .75 = .08, Ans.
3. .11 ÷ 1.10 = 10 per cent., Ans.
4. (402, note.) .12 ÷ .8 = 1.50; 1.50 — 1.00 = .50, Ans.
5. .09 ÷ 1.25 = $.07\frac{1}{5}$; .06 ÷ .75 = .08; .08 — $.07\frac{1}{5}$ = $\frac{4}{5}$ per cent.; therefore, it is the better investment by $\frac{4}{5}$ per cent. to buy the 6 per cent. stock.
6. .05 ÷ .07 = $.71\frac{3}{7}$; 1.00 — $.71\frac{3}{7}$ = $.28\frac{4}{7}$, Ans.
7. .05 ÷ .80 = $.06\frac{1}{4}$; $20000 × $.06\frac{1}{4}$ = $1250; $20000 × .06 = $1200; $1250 — $1200 = $50, Ans.

BROKERAGE AND COMMISSION.

(ART. **407**, p. **299.**)

2. $18768 × $.01\frac{3}{4}$ = $328.44, Ans.
3. $112.25 × 12 = $1347; 1347 × $.00\frac{1}{4}$ = 3.36\frac{3}{4}$, Ans.
4. $12.25 × 700 = $8575; $8575 × $.01\frac{3}{4}$ = 150.06\frac{1}{4}$, Ans.
5. 50 ÷ 10000 = .005 = $\frac{1}{2}$ per cent., Ans.
6. $8.95 × 173 × $.01\frac{7}{8}$ = 29.03\frac{5}{32}$, Ans.
7. $107.75 × 37 × $.00\frac{3}{8}$ = 14.95\frac{1}{32}$, Ans.
8. 395£. 15s. 5d. = 94985d.; 94985 × $.02\frac{1}{4}$ = 2137.1625d. = 8£. 18s. $1\frac{65}{400}$d., Ans.
9. 5.46 ÷ 364 = $.01\frac{1}{2}$, Ans.

(ART. **408**, p. **301.**)

2. $1976 ÷ 1.04 = $1900; $1976 — $1900 = $76, Ans.
3. $8341.50 ÷ 1.005 = $8300; $8300 ÷ 83 = 100 shares; $8341.50 — $8300 = $41.50 brokerage, Ans.
4. $8960 ÷ 1.02 = 8784.31\frac{19}{51}$; $8960 — 8784.31\frac{19}{51}$ = 175.68\frac{32}{51}$, Ans.

5. $5150 ÷ 1.03 = $5000; $5150 — $5000 = $150, Ans.

6. $285 ÷ .015 = $19000, Ans.

7. $$740.83\frac{3}{4}$ — $$7.50$ = $$733.33\frac{3}{4}$ = the bill minus the cartage; $$733.33\frac{3}{4}$ ÷ 1.015 = $722.50; $$733.33\frac{3}{4}$ — $722.50 = $$10.83\frac{3}{4}$ = commission; 250 × 34 = 8500 lb. of sugar; $722.50 ÷ 8500 = $$0.08\frac{1}{2}$ per pound, Ans.

8. 987£. 18s. 6d. × .0225 = 22£. 4s. $6\frac{159}{200}$d.; 987£. 18s. 6d. — 22£. 4s. $6\frac{159}{200}$d. = 965£. 13s. $11\frac{41}{200}$d.; 965£. 13s. $11\frac{41}{200}$d. × 100 = 96569£. 13s. $4\frac{1}{2}$d.; 96569£.13s. $4\frac{1}{2}$d. ÷ $101\frac{7}{8}$ = 947£. 18s. $5\frac{449}{815}$d.; 947£. 18s. $5\frac{449}{815}$d. = $227501\frac{449}{815}$d.; 1£. 3s. 8d. = 284d.; $227501\frac{449}{815}$ ÷ 284 = $801\frac{3576}{57865}$yds., Ans.

ACCOUNT OF SALES.

(Art. **410**, p. **303.**)

(2.)

Amount of sales, ...		$2671.40
Freight, ...	$185.50	
Commission at 3 per cent. on $2671.40,	74.57	
Cartage, cooperage, &c., ...	26.00	
Storage and insurance, ...	63.24	
Amount of charges, ...		$349.31
Net proceeds to J. B. & Co., ...		$2322.09

PROFIT AND LOSS.

(Art. **412**, p. **304.**)

2. $5 × .80 = $4, Ans.

3. $8.50 × 1.10 = $9.35, Ans.

4. $\$0.42 \times .95 = \$0.399$, Ans.

5. $\$2500 \times 1.20 = \$3000$, Ans.

(ART. **413**, p. **304**.)

2. $\$4 \div .80 = \$5$, Ans.

3. $\$96 \div 1.20 = \$80$; $\$80 \div 10 = \$8$, Ans.

4. $\$12.50 \div 1.17 = \$10.683\frac{89}{117}$, Ans

5. $\$6.125 \div .875 = \$7.00$, Ans.

(ART. **414**, p. **305**.)

2. $\$7 - \$6.125 = \$0.875$; $.875 \div 7 = 12\frac{1}{2}$ per cent., Ans.

3. $\$225 - \$200 = \$25$; $25 \div 200 = 12\frac{1}{2}$ per cent., Ans.

4. $500 \div 2500 = 20$ per cent., Ans.

5. $\$64.86\,4\frac{32}{37}$ was given for the cloth, and $\$2.50 \times 24 = \$60.00$ was received for it; therefore the loss was $\$64.86\,4\frac{32}{37} - \$60.00 = \$4.86,4\frac{32}{37}$, or $\frac{4864\frac{32}{37}}{64864\frac{32}{37}}$ of its value, $= .075 = 7\frac{1}{2}$ per cent., Ans.

(ART. **415**, p. **306**.)

2. $\$7 \div 1.75 = \$4$ prime cost; $\$4 - \$3 = \$1$, loss on 1yd.; $1 \div 4 = .25$ loss, Ans.

3. $\$1.25 \div .85 = \$1.470\frac{10}{17}$ par value of corn; $\$1.647\frac{1}{17} - 1.470\frac{10}{17} = .176\frac{8}{17}$ gain per bushel if sold for $\$1.647\frac{1}{17}$; $.176\frac{8}{17} \div 1.470\frac{10}{17} = .12$, Ans.

4. $\$1.647\frac{1}{17} \div 1.12 = \$1.470\frac{10}{17}$; $\$1.470\frac{10}{17} - \$1.25 = \$0.220\frac{10}{17}$ loss per bushel if sold at $\$1.25$; $.220\frac{10}{17} \div 1.470\frac{10}{17} = 15$ per cent., Ans.

5. $\$75 \div .75 = \$100$ cost of the horse; $\$100 \times 1.30 = \$130$ real value of horse; $\$130 - \$75 = 55$, Ans.

6. $\$1.25 \div .75 = \$1.66\frac{2}{3}$ cost per pound; $\$1.66\frac{2}{3} - \$1.40 = .26\frac{2}{3}$ loss per lb. if sold at $\$1.40$; $\$.26\frac{2}{3} \div 1.66\frac{2}{3} = 16$ per cent. loss, Ans.

MISCELLANEOUS EXAMPLES.

(PAGE **306.**)

1. \$100 × 1.15 = \$115; \$115 — \$100 = \$15 gain on 1 share; \$120 ÷ 15 = 8 shares, Ans.

2. \$250 × 1.09 = \$272.50 cost per share; \$272.50 — \$25 = \$247.50 = what one share was sold for; \$250 — \$247.50 = \$2.50 loss by each share on the par value; 2.50 ÷ 250 = .01 discount, Ans.

3. \$19200 ÷ .96 = \$20000; \$20000 × .07 = \$1400 = income from 7 per cent. stocks; \$19200 ÷ .80 = \$24000; \$24000 × .05 = \$1200 = income from 5 per cent. stocks; \$1400 — \$1200 = \$200 greater income from 7 per cent. stocks, Ans.

4. If each share of \$250 should pay 10 per cent., the dividend would be \$25; then, by proportion, \$25 : \$15 : : \$250 : \$150, Ans.

5. \$1.00 — \$0.20 = \$0.80; \$0.80 × .015 = .012 commission on each bill of \$1; \$0.80 + \$0.012 = \$0.812 = the cost per cent. of the bills; .812 × \$200 = \$162.40 the amount lost on the worthless bills; \$162.40 + \$364 = \$526.40 = sum which must accrue from the bills sold at par to make a net gain of \$364; \$526.40 ÷ .188 (the gain on each bill of \$1 sold at par) = \$2800 = amount of bills sold at par; \$2800 + \$200 (amount of the worthless bills) = \$3000, Ans.

6. \$5640 — \$76.50 = \$5563.50 = proceeds of cotton *minus* the storage; \$5563.50 — \$5422.50 (sum remitted) = \$141 commission; \$141 ÷ 5640 = .02½ the per cent. of commission, Ans.

7. The horse was bought for 70 per cent of his real worth; and .70 × .75 = .525; that is, the horse was sold for 52½ per cent. of his original worth, which is a loss of 1.00 — .525 = 47½ per cent.; but by the question 47½ per cent.

of the original value = $\$55$; therefore $47\frac{1}{2} : \$55 :: 52\frac{1}{2} : \$60.78\frac{18}{19}$, the sum received for the horse, Ans.

8. $\$35 \div 1.04 = \$33.65,3\frac{11}{13}$ present worth of $\$35$; $\$33.65,3\frac{11}{13} - \$30 = \$3.65,3\frac{11}{13}$ gain, Ans.

9. 63gal. — 15gal. = 48gal.; $\$2.21,6\frac{2}{3} \times 48 = \$106.40$, price sold at; $\$112 - \$106.40 = \$5.60$ loss; $\$112 : \$5.60 :: 100 : 5$ per cent. loss, Ans.

10. 63gal. — 15gal. = 48gal. left: $\$2.21,6\frac{2}{3} \times 48 = \$106.40$; $\$106.40 \div .95 = \$112$, Ans.

11. $\$9.00 \times .90 = \$8.10$, what 1bbl. of the damaged flour is sold for; $\$8.10 \times 25 = \$202.50$, proceeds from 25bbl. of damaged flour; $\$9.50 \times 25 = \$237.50$; $\$202.50 + \$237.50 = \$440$, the avails of the flour; $\$9 \times 50 = \$450$, cost of the flour; $\$450 - \$440 = \$10$ loss, Ans.

12. $\$6135 \div 1.02\frac{1}{4} = \$6000$ = sum paid for the flour; $\$6000 \times 1.20 = \$7200$; $\$7200 - \$6135 = \$1065$; $\$1065 - \$31.63$ (storage, &c.) = $\$1033.37$, Ans.

13. $\$102.50 \times 90 = \$9225 \div 2\frac{1}{2} = \$3690$, the sum remitted; $\$3690 - \$90 = \$3600 \div 95 =$ 37T. 17cwt. 3qr. $14\frac{9}{19}$lb., Ans.

PARTNERSHIP.

(Art. **417**, p. **309**.)

2. $\$4000 \div 10000 = 40$ per cent. gain; $\$3000 \times 40 = \$1200$, Jones' part.
$\$2000 \times .40 = \$800$, Weston's part.
$\$5000 \times .40 = \$2000$, Sprague's part.

3. $\$18780 : \$6780 :: \$2000 : \$722.044$, C's part.
$\$18780 : \$12000 :: \$2000 : \$1277.956$, D's part.

4. Harvey $\$2500 \times .05 = \$125$, Harvey's loss.
Blake $\$3000 \times .05 = \$150$, Blake's loss.
Horsford $\$5500 \times .05 = \$275$, Horsford's loss.

5. \$332.50 — \$120 = \$212.50 to be divided between Elliott and Mahew; \$850 : \$212.50 : : \$500 : \$125, Elliott's share; \$850 : \$212.50 : : \$350 : \$87.50, Mahew's share, Griswold's gain will bear the same relation to his stock that Mahew's or Elliott's does; therefore \$125 : \$500 : : \$120 : \$480, the value of Griswold's stock.

6. \$5000 + \$6500 + \$7500 = \$19000, amount of stock; \$19000 × .40 = \$7600; \$7600 × .90 = \$6840, net gain.

$$\left.\begin{array}{l}\$19000 : \$5000 :: \$6840 : \$1800, \text{A's gain,}\\ \$19000 : \$6500 :: \$6840 : \$2340, \text{B's gain,}\\ \$19000 : \$7500 :: \$6840 : \$2700, \text{C's gain,}\end{array}\right\}\text{Ans.}$$

7. \$40000 capital; \$2000 + \$4500 + \$2500 + \$1500 = \$10500 = whole gain.

$$\left.\begin{array}{l}\$10500 : \$2000 :: \$40000 : \$\ \ 7619.04\tfrac{16}{21} \text{ A's,}\\ \$10500 : \$4500 :: \$40000 : \$17142.85\tfrac{15}{21} \text{ B's,}\\ \$10500 : \$2500 :: \$40000 : \$\ \ 9523.80\tfrac{20}{21} \text{ C's,}\\ \$10500 : \$1500 :: \$40000 : \$\ \ 5714.28\tfrac{12}{21} \text{ D's,}\end{array}\right\}\text{Ans.}$$

8. \$7500 gains \$1000 + \$500 = \$1500, which is 20 per cent. on the principal; hence,

$$\left.\begin{array}{l}\$2000 \times .20 = \$400, \text{ A's gain,}\\ \$3000 \times .20 = \$600, \text{ B's gain,}\\ \$400 + \$600 + \$500 = \$1500, \text{ C's gain,}\end{array}\right\}\text{Ans.}$$

9. Let A = 1.00; then B = 1.50; C = 1.25; \$30000 × .25 = \$7500 whole profits; A + B + C = 3.75.

$$\left.\begin{array}{l}3.75 : 1.00 :: \$7500 : \$2000, \text{ A's gain,}\\ 3.75 : 1.50 :: \$7500 : \$3000, \text{ B's gain,}\\ 3.75 : 1.25 :: \$7500 : \$2500, \text{ C's gain,}\end{array}\right\}\text{Ans.}$$

10. The fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, are as the fractions $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, or as 20, 15, 12; 20 + 15 + 12 = 47; $\frac{20}{47}$ = Walker's, $\frac{15}{47}$ = Edwards', $\frac{12}{47}$ = Armstrong's share of the gain; but Armstrong's is divided between Walker and Edwards, and Walker's share is to Edwards' as 20 to 15; hence

Walker's share is $\frac{20}{47} + \frac{20}{35}$ of $\frac{12}{47} = \frac{4}{7}$; and Edwards' share is $\frac{15}{47} + \frac{15}{35}$ of $\frac{12}{47} = \frac{3}{7}$; then

$$\left.\begin{array}{l} \frac{4}{7} \text{ of } \$50000 = \$28571.42\tfrac{6}{7} \text{ Walker's,} \\ \frac{3}{7} \text{ of } \$50000 = \$21428.57\tfrac{1}{7} \text{ Edwards',} \end{array}\right\} \text{Ans.}$$

(ART. **418**, p. **311.**)

2\.

BY ANALYSIS.

$3200 for 12 months is the same as $38400 for 1 month; and $4200 for 8 months is the same as $33600 for 1 month. Therefore the whole stock may be considered $38400 + $33600 = $72000. Goodwin's share of the profit will therefore be $\frac{38400}{72000} = \frac{8}{15}$ of $240 = $128; Blunt's share will be $\frac{33600}{72000} = \frac{7}{15}$ of $240 = $112, Ans.

FORM OF STATEMENT.

$$\begin{array}{l} \$3200 \times 12 = \$38400, \text{ G's product.} \\ \$4200 \times \ \ 8 = \underline{\$33600}, \text{ B's product.} \\ \qquad\qquad\quad\ \$72000 \end{array}$$

$$\left.\begin{array}{l} \$72000 : \$38400 :: \$240 : \$128, \text{ G's gain,} \\ \$72000 : \$33600 :: \$240 : \$112, \text{ B's gain,} \end{array}\right\} \text{Ans.}$$

3\.

BY ANALYSIS.

24 oxen for 8 weeks is 1 ox for 192 weeks; 18 oxen for 12 weeks is 216 oxen for 1 week; and 12 oxen for 10 weeks is 1 ox for 120 weeks. The amount of pasturing is 192 + 216 + 120 = 528 weeks. A's share is $\frac{192}{528} = \frac{4}{11}$ of $26.40 = $9.60; B's share is $\frac{216}{528} = \frac{9}{22}$ of $26.40 = $10.80; C's share is $\frac{120}{528} = \frac{5}{22}$ of $26.40 = $6.00, Ans.

FORM OF STATEMENT.

$$\begin{array}{l} 24 \text{ oxen} \times \ \ 8 = 192 \text{ oxen.} \\ 18 \text{ oxen} \times 12 = 216 \text{ oxen.} \\ 12 \text{ oxen} \times 10 = \underline{120} \text{ oxen.} \\ \qquad\qquad\qquad\ \ 528 \text{ oxen.} \end{array}$$

$$\left.\begin{array}{l} 528 : 192 :: \$26.40 : \$\ \ 9.60 \text{ what A pays,} \\ 528 : 216 :: \$26.40 : \$10.80 \text{ what B pays,} \\ 528 : 120 :: \$26.40 : \$\ \ 6.00 \text{ what C pays,} \end{array}\right\} \text{Ans.}$$

(4.)

$$\begin{array}{r}
\$300 \times 5 = \$1500 \\
\$400 \times 8 = \$3200 \\
\$500 \times 3 = \underline{\$1500} \\
\$6200
\end{array}$$

$$\left.\begin{array}{l}
\$6200 : \$1500 :: \$100 : \$24.19\tfrac{11}{31} \text{ Barclay's,} \\
\$6200 : \$3200 :: \$100 : \$51.61\tfrac{9}{31} \text{ Hickman's,} \\
\$6200 : \$1500 :: \$100 : \$24.19\tfrac{11}{31} \text{ Oliver's,}
\end{array}\right\} \text{Ans.}$$

5. $\$21 + \$40 + \$24 = \$85$, whole gain; and
$\frac{21}{85}$ = A's for 7mo., and $\frac{21}{85} \div 7 = \frac{3}{85}$, A's for 1mo.
$\frac{40}{85}$ = B's for 8mo., and $\frac{40}{85} \div 8 = \frac{5}{85}$, B's for 1mo.
$\frac{24}{85}$ = C's for 12mo., and $\frac{24}{85} \div 12 = \frac{2}{85}$, C's for 1mo.
$\frac{3}{85} + \frac{5}{85} + \frac{2}{85} = \frac{10}{85}$; hence

$$\left.\begin{array}{l}
\frac{10}{85} : \frac{3}{85} :: \$1000 : \$300 = \text{A's capital,} \\
\frac{10}{85} : \frac{5}{85} :: \$1000 : \$500 = \text{B's capital,} \\
\frac{10}{85} : \frac{2}{85} :: \$1000 : \$200 = \text{C's capital,}
\end{array}\right\} \text{Ans.}$$

6. Let 100 = gain of each; $100 + 100 = 200$, gain of both.
$\frac{100}{200}$ = White's gain in 12mo., and $\frac{100}{200} \div 12 = \frac{2}{48}$, White's gain in 1mo.
$\frac{100}{200}$ = D's gain in 8mo., and $\frac{100}{200} \div 8 = \frac{3}{48}$, D's gain in 1mo.
$\frac{2}{48} + \frac{3}{48} = \frac{5}{48}$, gain of both in 1mo., or in any equal time.

$$\left.\begin{array}{l}
\frac{5}{48} : \frac{2}{48} :: \$6300 : \$2520, \text{ White's capital,} \\
\frac{5}{48} : \frac{3}{48} :: \$6300 : \$3780, \text{ Daniel's capital,}
\end{array}\right\} \text{Ans.}$$

(7.)

$$\begin{array}{r}
\$4000 \times 4 = 16000 \\
\underline{500} \qquad\qquad \\
4500 \times 12 = 54000 \\
\underline{1000} \qquad\qquad \\
3500 \times 4 = \underline{14000} \\
\text{A's product} = 84000
\end{array}
\qquad
\begin{array}{r}
\$3000 \times 10 = 30000 \\
\underline{1500} \qquad\qquad \\
1500 \times 4 = 6000 \\
\underline{3000} \qquad\qquad \\
4500 \times 6 = \underline{27000} \\
\text{B's product} = 63000
\end{array}$$

$2000 × 6 = 12000
2000
4000 × 8 = 32000
2000
6000 × 2 = 12000
1500
4500 × 4 = 18000
C's product = 74000

A's product, 84000
B's product, 63000
C's product, 74000
221000

$221000 : $84000 : : $4420 : $1680, A's gain,
$221000 : $63000 : : $4420 : $1260, B's gain, Ans.
$221000 : $74000 : : $4420 : $1480, C's gain,

(8.)

$12000 × 6 = 72000
2500
9500 × 3 = 28500
1000
8500 × 3 = 25500
G's product = 126000

$18000 × 6 = 108000
2500
15500 × 3 = 46500
6000
9500 × 3 = 28500
T's product = 183000

126000 + 183000 = 309000, sum of products.

$8500 + 9500 = 18000; 18000 — 15000 = $3000, whole loss.

$3000 × $\frac{126000}{309000}$ = 1223.30\frac{10}{103}$, G's loss; $3000 × $\frac{183000}{309000}$ = 1776.69\frac{93}{103}$, T's loss.

$8500 — 1223.30\frac{10}{103}$ = 7276.69$\frac{93}{103}$, G's share; $2500 — 1776.69\frac{279}{309}$ = 7723.30\frac{10}{103}$, T's share.

(9.)

$4000 × 6 = 24000
2000
2000 × 18 = 36000
J's product = 60000

$3500 × 8 = 28000
1500
2000 × 16 = 32000
E's product = 60000

$2500 × 10 = 25000
2000
4500 × 14 = 63000
B's product = 88000

J's product 60000
E's " 60000
B's " 88000
208000

$$\left.\begin{array}{l} 208000 : 60000 :: \$1041.80 : \$300.51\tfrac{12}{13},\ \text{J's}, \\ 208000 : 60000 :: \$1041.80 : \$300.51\tfrac{12}{13},\ \text{E's}, \\ 208000 : 88000 :: \$1041{:}80 : \$440.76\tfrac{2}{13},\ \text{B's}, \end{array}\right\}\text{Ans.}$$

10. This problem may be solved by analysis, thus: $10000 gain $22.50 in 12mo., and gain $\frac{2250}{12}$ = $187.50 in 1mo., and $\frac{187.50}{10000}$ = .01875, the gain of $1 in 1mo.; and $2800 × .01875 = $52.50 = Parkman's gain in 1mo.; and 420 ÷ 52.50 = 8mo., the time Parkman's money was in trade; and $3600 × .01875 = $67.50, Delano's gain in 1mo.; hence, 405 ÷ 67.50 = 6, the number of months Delano's money was in trade. To get the rate per cent., divide the gain for 1y. by the capital, 2250 ÷ 10000 = .22½ per cent., Ans.

The problem may be solved by Art. 360. Obtaining the rate per cent. as before, divide the gain by the interest of the principal for one year.

$2800 × .22½ = $630; 420 ÷ 630 = ⅔y. = 8mo., P's time.
$3600 × .22½ = $810; 405 ÷ 810 = ½y. = 6mo., D's time.

BANKRUPTCY.

(Art. **420**, p. **313.**)

2. $52384 — $1584 = $50800; 50800 ÷ 63500 = .80; $8361.55 × .80 = $6689.24, amount received by Dayton, Ans.

3. $600 + $760 + $840 + $800 = $3000; $2275 ÷ 3000 = $0.75⅚, the amount paid on $1;

$$\left.\begin{array}{l} \$600 \times .75\tfrac{5}{6} = \$455.00,\ \text{A's dividend}, \\ \$760 \times .75\tfrac{5}{6} = \$576.33\tfrac{1}{3},\ \text{B's}\ \text{``} \\ \$840 \times .75\tfrac{5}{6} = \$637.00,\ \text{C's}\ \text{``} \\ \$800 \times .75\tfrac{5}{6} = \$606.66\tfrac{2}{3},\ \text{D's}\ \text{``} \end{array}\right\}\text{Ans.}$$

4. $400 + $300 + $1000 = $1700; 600 ÷ 1700 = .35$\frac{5}{17}$;

$$\left.\begin{array}{l} \$\ \ 400 \times .35\tfrac{5}{17} = \$141.17\tfrac{11}{17},\ \text{A's share}, \\ \$\ \ 300 \times .35\tfrac{5}{17} = \$105.88\tfrac{4}{17},\ \text{B's}\ \text{``} \\ \$1000 \times .35\tfrac{5}{17} = \$352.94\tfrac{2}{17},\ \text{C's}\ \text{``} \end{array}\right\}\text{Ans.}$$

5. $180000 + $40000 + $12875 = $232875, and this amount contains both the sum distributed to the creditors and the 3½ per cent. charges; therefore, $232875 ÷ 1.03½ = $225000 = the amount distributed to the creditors; and 225000 ÷ 300000 = .75, the per cent. paid; $1360.60 × .75 = $1020.45, amount received by Henderson, Ans.

TAXES.

(Art. 425, p. 316.)

2. $1310 × .005 = $ 6.55, amount on F's property.
   $1.50 × 7 = $10.50, Forster's poll tax.
   $17.05, Forster's tax, Ans.
3. $415.35 × .005 = $2.07675, Ans.
4. $3506300 = ———, taxable property;
   $14018.90 — $3500 = $10518.90, to be assessed on property;
   $10518.90 ÷ 3506300 = .003 = tax on $1;
   $29010 × .003 = $87.03 = A's property tax;
   $87.03 + $2 = $89.03 = amount of A's tax, Ans.

(Art. 426, p. 316.)

2. $10450 ÷ .95 = $11000, Ans.
3. $9700 ÷ .97 = $10000 = assessment; $10000 — $9700 = $300 = cost of collection, Ans.
4. $756000 = ———, taxable property;
   $18000 ÷ .97 = $18556.70 = amount of tax to be raised;
   $1.50 × 450 = $675 = amount assessed on the polls;
   $18556.70 — $675 = $17881.70 = amount to be assessed on the property;
   $17881.70 ÷ 756000 = $0.0236 (nearly), tax on each dollar.

$15600 × .0236 = $368.16, A's property tax;
$1.50 × 3 = $4.50, A's poll tax;
$368.16 + $4.50 = $372.66, amount of A's tax, Ans.

(Art. **427**, p. **317.**)

2. $500 + $150 + $30 = $680, amount of expenditures;
$680 − $350 = $330, amount of rate money;
$330 ÷ 5500 = $0.06, rate per day;
60 + 60 + 30 = 150 days' attendance for A;
150 × 0.06 = $9, A's bill, Ans.

3. $150 + $18.50 = $168.50, amount of expenditures;
$168.50 — $63.50 = $105, amount of rate money;
$105 ÷ 3000 = $0.035, rate per day for each pupil;
121 × 0.035 = $4.235, A's rate bill, }
173 × 0.035 = $6.055, B's rate bill, } Ans.

GENERAL AVERAGE.

(Art. **432**, p. **320.**)

(2.)

CONTRIBUTORY INTERESTS.		LOSS FOR GENERAL BENEFIT.	
Vessel,	$31500	Thrown overboard,	$6500
Cargo,	6235	Repairs less ⅓,	700
Freight less ½,	2080	Cost of detention,	142
Entire contrib. int.,	$39815	Entire loss,	$7342

$7342 ÷ 39815 = .18½ (nearly), the loss per cent.;
$31500 × .18½ = $5827.50, am't payable by vessel;
2080 × .18½ = 384.80, am't payable by freight;
2145 × .18½ = 396.825, am't pay'e by Manning & Bro.;
1460 × .18½ = 270.10, am't pay'e by Anderson & Fiske;
960 × .18½ = 177.60, am't payable by Smidt & Huber;
670 × .18½ = 123.95, am't pay'e by Greenwood & Co.;
1000 × .18½ = 185.00, am't payable by A., R., & H.

EQUATION OF PAYMENTS.

(ART. **435**, p. **322**.)

3. \$200 to be paid in 3 months is the same as \$100 in 6 months; and \$300 in 5 months is the same as \$100 in 15 months; and \$500 in 10 months is the same as \$100 in 50 months. If, then, \$100 is to be paid in $6 + 15 + 50 = 71$ months, \$1000 should be paid in $\frac{1}{10}$ of 71 months $= \frac{71}{10} =$ 7mo. 3da., Ans.

OPERATION.

$\$\ 200 \times 3 = 600$

$\$\ 300 \times 5 = 1500$

$\$\ \underline{500} \times 10 = \underline{5000}$

$\$1000 \qquad 1000)7100(7\frac{1}{10}$ months, Ans.

$\underline{7000}$

100

4. $\$2000 \times$

$500 \times 12 = 6000$

$\underline{4000} \times 24 = \underline{96000}$

$6500 \qquad 102000$

$102000 \div 6500 = 15\frac{9}{13}$mo. = 15mo. $20\frac{10}{13}$da., Ans.

5. \$50 to be paid in 2 months is the same as \$100 in 1 month; \$150 in 8 months is the same as \$100 in 12 months. The equated time, then, will be as if \$100 were to be paid in 1 month, \$100 in 5 months, and \$100 in 12 months; that is, \$100 in $1 + 5 + 12 = 18$ months; consequently \$300 in $\frac{1}{3}$ of 18 months = 6 months, Ans.

OPERATION.

$\$\ 50 \times 2 = 100$

$\$100 \times 5 = 500$

$\underline{\$150} \times 8 = \underline{1200}$

$\$300 \qquad 300)1800(6$ months, Ans.

$\underline{1800}$

6. The first bill due is that for \$600, on May 1.

$$\begin{array}{llrlrcr}
\text{Due} & \text{May} & 1, & \$600 & & & \\
\text{"} & \text{July} & 7, & 370 & \times\ 67 & = & 24790 \\
\text{"} & \text{July} & 15, & 560 & \times\ 75 & = & 42000 \\
\text{"} & \text{Oct.} & 20, & \underline{420} & \times\ 172 & = & \underline{72240} \\
 & & & 1950 & & & 139030
\end{array}$$

139030 ÷ 1950 = 71 days, nearly; May 1 + 71 days = July 11, Ans.

7. The first bill due is that for \$675.25, on June 1, 1855.

$$\begin{array}{llrlrcr}
\text{Due} & \text{June} & 1, & \$675 & & & \\
\text{"} & \text{Nov.} & 4, & 376 & \times\ 156 & = & 58656 \\
\text{"} & \text{"} & 25, & 822 & \times\ 177 & = & 145494 \\
 & & 1856. & & & & \\
\text{Due} & \text{June} & 1, & 961 & \times\ 366 & = & 351726 \\
\text{"} & \text{Apr.} & 1, & 145 & \times\ 305 & = & 44225 \\
\text{"} & \text{Aug.} & 10, & 811 & \times\ 436 & = & 353596 \\
\text{"} & \text{"} & 12, & 568 & \times\ 438 & = & 248784 \\
\text{"} & \text{"} & 15, & \underline{370} & \times\ 441 & = & \underline{163170} \\
 & & & 4728 & & & 1365651
\end{array}$$

1365651 ÷ 4728 = 289 days; June 1, 1855 + 289 days = March 16, 1856, Ans.

(Art. **436**, p. **324**.)

2. As $\frac{1}{2}$ of \$144 is paid 7 months before it is due, it is evident the remainder, \$72, is payable in 14 months; but \$48 of this sum is paid in 4 months, which is 10 months before it is due. There will then remain \$24 unpaid; and the question will be, How long may \$24 be kept to pay the interest of \$72 for 10 months? \$72 for 10 months is the same as \$1 for 720 months; and \$1 for 720 months is the same as \$24 for $\frac{1}{24}$ of 720 months = 720 ÷ 24 = 30 months. To the 30 months we add the 4 months, and we have the whole equated time; thus, 30 + 4 = 34 months = 2y. 10mo., Ans.

OPERATION.

$$\begin{array}{llll}
\underline{144 \times 7} = 1008 & & & \\
72 & & & \\
\underline{48} \times 4 = \underline{192} & & & \\
\overline{120} & 24)816(34\text{mo.} = 2\text{y. }10\text{mo., Ans.} & & \\
\overline{24} & \quad\;\; \underline{72} & & \\
 & \quad\;\;\; 96 & & \\
 & \quad\;\;\; \underline{96} & &
\end{array}$$

3. The payments in this problem were not due before Nov. 23, 1856; the days are reckoned from the time each payment was made to the end of 8 months, Nov. 23.

		$8000				
Paid April	5,	1200	×	232	=	278400
" July	4,	1500	×	142	=	213000
" Sept.	25,	1800	×	59	=	106200
" Oct.	1,	1000	×	53	=	53000
" Nov.	20,	500	×	3	=	1500
		6000				652100
Balance unpaid,		$2000				

652100 ÷ 2000 = 326 days; Nov. 23, 1856 + 326 days = [Oct. 15, 1857, Ans.

4. $\$133\frac{1}{3}$ in 2 months is the same as $1 in $266\frac{2}{3}$ months; and $\$266\frac{2}{3}$ in 3 months is the same as $1 in 800 months; $400 in 6 months is the same as $1 in 2400 months; and $1 in $266\frac{2}{3} + 800 + 2400 = 3466\frac{2}{3}$ months is the same as $800 for $\frac{1}{800}$ of $3466\frac{2}{3}$ months $= 3466\frac{2}{3} \div 800 = 4\frac{1}{3}$ months. And if $\frac{1}{2}$ of $800 be paid down, the remainder, $400, may be kept twice $4\frac{1}{3}$ months $= 8\frac{2}{3}$ months, Ans.

OPERATION.

$$\begin{array}{lcl}
\$133\frac{1}{3} \times 2 = & 266\frac{2}{3} & \\
266\frac{2}{3} \times 3 = & 800 & \\
\underline{400} \times 6 = & \underline{2400} & \\
800 & 400)3466\frac{2}{3}(8 \text{ months.} & \\
\underline{400} & \underline{3200} & \\
400 & 266\frac{2}{3} & \\
 & \underline{30} & \\
 & 400)8000(20 \text{ days.} & \\
 & \underline{800} & \\
 & 0 &
\end{array}$$

5. The payments made in this bill were not due before March 25, 1857, 6 months from Sept. 25, 1856.

1856.			3051		
	Paid Oct.	4,	476 ×	172 =	81872
	" Nov.	12,	375 ×	133 =	49875
	" Dec.	5,	800 ×	110 =	88000
1857.					
	" Jan.	1,	200 ×	83 =	16600
			1851		236347
Balance unpaid,			$1200		

236347 ÷ 1200 = 197 days, nearly; March 25, 1857 + 197 [days = Oct. 8, 1857, Ans.

AVERAGING OF ACCOUNTS.

(ART. **438**, p. **327**.)

2. The first item in this account was due July 4, 1855.

Days reckoned from July 4, 1855.

Date	Amount	Days	Product
July 4,	$376		
Aug. 20,	816 ×	47 =	38352
Aug. 29,	178 ×	56 =	9968
Sept. 25,	387 ×	83 =	32121
Dec. 5,	419 ×	154 =	64526
	$2176		144967

Days reckoned from July 4, 1855.

Date	Amount	Days	Product
Aug. 10,	$316 ×	37 =	11692
Sept. 1,	675 ×	59 =	39825
Sept. 25,	512 ×	83 =	42496
Nov. 20,	162 ×	139 =	22518
Dec. 1,	100 ×	150 =	15000
	$1765		131531

$2176 — $1765 = $411, difference of items; 144967 — 131531 = 13436, difference of products; 13436 ÷ 411 = 32 + days; July 4, 1855 + 33 days = Aug. 6, 1855, Ans.

3. The first item due in this account is the cash payment of $700, April 1; from this date the time is reckoned.

Due 1856.					
Sept. 1,	$721	×	153	=	110313
Sept. 20,	815	×	172	=	140180
Oct. 11,	588	×	193	=	113484
Oct. 30,	300	×	212	=	63600
Dec. 15,	625	×	258	=	161250
1857.					
Jan. 18,	560	×	292	=	163520
*Feb. 28,	685	×	333	=	228105
Mar. 25,	365	×	358	=	130670
	$4659				1111122

Due 1856.					
April 1,	$700				
Nov. 30,	570	×	243	=	138510
July 20,	500	×	110	=	55000
Sept. 25,	100	×	177	=	17700
1857.					
Mar. 30,	750	×	363	=	272250
April 30,	330	×	394	=	130020
May 20,	500	×	414	=	207000
	$3450				820480

1111122 — 820480 = 290642, balance of products;
$4659 — $3450 = $1209, balance of the items;
290642 ÷ 1209 = 240 days; 240 days forward from April 1, 1856 = November 27, 1856, Ans.

4. In this problem the time is reckoned from the first date in the account to the date of each item, it being due from date.

1856.	(Time from March 1.)				
Mar. 1,	$36.25				
April 7,	18.15	×	37	=	671.55
June 15,	48.26	×	106	=	5115.56
July 21,	91.20	×	142	=	12950.40
Aug. 1,	30.00	×	153	=	4590.00
	$223.86				23327.51

1856.	(Time from April 1.)				
April 1,	$48.25				
May 20,	90.10	×	49	=	4414.90
June 17,	12.50	×	77	=	962.50
July 4,	20.00	×	94	=	1880.00
July 10,	25.00	×	100	=	2500.00
	$195.85				9757.40

23327.51 ÷ 223.86 = 104 days; March 1 + 104 days = June 13;

9757.40 ÷ 195.85 = 50 days, nearly; April 1 + 50 days = May 21;

---

* See note 2, p. 350, National Arithmetic.

June 13 is the average time of the debit items becoming due; and May 21 is the average time of the credit items becoming due. From May 21 to June 13 = 23 days = difference between the average dates; then the amount of the smaller side, 195.85 × 23 = 4504.55; 4504.55 ÷ 28.01 (= 223.86 — 195.85, balance of the account) = 161 days nearly; hence, June 13 (the average date of the larger size) + 161 days = Nov. 21, 1856, Ans.

$223.86 — $195.85 = $28.01, balance due Nov. 21; $28.01 ÷ 1.01 (the amount of $1 from Sept. 21, 1856, to Nov. 21, 1856) = $27.73, the cash value of balance, Sept. 1, 1856, Ans.

5. In the adjustment of this account, the time is reckoned for the debit items from the first date; after ascertaining the average time of payment, the 8 months of credit are added.

1856.			1857.		
May 1,	$300		Jan. 1,	$500	
July 7,	760 × 67 =	50920	*Oct. 18,	482 × 290 =	139780
Sept. 11,	417 × 133 =	55461	Mar. 19,	750 × 77 =	57750
Nov. 25,	288 × 208 =	59904	April 1,	210 × 90 =	18900
Dec. 20,	571 × 233 =	133043	May 25,	100 × 144 =	14400
	$2336	299328		$2042	230830

299328 ÷ 2336 = 128 days; May 1 + 128 days = Sept. 6, 1856.

230830 ÷ 2042 = 113 days; Jan. 1 + 113 days = April 24, 1857.

Sept. 6, 1856, is the average date of the debits; and Sept. 6, 1856 + 8 mo. = May 6, 1857, the average time of the debits becoming due; April 24 is the average time of the credits becoming due. The difference between April 24 and May 6 is 12 days; 2042 (amount of smaller side) × 12 = 24504; 24504 ÷ 294 (balance of the account)

---

* Due Oct. 18.

= 83 days; May 6 (average time of the larger side) + 83 days = July 28, 1857, Ans.

Also, what will be the value of the balance on Nov. 28, 1857? $294 is the balance; from July 28 to Nov. 28 are 4 months; the amount of $294 for 4 months = $294 × 1.02 = $299.88, Ans.

6. In this problem the first item of debits becoming due is that of $560, on June 1; and the first item of credits becoming due is that of $100, cash paid June 20.

Due June 1, $560.			Due June 20, $100.		
846 ×	71 =	60066	820 ×	119 =	97580
728 ×	111 =	80808	350 ×	79 =	27650
400 ×	59 =	23600	800 ×	56 =	44800
560 ×	345 =	193200	630 ×	271 =	170730
600 ×	200 =	120000	400 ×	130 =	52000
500 ×	343 =	171500	750 ×	254 =	190500
$4194		649174	$3850		583260

649174 ÷ 4194 = 155 days; June 1 + 155 days = Nov. 3, 1856.

583260 ÷ 3850 = 151 days; June 20 + 151 days = Nov. 18, 1856.

Nov. 18 — Nov. 3 = 15 days, the difference between the average date of the debits and credits becoming due; 3850 (the amount of the smaller side) × 15 = 57750; 57750 ÷ 344 (the balance of the account) = 168 days. Since the amount of the larger side becomes due first, Nov. 3, the 168 days must be counted backward from that date; therefore, Nov. 3, 1856 — 168 days = May 19, 1856, Ans.

The time from this date to the time of settlement, 12mo. 5da.; therefore, $344 × 1.060$\frac{5}{6}$ = $364.93, the value of the balance, May 24, 1857, Ans.

(Art. 439, p. 331.)

Alfred Hicks *in account with* Keen & Lee.

1857.	1857.
Due July 1, $156 × 153 = 23868	Due Feb. 1, $120 × 303 = 36360
" Feb. 3, 100 × 301 = 30100	" July 20, 420 × 134 = 56280
" July 20, 317 × 134 = 42478	" Nov. 1, 300 × 30 = 9000
" " 30, 162 × 124 = 20088	" " 1, 50 × 30 = 1500
" May 15, 100 × 200 = 20000	1858.
1858.	" Feb. 20, 213 × 81 = 17253*
" Jan. 10, 100 × 40 = 4000*	$890 6)120393
$835 6)140534	100 20.065
213 23.422	$990
$1048	

$1048 — $990 = $58 = balance of items.

$23.422 — $20.065 = $3.36 = balance of interest.

$58 + $3.36 = $61.36, balance due Dec. 1, 1857.

* These products becoming due after settlement, are transferred each to its opposite side; but the items themselves must be added in their own places.

4. In finding the balance of this account, the time is reckoned from the date of each item, since it is due from date.

$200 × 264 = 52800	$300 × 248 = 74400
300 × 198 = 59400	350 × 222 = 77700
250 × 181 = 45250	400 × 205 = 82000
600 × 175 = 105000	320 × 144 = 46080
400 × 125 = 50000	600 × 109 = 65400
500 × 103 = 51500	100 × 95 = 9500
100 × 69 = 6900	200 × 52 = 10400
120 × 14 = 1680	150 × 25 = 3750
$2470 372530	$2420 369230

372530 — 369230 = 3300; 3300 ÷ 6000 = $0.55, balance of interest at 6 per cent.; $0.55 ÷ 6 = $\$0.09\frac{1}{6}$; $\$0.09\frac{1}{6} \times 7 = \$0.64$, balance of interest at 7 per cent.;

Items of debit, $2470 Balance of items, $50

Items of credit, 2420 Balance of interest, 0.64

Balance of items, $50; Bal. due Mar. 25, 1857, $50.64, Ans.

ACCOUNTS OF STORAGE.

(Art. **441**, p. **333**.)

2. 1857.		chests.		da.		prod.
May 16,	Rec.	4560	×	14	=	63840
" 30,	Deliv.	564				
	Bal.	3996	×	2	=	7992
June 1,	Deliv.	904				
	Bal.	3092	×	38	=	117496
July 9,	Deliv.	1000				
	Bal.	2092	×	25	=	52300
Aug. 3,	Deliv.	1500				
	Bal.	592	×	13	=	7696
" 16,	Deliv.	592				30)249324

Chests chargeable for 1mo., 83108

8310.8 × $0.03 = $249.32,4, cost of storage, Ans.

3. 1857.		Bales.		da.		prod.
Jan. 1,	Rec.	2310	×	15	=	34650
" 16,	"	120				
		2430	×	16	=	38880
Feb. 1,	Rec.	300				
		2730	×	11	=	30030
Feb. 12,	Deliv.	1000				
		1730	×	17	=	29410
Mar. 1,	Deliv.	600				
		1130	×	33	=	37290
Apr. 3,	Deliv.	400	(carried forward).			

Apr. 3, Deliv. 400 (brought forward).
730 × 7 = 5110
Apr. 10, Deliv. 312
418 × 21 = 8778
May 1, In store, 418ba. 30)184148
Bales chargeable for 1 month, 6138
6138 × $0.05. = $306.90, cost of storage up to May 1.

MISCELLANEOUS EXAMPLES.

(PAGE **333.**)

1. The ratio of $1120, A's gain, to $3500, A's stock = $\frac{1120}{3500} = \frac{8}{25}$.
   The ratio of $880, B's gain, to $2200, B's stock = $\frac{880}{2200} = \frac{10}{25}$.
   The ratio of $1200, C's gain, to $2500, C's stock = $\frac{1200}{2500} = \frac{12}{25}$.
   $\frac{10}{25} - \frac{8}{25} = \frac{2}{25}$ = the gain of 2mo., B's time more than A's;
   $\frac{2}{25} : \frac{8}{25}$ :: 2mo. : 8mo., A's time; 8 + 2 = 10mo., B's time;
   $\frac{2}{25} : \frac{12}{25}$ :: 2mo. : 12mo., C's time, Ans.
2. $540 + $660 = $1200; $540 ÷ 1200 = 45cts. on $1; $15000 × .45 = $6750, paid by the bankrupt; then, $15000 — $6750 = $8250, Ans.
3. $64.50 — $1.50 = $63, A's property tax; $50.50 — $1.50 = $49, B's property tax; $63 ÷ .007 = $9000, A's property; $49 ÷ .007 = $7000, B's property, Ans.
4. $20 + $30 + $150 = $200, total expenses; $200 — $50 = $150, raised by tax; $150 ÷ .03 = 5000, days of attendance, Ans.
5. Kimball's whole loss was made up *minus* a 5 per cent. tax on his share of the cargo; therefore, $7000 × .05 = $350, Ans.

6. \$50 for 4mo. is \$1 for 200mo.; and \$100 for 8mo. is \$1 for 800mo. If, then, \$1 is to be paid in $200 + 800 = 1000$ months, \$150 should be paid in $\frac{1}{150}$ of 1000mo. $= \frac{1000}{150} = 6\frac{2}{3}$mo. It appears, then, that \$150 is paid $6\frac{2}{3}$mo. before it is due. The question now is, In how much *less* than 10mo. should the \$250 be paid? As we have before shown that \$1 is to be paid in 1000mo., \$250 should be paid in $\frac{1}{250}$ of 1000mo. $= 1000 \div 250 = 4$mo. We find, then, that the time in which B is to pay A is 4mo. *less* than it otherwise would have been, on account of A's making present payment. We therefore subtract 4mo. from 10, and find the true answer; thus $10 - 4 = 6$mo., Ans.

OPERATION.

$$\begin{array}{ll} 50 \times 4 = 200 & \\ \underline{100} \times 8 = \underline{800} & \\ 150 & 150\overline{)1000}(6\frac{2}{3}\text{mo.} \\ & \underline{1000} \end{array}$$

$150 \times 6\frac{2}{3} = 1000 \div 250 = 4$mo.

$10 - 4 = 6$mo., Ans.

(7.)

$\frac{1}{3} = \frac{4}{12}$

$\frac{1}{4} = \frac{3}{12} \times 6 = \frac{18}{12}$

$\frac{1}{6} = \frac{2}{12} \times 8 = \frac{16}{12}$

$\frac{1}{4} = \frac{3}{12} \times 12 = \frac{36}{12}$

$\frac{4}{12} + \frac{3}{12} + \frac{2}{12} + \frac{3}{12} = \frac{12}{12}$

$\frac{18}{12} + \frac{16}{12} + \frac{36}{12} = \frac{70}{12}$

$\frac{70}{12} \div \frac{12}{12} = 5\frac{10}{12}$mo. =

5mo. 25da., Ans.

8. \$2000 + his services are worth \$800 per year; but \$4000 + his services are worth \$1100 per year; therefore, \$2000 are worth \$300 per year; hence his services without the \$2000 are worth \$800 — \$300 = \$500, Ans.

9. $\$100 \times 4 = 400$

$\$100 \times 8 = 800$

$\underline{\$200} \times 12 = \underline{2400}$

$\$400 \qquad 3600$

$3600 \div 400 = 9$mo. for \$400;

200 : 400 : : 9 : 18mo. for \$200; that is, \$200 running 18mo. is equal to \$400 running 9mo. If now 18 be divided into two parts,

one of which shall be twice as great as the other, we have 12 and 6; hence, if one note of \$200 run 12mo., and the other note of \$200 run 6mo., it will be equivalent to \$200 in 18mo., or to \$400 in 9mo.; therefore, in 6mo. and 12mo., Ans.

10. The question involved in this problem is, In what time will the interest of \$360 be \$3.78; $\$360 \times .06 = \$21.60$, interest of \$360 for 1y.; then, $3.78 \div 21.60$ will be the time in years (Art. 360) $= \frac{378}{2160} = \frac{7}{40}$y. = 2.1mo. = 63da., Ans.

INSURANCE.

(Art. **445**, p. **335.**)

2. $\$896 \times .12 = \$107.52$, Ans.

3. $\$17289 \times .01\frac{1}{4} = 216.11\frac{1}{4}$, Ans.

4. $\$35000 \times \frac{3}{4} \times .03\frac{1}{4} = \$682.50$; and $\$75000 \times \frac{2}{3} \times .02\frac{1}{2} = \$1250$; $\$1250 + \$682.50 = \$1932.50$, the amount of premium, Ans.

5. The loss was what the insurance cost; therefore, $\$3675 \times .04\frac{7}{8} = \$179.15\frac{5}{8}$, the premium; $\$179.15\frac{5}{8} + \$1$, the cost of the policy $= \$180.15\frac{5}{8}$, Ans.

6. $690.90 \div 9870 = .07$, Ans.

7. $.04\frac{1}{4} + .03\frac{7}{8} + .04\frac{2}{3} + .05 + .05\frac{1}{3} = .23\frac{1}{8}$; $\$47600 \times .23\frac{1}{8} = \$11007.50$, Ans.

(Art. **446**, p. **336.**)

2. $\$26250 \div .87\frac{1}{2} = \$30000$, Ans.

3. $\$6370 \div .98 = \$6500$, Ans.

4. In this problem the rate per cent. and interest are given, by which to find the principal; then (Art. 358), $\$50 \div .01\frac{1}{4} = \$4000$, Ans.

5. As the policy is 10 per cent. of \$600, the property must be 90 per cent. of \$600; therefore $\$600 \times .90 = \$540$, Ans.

6. 1.00 — .03 = .97; $1000 ÷ .97 = $1030.927+
1.00 — .05 = .95; $1030.927 ÷ .95 = $1085.187+
1.00 — .06 = .94; $1085.187 ÷ .94 = $1154.454+.
1.00 — .07 = .93; $1154.454 ÷ .93 = $1241.348+
Ans.

LIFE INSURANCE.

(Art. 454, p. 340.)

2. $8000 × .0114 = **$91.20, Ans.**

3. $78000 × .0491 = $3829.80, Ans.

4. $2000 × .0088 = $17.60, Ans.

5. $12000 × .0373 = $447.60, the annual premium; the insured died in the third year, so that this premium was paid for three years; therefore, $447.60 × 3 = $1342.80, and $12000 — $1342.80 = $10657.20, Ans.

6. 80y. — 50. = 30y., for which the annual premium was paid; $5000 × .046 = $230; $230 × 30 = $6900, the amount of premiums paid; therefore $6900 — $5000 = $1900, Ans.

7. The number of years the premium is paid is 4;
$4000 × $\frac{5.20}{100}$ = $208, the annual premium;
The interest of $1 for 4y., and 3y., and 2y., and 1y., is respectively .24 + .18 + .12 + .06 = .60; and $208 × .60 = $124.80, the amount of interest; $208 × 4 = $832, the amount of premium; $124.80 + $832 = $956.80; then, $4000 — $956.80 = $3043.20, Ans.

8. $10000 × .02 = $200, the annual premium.
The last annual premium paid is not charged with interest; to shorten the work, we take the amount of $1 at compound interest from the table, for the periods of 11y., and 10y., and 9y., etc.; these added together give $15.869947, which is the sum of the amounts of an annual premium of $1 for 11y.; a premium of $200 will be 200

times this = \$ 15.869947 × 200 = \$ 3173.9894; to this add the premium for the 12th year, which is not chargeable with interest; \$ 3173.9894 + \$ 200 = \$ 3373.9894; and \$ 10000 − \$ 3373.9894 = \$ 6626.01, Ans.

Note. — The work of this problem might be much shortened by referring to the table of Annuities later in the book.

CUSTOM-HOUSE BUSINESS.

(Art. **460**, p. **343.**)

2. \$ 4350 × .08 = \$ 348, Ans.

3. \$ 7890 × .15 = \$ 1183.50; \$ 1183.50 × .19 = \$ 224.865, Ans.

4. 986 × 4.84 × .24 = \$ 1145.3376, Ans.

5. 112.50 ÷ 750 = 15 per cent., Ans.

6. 60 × .95 × 70 = 3990fr., value of champagne;
36 × .95 × 35 = 1197fr., value of port;
50 × (31 — 1) × 4 = 6000fr., value of sherry;
11187

11187 × .30 = 3356.1fr. duties; 3356.1 × .18$\frac{6}{10}$ = \$ 624.2346, Ans.

7. \$ 53.76 ÷ .08 = \$ 672; 60 × (112 — 12) = 6000lb.; \$ 672 ÷ 6000 = 11$\frac{1}{5}$ cts., Ans.

8. 40 × 63 × .98 = 2469.6gal. molasses charged with duty;
24 × 400 × .90 = 8640lb. sugar charged with duty;
2469.6 × 3 × 10 × .24 = \$ 177.81
8640 × 1 × 5 × .24 = 103.68
260 × 2 × .08 = 41.60
410 × 7 × .30 = 861.00
\$ 1184.09, Ans.

9. 270 × \$ 50 = \$ 13500; \$ 16740 — \$ 13500 = \$ 3240; 3240 ÷ 13500 = .24, Ans.

10.
$$\begin{array}{rrl}
\text{Whole invoice} & \underline{\$20560} & \\
 & 3000 \times .04 = & \$120.00 \\
 & 4200 \times .08 = & 336.00 \\
 & 2100 \times .15 = & 315.00 \\
 & \underline{6000} & \\
 & \$15300 & \\
\text{Bal. of invoice} & \$5260 \times .30 = & \underline{1578.00} \\
 & & \$2349.00,\ \text{Ans.}
\end{array}$$

11.
$$\begin{array}{rrl}
10 \times 40 \times \frac{5}{20} \times & .24 = & 24\ \text{£.} \\
200 \times \frac{4}{20} \times & .19 = & 7.6\ \text{£.} \\
100 \times \frac{2\frac{1}{2}}{20} \times & .15 = & 1.875\text{£.} \\
60 \times .04 & = & \underline{2.4\ \text{£.}} \\
 & & 35.875\text{£.} \times 4.84 = \$173.635,\ [\text{Ans.}
\end{array}$$

12. $32 \times 32 = 1024$yds.; \$$122.88 \div .24 =$ \$512, invoice value of the cloth; \$$512 \div 1024 =$ \$0.50 per yard; \$512 + \$122.88, the cost of duties, + \$40.96, other charges, = \$675.84; \$$675.84 \div 1024 =$ \$0.66, cost per yard, Ans.

COINS AND CURRENCIES.

(Art. **470**, p. **348.**)

3. 46£. 16s. 6d. = 46.825£.; $46.825 \div \frac{2}{5} =$ \$$117.06\frac{1}{4}$, Ans.

4. $1032 \times \frac{3}{8} = 387$£., Ans.

5. $515.70 \div 4 = 128.925$£.; 128.925£. = 128£. 18s. 6d., Ans.

6. $160.50 \times .18\frac{6}{10} =$ \$29.853, Ans.

7. $728.41 \div 4.84 = 150\frac{241}{484}$£. = 150£. 9s. $11\frac{61}{121}$d., Ans.

8. 12£. 12s. = 12.6£.; $12.6 \div \frac{7}{30} =$ \$54, Ans.

9. 128£. 18s. 6d. = 128.925£.; 128.925 × 4 = $515.70, Ans.

10. 740.45 × 0.75 = 555.33\frac{3}{4}$, Ans.

11. 46£. 16s. 6d. = 46.825£.; 46.825 ÷ $\frac{2}{5}$ = 117.06\frac{1}{4}$; 117.06$\frac{1}{4}$ × $\frac{3}{10}$ = 35.118$\frac{3}{4}$£. = 35£. 2s. 4$\frac{1}{2}$d., Ans.

12. 151 × 1.12 = $169.12; 169.12 ÷ .10 = 1691.2 reals plate, Ans.

13. 1000 × 1.06 = $1060; 1060 ÷ 18$\frac{6}{10}$ = 5698$\frac{86}{93}$ francs, Ans.

14. 30 × 30 × 15 = 13500s. sterling = 675£.; 675 × 4.84 = $3267; 3267 ÷ 4 = 816.75£. = 816£. 15s. Canada currency, Ans.

15. 410 × .80 = $328; 328 ÷ .69 = 475$\frac{25}{69}$ thalers, Ans.

EXCHANGE.

(Art. **493**, p. **354.**)

2. $1950.50 × 1.00$\frac{1}{4}$ = $1955.37, Ans.

3. $2000 × 1.005 = $2010, Ans.

4. $1744.40 ÷ .98 = $1780, face of draft, Ans.

5. $600 × .99 = $594; $594 — $6.30 (the interest of $600 for 63 days) = $587.70, Ans.

6. $9256.40 × 1.00$\frac{3}{8}$ = 9291.11; $9291.11 — $50.91 (the interest of $9256.40 for 33 days) = $9240.20, Ans.

7. Find the value of $1 at $\frac{3}{8}$ per cent. premium, and interest off for 33 days at 6 per cent.; $1 × 1.00$\frac{3}{8}$ = $1.00375; $1.00375 — $0.0055 = $0.99825, the value of $1 by the conditions of the question; hence, as $0.99825 is to $1, so is $9240.20 to that sum of which $9240.20 is the value or yield; therefore, $9240.20 ÷ .99825 = $9256.39+, Ans.

8. The yield of $1, by the conditions of the question, is $0.982; then, $1190.184 ÷ .982 = $1212, Ans.

(ART. **499**, p. **357**.)

3. $3676 \div 5\frac{20}{100} = \$706.92\frac{4}{13}$, Ans.

4. $3000\frac{10}{16} \times 0.35 = \$1050.21\frac{7}{8}$, Ans.

5. $81727.75 \div 4.867 = 16792.223$£. = 16792£. 4s. $5\frac{1}{2}$d., Ans.

6. $79000 \times .186 = \$14694$, the par value of 79000 francs; $\$14694 - \$14400.12 = \$293.88$; $293.88 \div 14694 = .02$, Ans.

7. $5763.75 \times 0.74 = \$4265.175$, Ans.

8. $550.66 \div 1.10 = 500\frac{6}{10}$ millreas = 500 millreas 600 reas, Ans.

9. $10000 \div .68 = 14705\frac{60}{68}$ thalers = 14705 thalers 26 groschen $5\frac{1}{17}$ pfennings, Ans.

10. $400\frac{12}{48} \times 1.06 = \$424.265$, Ans.

11. 300 thalers 20 groschen 0 pfennings = $300.6\frac{17}{18}$ thalers; $300.6\frac{17}{18} \times .69 = \$207.47\frac{11}{12}$, Ans.

12. $5694 \div .40 = 14235$ rupees, Ans.

13. $615.60 \div .80 = 769.5$ ducats = 769 ducats 5 carlini, Ans.

14. $7300 \div 1500 = \$4.86\frac{2}{3}$; $4.86\frac{2}{3} - 4.44\frac{4}{9} = .42\frac{2}{9}$; $.42\frac{2}{9} \div 4.44\frac{4}{9} = .09\frac{1}{2}$, Ans.

15. 868£. 17s. 6d. = 868.875£.; $868.875 \times 23.60 = 20505.45$ francs = 20505 francs 45 centimes, Ans.

16. $2640 \div .18 = 14666.66\frac{2}{3}$ lire = 14666 lire $66\frac{2}{3}$ centesimi, Ans.

17. $17280 \div .186 = 92903\frac{7}{31}$ francs; $92903\frac{7}{31} - 88128 = 4775\frac{7}{31}$; $4775\frac{7}{31} \div 88128 = 5\frac{1985}{4743}$ per cent., Ans.

18. English money at $8\frac{1}{2}$ per cent. premium $= \frac{40}{9} \times 1.085 = \frac{43.4}{9} = \$4.82\frac{2}{9}$; $17000 \times 4.82\frac{2}{9} = \$81977.777$, Ans.

ARBITRATION OF EXCHANGES.

(Art. **504**, p. **361**.)

(2.)

1m. = 5.95fr.
5.20fr. = $1
$3500 = —— m.

$$\frac{1 \times 5.20 \times 3500}{1 \times 5.95} = 3058.823\tfrac{9}{17} \text{ millreas} = 3058 \text{ millreas}$$

$823\tfrac{9}{17}$ reas, Ans.

(3.)

EXCHANGE DIRECT.

$$4\tfrac{8}{9}$ = 1£.
2000£. = $9777.777

EXCHANGE THROUGH PARIS.

$1 = 5.20fr.
24fr. = 1£.
2000£. = $9230.769

EXCHANGE THROUGH HAMBURG.

$0.35 = 1 mar. banc.
$13\tfrac{3}{4}$ m. b. = 1£.
2000£. = $9625.

Exchange through Paris is better than exchange direct by $547.008 ; and better than exchange through Hamburg by $394.231.

4. $5000.00
12.50, $\tfrac{1}{4}$ per cent. discount added;
$5012.50, exchange value of $5000 at New Orleans, to be paid in New York;
$5012.50
25.0625, $\tfrac{1}{2}$ per cent. of $5012.50 subtracted;
$4987.4375, exchange value of $5012.50 at St. Louis, or New Orleans. The exchange being made through New Orleans, $4987.4375 in St. Louis will pay a debt of $5000 in New York; $5000 × 1.015 = $5075 = the amount that must be paid by direct exchange; therefore, $5075 — $4987.4375 = 87.56\tfrac{1}{4}$, Ans.

5. $\$4.84\frac{4}{9} = 1£.$

$1£. = \frac{120}{17}$ thalers.

$9760\text{th.} = \$6698.25+;\ \$6698.25 \times 1.005 = \$6731.74+$ [Ans.

6. $\$0.34 \quad = 1$ mark banco.

$2\frac{1}{2}$ m. b. $= 1$ ruble.

—— $= \$650.$

$$\frac{650}{.34 \times 2\frac{1}{2}} = 764\frac{12}{17} \text{ rubles} = 764 \text{ rubles } 70\frac{10}{17} \text{ kopecks,}$$ [Ans.

7. By the question, $1 on Boston = $1.00¼ on Philadelphia, by direct exchange; and $1 on Boston = $1 on Chicago, and $1 on Chicago = $0.98 on Philadelphia; consequently, $1 on Boston = $0.98 on Philadelphia, by circuitous exchange; therefore, exchange on Boston through Chicago is better than direct exchange by the difference between $1.00¼ and $0.98 = 2¼ per cent., Ans.

8.
$$\left.\begin{array}{l} 8 \text{ guilders} = 1 \text{ Amsterdam ell,} \\ 1 \text{ ell} \quad = \frac{27}{36} = \frac{3}{4}\text{yd., and} \\ 8 \text{ guilders} = \$3.20, \text{ and} \\ \$3.20 = \frac{3}{4}\text{yd., or } \$4.26\frac{2}{3} = 1\text{yd.} \end{array}\right\} \text{Amsterdam.}$$

$$\left.\begin{array}{l} 3\frac{1}{2} \text{ thalers} = 1 \text{ Berlin ell,} \\ 1 \text{ ell} \quad = \dfrac{25\frac{1}{2}}{36} = \frac{17}{24}\text{yd., and} \\ 3\frac{1}{2} \text{ thalers} = 3\frac{1}{2} \times .66 = \$2.31, \\ \$2.31 = \frac{17}{24}\text{yd., or } \$3.26\frac{2}{17} = 1\text{yd.} \end{array}\right\} \text{Berlin.}$$

$$\left.\begin{array}{l} 1£. = \$4.86\frac{6}{9};\ \frac{15}{20} \text{ of } \$4.86\frac{6}{9} = \$3.65, \\ \$3.65 = 1\text{yd., bought in England.} \end{array}\right\} \text{England.}$$

Therefore, Berlin is better for the purchase than Amsterdam or England, by the difference between $\$3.26\frac{2}{17}$ and $\$4.26\frac{2}{3}$, or $3.65 respectively.

(9.)

$$\begin{aligned} \$4.80 &= 1£. \\ 1£. &= 25\tfrac{1}{4} \text{ francs.} \\ 7000 \text{ francs} &= \$ \text{———} \end{aligned}$$

$$\frac{4.80 \times 7000}{25\frac{1}{4}} = \$1330.69, \text{ Ans.}$$

(10.)

$$\begin{aligned} 1£. &= 13\tfrac{1}{3} \text{ guilders.} \\ 1\text{g.} &= 2\tfrac{1}{5} \text{ francs.} \\ \text{— francs} &= 1000£. \end{aligned}$$

$\dfrac{13\frac{1}{3} \times 2\frac{1}{5} \times 1000}{1} = 29333\frac{1}{3}$ francs; $29333\frac{1}{3} - 146\frac{2}{3}$ ($\frac{1}{2}$ per cent. commission) $= 4186\frac{2}{3}$ whole gain; $4186\frac{2}{3} \div 25000$ ($1000£. \times 25 = 25000$ francs) $= 16\frac{56}{75}$ gain per cent., Ans.

ALLIGATION.

(Art. **506**, p. **363**.)

(2.)

$$\begin{array}{rcr} \$1.00 \times 19 & = & \$19.00 \\ .66 \times 40 & = & 26.40 \\ .50 \times \underline{11} & = & \underline{5.50} \\ 70) & & \$50.90 \\ & & \overline{\$0.727\tfrac{1}{7}}, \text{ Ans.} \end{array}$$

(3.)

$$\begin{array}{r} 22 \times 3 = 66 \\ 20 \times 3 = \underline{60} \\ 6)\ \ 126 \\ \overline{21}, \text{ Ans.} \end{array}$$

(4.)

$$\begin{array}{rcr} 70 \times 20 & = & \$14.00 \\ 60 \times 15 & = & 9.00 \\ 40 \times \underline{80} & = & \underline{32.00} \\ 115) & & 55.00 \\ & & \overline{\$0.47\tfrac{19}{23}}, \text{ Ans.} \end{array}$$

(ART. **509**, p. **365**.)

(2.)

$$80\left\{\begin{array}{l}\left.\begin{array}{l}\text{1bu. at } 45\text{, gain } 35\\ \text{1bu. at } 75\text{, gain } 5\end{array}\right\}=40\text{ gain.}\\ \left.\begin{array}{ll}\text{1bu. at } 100\text{, loss } 20 & =20\text{ loss}\\ \hline \text{1bu. at } 100\text{, loss } 20 & =20\text{ loss}\end{array}\right\}=40\text{ loss.}\end{array}\right.$$

1bu. of barley, 1bu. of rye, and 2 of wheat, Ans.

(3.)

$$20\left\{\begin{array}{l}\left.\begin{array}{l}\text{1 of } 24\text{, loss } 4\\ \text{1 of } 23\text{, loss } 3\end{array}\right\}=7\text{ loss.}\\ \left.\begin{array}{ll}\left.\begin{array}{l}\text{1 of } 19\text{, gain } 1\\ \text{1 of } 15\text{, gain } 5\end{array}\right\} & =6\text{ gain.}\\ \hline \text{1 of } 19\text{, gain } 1 & =1\text{ gain}\end{array}\right\}=7\text{ gain.}\end{array}\right.$$

1 part of 24, 1 of 23, 1 of 15, and 2 of 19, Ans.

(4.)

$$75\left\{\begin{array}{l}\left.\begin{array}{l}\text{1 gal. at } 0\text{, gain } 75\\ \text{1 gal. at } 60\text{, gain } 15\end{array}\right\}=90\text{ gain.}\\ \left.\begin{array}{l}\text{1 gal. at } 80\text{, loss } 5\\ \text{1 gal. at } 120\text{, loss } 45\\ \hline \text{8 gal. at } 80\text{, loss } (8\times 5)\ 40\end{array}\right\}=90\text{ loss.}\end{array}\right.$$

1 gal. of water, 1 of 60 cts., 1 of 120 cts., and 9 of 80 cts, Ans.

(ART. **510**, p. **366**.)

(2.)

$$100\left\{\begin{array}{l}\text{20 at } 0\text{, } 2000\text{ gain}=2000\text{ gain.}\\ \left.\begin{array}{l}\text{1 at } 175\text{, } 75\text{ loss}\\ \text{1 at } 125\text{, } 25\text{ loss}\\ \hline \text{19 at } 175\text{, } 1425\text{ loss}\\ \text{19 at } 125\text{, } 475\text{ loss}\end{array}\right\}=2000\text{ loss.}\end{array}\right.$$

20 gallons of each price, Ans.

(3.)

$$190\left\{\begin{array}{l}
\left.\begin{array}{l}1\text{ at }180,\text{ gain }10\\10\text{ at }170,\text{ gain }200\end{array}\right\}=210\text{ gain.}\\
\left.\begin{array}{l}4\text{ at }220,\text{ loss }120\\1\text{ at }200,\text{ loss }10\\\hline 8\text{ at }200,\text{ loss }80\end{array}\right\}=210\text{ loss.}
\end{array}\right.$$

9 bushels at $2.00; 1 bushel at $1.80, **Ans.**

(4.)

$$\begin{array}{l}
\left.\begin{array}{llll}1&\text{at }8,&\text{gain}&4\frac{1}{2}\\3&\text{at }9\frac{1}{4},&\text{gain}&9\frac{3}{4}\\4&\text{at }10\frac{1}{2},&\text{gain}&8\end{array}\right\}=22\frac{1}{4}\text{ gain.}\\
\left.\begin{array}{llll}1&\text{at }13,&\text{loss}&\frac{1}{2}\\6&\text{at }13\frac{1}{2},&\text{loss}&6\\1&\text{at }14,&\text{loss}&1\frac{1}{2}\\\hline 4\frac{1}{2}&\text{at }13,&\text{loss}&2\frac{1}{4}\\8&\text{at }14,&\text{loss}&12\end{array}\right\}=22\frac{1}{4}\text{ loss.}
\end{array}$$

Ans. 1lb. at 8 cts., 5½lb. at 13 cts., and 9lb. at 14cts.

(5.)

$$50\left\{\begin{array}{l}
10\text{ of }58,\text{ loss }80\quad=80\text{ loss.}\\
\left.\begin{array}{l}1\text{ of }45,\text{ gain }5\\\hline 15\text{ of }45,\text{ gain }75\end{array}\right\}=80\text{ gain.}
\end{array}\right.$$

Ans. 16 bushels of barley.

(Art. **511,** p. **367.**)

(2.)

$$15\left\{\begin{array}{l}
\left.\begin{array}{l}1\text{ at }8,\text{ gain }7\\1\text{ at }10,\text{ gain }5\\1\text{ at }12,\text{ gain }3\end{array}\right\}=15\text{ gain.}\\
\left.\begin{array}{l}1\text{ at }20,\text{ loss }5\\\hline 2\text{ at }20,\text{ loss }10\end{array}\right\}=15\text{ loss.}
\end{array}\right.$$

$1+1+1+3=6$; sugar at 8, 10, and 12 cents, will be each 1 part in 6; and sugar at 20 cents will be 3 parts in 6; $\frac{1}{6}$ of $200=33\frac{1}{3}$lb. of 8, 10, and 12 cents; $\frac{3}{6}$ of 200 = 100lb. of 20 cents, Ans.

(3.)

$$1.90\left\{\begin{array}{l}\text{10bu. at 1.70, gain 2.00}\\ \text{4bu. at 2.20, loss 1.20}\\ \hline \text{8bu. at 2.00, loss 0.80}\end{array}\right.\begin{array}{l}=\text{2.00 gain.}\\ \left.\begin{array}{l}\\ \end{array}\right\}=\text{2.00 loss.}\end{array}$$

$10 + 4 + 8 = 22$bu.; $48 - 22 = 26$bu. Then, as the gain on 1bu. at \$1.80 equals the loss on 1bu. at \$2.00, $\frac{1}{2}$ of $26 = 13$bu. = the quantity at \$1.80, and $13 + 8 = 21$bu. = the quantity at \$2.00, Ans.

(4.)

5lb. = 60oz.; 5oz. at 22 carats give a loss equal the gain on the 5oz. at 18 carats; and $5 + 5 = 10$oz.; $60 - 10 = 50$oz. Then,

$$20\left\{\begin{array}{l}\text{1oz. at 15, gain 5}\\ \text{1oz. at 17, gain 3}\\ \text{4oz. at 22, loss 8}\end{array}\right.\begin{array}{l}\left.\begin{array}{l}\\ \end{array}\right\}=\text{8 gain.}\\ =\text{8 loss.}\end{array}$$

$1 + 1 + 4 = 6$; $\frac{1}{6}$ of $50 = 8\frac{1}{3}$oz. = the quantity at 15 carats, and at 17 carats; and $\frac{4}{6}$ of $50 = 33\frac{1}{3}$oz.; $33\frac{1}{3} + 5 = 38\frac{1}{3}$oz. = the quantity at 22 carats, Ans.

(5.)

As 20 animals were bought for \$20, the average price of them was \$1.00.

$$\begin{array}{rl|l}
 & \text{2 sheep at 400, loss 600} & \\ \cline{2-2}
 & \text{1 sheep at 400, loss 300} & =\text{900 loss.}\\ \cline{3-3}
100 & \text{1 lamb at 50, gain 50} & \\
 & \text{1 kid at 25, gain 75} & \\ \cline{2-2}
 & \text{1 kid at 25, gain 75} & =\text{900 gain.}\\
 & \text{14 lambs at 50, gain 700} &
\end{array}$$

Ans. 3 sheep, 15 lambs, 2 kids.

MISCELLANEOUS EXAMPLES.

(PAGE **368.**)

(1.)

$$97\tfrac{1}{2}\left\{\begin{array}{l}\text{9 at 80, gain }157\tfrac{1}{2}\\ \text{1 at 120, loss }22\tfrac{1}{2}\end{array}\right.$$
$$\left.\begin{array}{l}\text{1 at 120, loss }22\tfrac{1}{2}\\ \hline \text{6 at 120, loss 135}\end{array}\right\}=157\tfrac{1}{2}\text{ loss.}$$

$6 + 1 = 7$ men at \$1.20, Ans.

2. 5000 rix dollars 12 skillings = $5000\frac{1}{4}$ rix dollars; $5000\frac{1}{4} \times 1.06 = \$5300.265$, Ans.

3. If Sanford remits the amount, he buys a bill at Liverpool on New Orleans, and pays 10 per cent. premium; but if Lassale at New Orleans draws on Liverpool for the amount, he receives the amount at only 8 per cent. premium; therefore the difference is the difference between 1£. at 10 per cent. premium ($\$4.88\frac{8}{9}$) and 1£. at 8 per cent. premium ($\$4.80$) $= \$4.88\frac{8}{9} - \$4.80 = \$0.08\frac{8}{9}$, and this difference multiplied by $1500\frac{3}{4}$; then $\$0.08\frac{8}{9} \times 1500\frac{3}{4} = \$133.40$, Ans.

4. $\$1.56$ is a gain of 20 per cent.; therefore $\$156 \div 1.20 = \$1.30$, the actual value of the mixture; hence,

$$130 \left\{ \begin{array}{l} 17 \text{ at } 126, \text{ gain } 68. \\ 7 \text{ at } \text{——} \text{ loss } 68. \end{array} \right.$$

That is, 7 gallons cost so much that the loss, when sold at 130, was 68; then $68 \div 7 = 9\frac{5}{7}$cts. loss per gallon; and $\$1.30 + \$0.09\frac{5}{7} = \$1.39\frac{5}{7}$, Ans.

5. 100 ounces 20 tari 10 graini = $100.68\frac{1}{3}$ ounces; $100.68\frac{1}{3} \times 2.40 = \$241.64 \div .16 = 1510.25$ lire = 1510 lire 25 centesimi, Ans.

6. 20 U. S. gal. = 1 eimer of Sweden.
3 eimers of S. = 4 eimers of Trieste.
24 eimers of T. = 9 ahms Danish.
33 ahms D. = 5 carri of Naples.
12 carri of N. = —— U. S. gal.

$$\frac{20 \times 3 \times 24 \times 33 \times 12}{1 \times 4 \times 9 \times 5} = 3168\text{gal.}; \quad 3168 \times \$0.80$$

(value of 1 ducat) = $\$2534.40$, cost of 12 carri of wine. Again,

20 U. S. gal. = 1 eimer of Sweden.
3 eimers of S. = 4 eimers of Trieste.
170 eimers of T. = —— U. S. gal.

$\frac{20 \times 3 \times 170}{1 \times 4}$=2550gal.; 2550 × 0.84\frac{7}{8}$ (the value of 1 florin 45 kreutzers in U.S. money) = 2164.31\frac{1}{4}$, cost of 170 eimers of Trieste; hence, 12 carri of wine will cost $2534.40 — 2164.31\frac{1}{4}$ = 370.08\frac{3}{4}$ *more* than 170 eimers of Trieste of wine, Ans.

7. 60s. = 3£.; 3 × 4.80 = $14.40; $14.40 ÷ 8.25 (bu. in 1qr.) = 1.74\frac{6}{11}$, the limited cost of a bushel delivered in Liverpool; this sum *less* the freight will be the limited cost of a bushel in Baltimore; 12d. = $\frac{12}{240}$ of 1£., of $4.80 = $0.24; then, 1.74\frac{6}{11}$ — $0.24 = 1.50\frac{6}{11}$, Ans.

8. 87 ÷ 1.16 = 75 cts., the actual value of the mixture. Since there is a difference of 12 cts. per pound in the two ingredients of the mixture, 11 pounds cancels eleven parts of the difference, and the 5 pounds cancels five parts of the difference; therefore, $\frac{11}{16}$ of 12 (the amount cancelled by the 11 pounds) must be deducted from 75 in finding the value of the 5 pounds per pound; 75 — $\frac{11}{16}$ of 12 = 66$\frac{3}{4}$ cts., the value of each pound of the 5 pounds; 75 + $\frac{5}{16}$ of 12 = 78$\frac{3}{4}$ cts., the value per pound of the 11 pounds, Ans.

INVOLUTION.

(Art. **515**, p. **370.**)

1. 512.
2. 1024.
3. $\frac{27}{64}$.
4. 50$\frac{46}{81}$.
5. $\frac{1}{243}$.
6. 15625.
7. 16$\frac{12144}{15625}$.
8. 282475249.
9. .000004100625.

	(Art. **516**, p. **370.**)	9.	96889010407
2.	2097152.	10.	2176782336.
3.	40353607.	11.	205891132094649+.
4.	60466176.	12.	363.691179+.
5.	281950621875.		
6.	$\frac{64}{729}$.	13.	$157\frac{283}{1024}$. 1.800943+. $116\frac{1325}{2401}$.
7.	22071204.		
8.	36372961.		

EVOLUTION.

EXTRACTION OF THE SQUARE ROOT.

	(Art. **525**, p. **376.**)	12.	$7\frac{1}{4}$.
5.	1856.	13.	$9\frac{3}{4}$.
6.	999.	14.	$19\frac{1}{19}$.
7.	$\frac{7}{12}$.	15.	1.4.
8.	15.3.	16.	81.
9.	$6\frac{1}{2}$.	17.	27.
10.	.027.	18.	64.
11.	4.16.	19.	3105671.

	(Art. **526**, p. **378.**)	4.	1.77482393+.
2.	2.5298+.	5.	19.3132079+.
3.	1.41421+.	6.	2.98831055+.

EXTRACTION OF THE CUBE ROOT.

(Art. **529**, p. **383.**)

3.	426.	9.	478.
4.	$\frac{5}{7}$.	10.	$\frac{11}{17}$.
5.	4.39.	11.	$\frac{39}{41}$.
6.	379.	12.	327.
7.	392.	13.	4968.
8.	.899.	14.	7583.

(ART. **530**, p. **383**.)

1.	1.2599+.	3.	1.442249+.
2.	2.2239+.	4.	2.08008382301904+.

EXTRACTION OF ANY ROOT.

(ART. **532**, p. **384**.)

(2.)	(3.)	(4.)
*3)998001	*2)262144	*3)43046721
3)332667	2)131072	3)14348907
*3)110889	2)65536	3)4782969
3)36963	*2)32768	3)1594323
*3)12321	2)16384	*3)531441
3)4107	2)8192	3)177147
*37)1369	*2)4096	3)59049
37)37	2)2048	3)19683
1	2)1024	*3)6561
3 × 3 × 3 × 37 = 999, [Ans.	*2)512	3)2187
	2)256	3)729
	2)128	3)243
	*2)64	*3)81
	2)32	3)27
	2)16	3)9
	*2)8	3)3
	2)4	1
	2)2	3 × 3 × 3 × 3 = 81, [Ans.
	1	

2 × 2 × 2 × 2 × 2 × 2 = 64, Ans.

(5.)

*3)14348907
3)4782969
3)1594323
3)531441
3)177147
*3)59049
3)19683
3)6561
3)2187
3)729
*3)243
3)81
3)27
3)9
3)3
1 3 × 3 × 3 = 27, Ans.

(6.)

*5)11390625
5)2278125
5)455625
5)91125
5)18225
5)3645
*3)729
3)243
3)81
3)27
3)9
3)3
1
5 × 3 = 15, Ans.

HORNER'S METHOD OF EXTRACTING ROOTS.

(Art. **533**, p. **387.**)

(3.)

0	0	$4\dot{1}67\dot{8}64\dot{8}56\dot{3}$(3467, Ans.	
3	9	27	
3	9	14673	
3	18	12304	
6	2700	2369648	
3	376	2117736	
90	3076	251912563	
4	392	251912563	
94	346800		
4	6156		
98	352956		
4	6192		(Brought up.)
1020	35914800		1032
6	72709		6
1026	35987509		10380
6			7
1032	(Carried up.)		10387

(4.)

0	0	$4\dot{3}61\dot{4}20\dot{8}$(352,
3	9	27 [Ans.
3	9	16614
3	18	15875
6	2700	739208
3	475	739208
90	3175	
5	500	
95	367500	
5	2104	
100	369604	
5		
1050		
2		
1052		

(5.)

0	0	1.05(1.016397, Ans.
1	1	1
1	1	50000
1	2	30301
2	30000	19699
1	301	18468
300	30301	1231
1	302	929
301	30603	302
1	18	279
302	3078	23
1	18	22
303	3096	1

(6.)

				184528125(45, Ans.
4	16	64	256	1024
4	16	64	256	82128125
4	32	192	1024	82128125
8	48	256	12800000	
4	48	384	3625625	
12	96	640000	16425625	
4	64	85125		
16	16000	725125		
4	1025			
200	17025			
5				
205				

(7.)

0	0	0	100(3.162278, **Ans.**
3	9	27	81
3	9	27	190000
3	18	81	113521
6	27	108000	7647900
3	27	5521	7360080
9	5400	113521	287820
3	121	5643	252672
120	5521	1191640	35148
1	122	35040	25294
121	5643	1226680	9854
1	123	35484	8853
122	5766	1262164	1001
1	74	120·	1012
123	5840	126336	
1	74	120	
124	5914	126456	
• •	74	1·	
	5988	12647	
	• •		

(8.)

0	0	0	0	6561(5.7995466,
5	25	125	625	3125 [Ans.
5	25	125•	625	343600000
5	50	375	2500	289192057
10	75	500	31250000	54407943
5	75	750	10063151	49025772
15	150	1250000	41313151	5382171
5	100	187593	11466854	5073030
20	25000	1437593	52780005	309141
5	1799	200529	169308·	282755
250	26799	1638122	5447308	26386
7	1848	213808	171936	22624
257	28647	1851930	5619244	3762
7	1897	292·	1745·	3394
264	30544	18812	563670	368
7	1946	292	1745	339
271	32490	19104	565415	29
7	• •	292	10·	
278		19396	56551	
7		• • •	10	
285			56561	
•			• • •	

APPLICATION OF POWERS AND ROOTS.

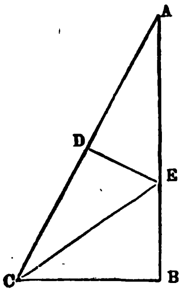
(PAGE 392.)

1. $\sqrt{141376} = 376$, Ans.
2. $1760 \times 9 = 15840$; $\sqrt{15840} = 125.857+$ feet, Ans.
3. $144 \times 144 = 20736$; $64 \times 64 = 4096$; $20736 - 4096 = 16640$; $\sqrt{16640} = 128.99+$ feet, Ans.
4. $20 \times 20 = 400$; $16 \times 16 = 256$; $12 \times 12 = 144$; $400 + 256 + 144 = 800$; $\sqrt{800} = 28.28+$ feet, Ans.
5. $40 \times 40 = 1600$; $1600 \div 4 = 400$; $\sqrt{400} = 20$ rods, Ans.
6. $60 : 90 :: 66^2 = 4356 : 6534$; $\sqrt{6534} = 80.83+$ rods, Ans.
7. 2h. : 3h. : : $(\frac{3}{4})^2 = \frac{9}{16} = .5625 : .84375$; $\sqrt{.84375} = .918+$inch, Ans.
8. $2^2 = 4 : 1.5^2 = 2.25$: : 50m. : 28m. $7\frac{1}{2}$sec., Ans.
9. $4 \times 4 \times 3 = 48 : 6 \times 6 \times 2 = 72$: : 4h. : 6h., Ans.
10. $40 \times 90 = 3600$; $\sqrt{3600} = 60$, Ans.
11. $31\frac{1}{4} \times 20 = 625$; $\sqrt{625} = 25$lb., Ans.
12. $2 : 3 :: 2400 : 3600$; $\sqrt{3600} = 60$ trees in length. $3 : 2 :: 2400 : 1600$; $\sqrt{1600} = 40$ trees in breadth. $60 - 1 = 59 \times 7 = 413$; $40 - 1 = 39 \times 7 = 273$; $413 \times 273 = 112749$ square yards, Ans.
13. $50 \div 2 = 25$; $25^2 = 625$; $625 - 600 = 25$; $\sqrt{25} = 5$; $25 + 5 = 30$ years the older; $25 - 5 = 20$ years the younger, Ans.
14. $128 \times 128 = 16384$; $72 \times 72 = 5184 + 16384 = 21568$; $\sqrt{21568} = 146.86+$ miles, Ans.
15. $100 \times 100 = 10000$; $70 - 5 = 65$; $65 \times 65 = 4225$; $10000 - 4225 = 5775$; $\sqrt{5775} = 75.993420+$; $80 \times 80 = 6400$; $50 - 5 = 45$; $45 \times 45 = 2025$; $6400 - 2025 = 4375$; $\sqrt{4375} = 66.143782+$; 75.993420

$+ 66.143782 = 142.137202 \times 142.137202 = 20202.984192388804$; $70 - 50 = 20$; $20 \times 20 = 400$; $20202.984192388804 + 400 = 20602.984192388804$; $\sqrt{20602.984192388804} = 143.537306+$ feet, Ans.

16. $\sqrt{16} = 4$; $44 + 4 = 48$; $48 \div 2 = 24$, the larger number; $44 - 4 = 40$; $40 \div 2 = 20$, the smaller, Ans.

17. Let A B represent the height of the tree, E the top of the stump, C the point on which the top of the tree will fall. As the tree will rest on the stump, it is evident that A E will be equal to C E. By drawing D E at right angles to A C, it is evident that A D will be equal to D C; that is, the line A C is bisected in D; therefore we have two similar triangles, A B C and A D E. This is evident from the fact that each of these triangles has one right angle, and that the angle A is common to both triangles. Therefore, as A B is to A C, so is A D to A E. If, then, we take A E from A B, the remainder, E B, will be the answer required.



FIRST OPERATION.

A B = 80; B C = 40; by Art. 535, $AC = \sqrt{AB^2 + BC^2} = \sqrt{80^2 + 40^2} = \sqrt{8000}$; and $AD = \frac{1}{2} AC = \sqrt{40^2 + 20^2} = \sqrt{2000}$. Then, by Art. 539, A B : A C : : A D : A E, and $AB^2 : AC^2 :: AD^2 : AE^2$; hence, 6400 : 8000 : : 2000 : 2500, and $\sqrt{2500} = 50$. Therefore, 80ft. — 50ft. = 30ft., Ans.

BY ALGEBRA.

Let x represent B E, $80 - x =$ C E, and $40 =$ C B. Then $\overline{80 - x}^2 - x^2 = 40^2$; $6400 - 16x + x^2 - x^2 = 1600$. And $160x = 6400 - 1600 = 4800$.

$x = 30$ feet $=$ B E, Ans.

SECOND OPERATION.

80 feet evidently equals the length of the part broken off and the stump together. The distance from the bottom of the tree to the point on which the top may fall, 40 feet, represents the base of a right-angled triangle, of which the part of the tree broken off is the hypothenuse, and the part left standing on the stump is the perpendicular. Then, according to ART. **553**, $40^2 = 1600 \div 80 = 20$; $80 - 20 = 60$; $60 \div 2 = 30$ feet, Ans.

18. $400 \times 400 = 160000$; $160 \times 160 = 25600$; $160000 - 25600 = 134400$; $\sqrt{134400} = 366.6 - 50 = 316.6$, Ans.

19. $70 \times 70 = 4900$; $40 \times 40 = 1600$; $30 \times 30 = 900$; $4900 - 1600 = 3300$; $\sqrt{3300} = 57.445+$; $4900 - 900 = 4000$; $\sqrt{4000} = 63.245 + 57.445 = 120.69+$ feet, Ans.

20. $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$: 1^2in. : : 450lb. : 45000lb., Ans.

21. 10 acres $= 435600$ square feet; $\sqrt{435600} = 660$ feet $=$ each side of the square plat; $660 \div 6 = 110 =$ the number of vines in each row, and also the number of rows, in the square order; $110 \times 110 = 12100 =$ the number of vines, in the square order.

In the quincunx order, the vines are disposed in the form of equilateral triangles; the distance between the rows, therefore, must equal the altitude of such triangles, having 6 feet for each of their equal sides. Hence, $6^2 - 3^2 = 27$; $\sqrt{27} = 5.196+$ feet $=$ the distance of the rows from each other; $660 \div 5.196 = 127 =$ the number of rows in the field; and the number of vines will be the same as in the square order. Therefore, $127 \times 110 = 13970 =$ the number of vines in the quincunx order; and $13970 - 12100 = 1870 =$ the number more of vines in the quincunx than in the square order, Ans.

In the square order no vine need be set *nearer* than 3 feet of the edge of the plat, and in the quincunx order nearer than $1\frac{1}{2}$ feet of the edge of the plat, and answer the conditions of the question.

22. $40 \times 40 = 1600$; $4 \times 4 = 16$; $1600 - 16 = 1584$;

$1584 \div 4 = 396$; $396 \times 3 = 1188$; $1188 + 16 = 1204$; $\sqrt{1204} = 34.698$; $40 - 34.698 = 5.302$; $5.302 \div 2 =$ 2.651in., the part A will grind off. $1204 - 396 = 808$; $\sqrt{808} = 28.425$; $34.698 - 28.425 = 6.273$; $6.273 \div 2 =$ 3.136+in., the part B will grind off. $808 - 396 = 412$; $\sqrt{412} = 20.297$; $28.425 - 20.297 = 8.128$; $8.128 \div 2 =$ 4.064in., part C grinds off. $412 - 396 = 16$; $\sqrt{16} = 4$; $20.297 - 4 = 16.297$; $16.297 \div 2 =$ 8.148+in., part D will grind off.

Ans. A, 2.651in.; B, 3.136+in.; C, 4.064in.; D, 8.148+in.

23. $49\frac{5}{8}$ feet $= 49.625$ feet; $49.625 \times 144 = 7146$ square inches; $1.5 \times 1.5 \times 2 = 4.5$; $7146 - 4.5 = 7141.5$; $7141.5 \div 6 = 1190.25$; $\sqrt{1190.25} = 34.5$; $34.5 + 1.5 = 36$ inches, Ans.

To understand the operation of this question, we will take six small square pieces of board, each of the same dimensions, and with these we will construct a cubic box; but, in so doing, we shall find that we need two small cubes, each of which is of the thickness of the board or plank used.

Now, if the box, after being completed, was a cube whose sides measured 36 inches each, and if the board was $1\frac{1}{2}$ inches thick, it would require 7146 square inches of the board to make the box, as the pupil can readily perceive.

But our box was made of six square pieces of board and two small cubes, each measuring, in the present case, $1\frac{1}{2}$ inches square. If we deduct the contents of these two squares, $= 1.5 \times 1.5 \times 2 = 4.5$ square inches, from the superficial contents of the board, $= 7146$ inches, we have $7146 - 4.5 = 7141.5$ square inches remaining; and if we divide these inches by 6, we have the superficial contents of one of the square boards of which we make our box. Thus, $7141.5 \div 6 = 1190.25$. The square root of this number, $\sqrt{1190.25} = 34.5$. will be the width of each board. To this number we must add the thickness of the board, 1.5 inches, and we have the answer, $34.5 + 1.5 = 36$ inches.

24. $22\frac{37}{72}$ feet $= 22\frac{37}{72} \times 144 = 3242$ square inches; $2.5 \times 2.5 = 6.25$ inches; $3242 - 6.25 = 3235.75$; $3235.75 \div 7 =$

462.25; $\sqrt{462.25} = 21.5$; $21.5 + 2.5 = 24$ inches, the width of the box; $24 \div 2 = 12$ inches, the height; $24 \times 2 = 48$ inches, the length.

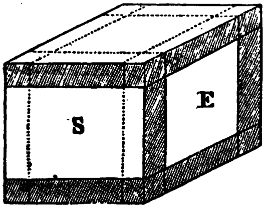
From each of these numbers we subtract 5 inches, the thickness of the two sides of the box; $12 - 5 = 7$; $24 - 5 = 19$; $48 - 5 = 43$. The inside dimensions of the box will therefore be 7, 19, and 43 inches; and its contents will therefore be $7 \times 19 \times 43 = 5719$ cubic inches, Ans.

To understand the above operation, we will construct a box that shall be twice as wide as its height, and twice as long as its width. With the materials used to construct this box the pupil will find that he can construct a cubical box whose sides will measure half the length of the former box. And, if it be constructed as the box in question 44, there will be material remaining sufficient to make one of the sides, after deducting the square of the thickness of the plank. Thus, if our box was 12 inches high, 24 inches wide, and 48 inches long, and was made of a plank 2½ inches thick, it would require one whose superficial contents were 3242 inches. From this sum we deduct the square of the thickness of the plank, $2.5 \times 2.5 = 6.25$ inches; $3242 - 6.25 = 3235.75$ inches. This sum, as we have before shown, is sufficient not only to construct the cubical box, but there will be sufficient remaining, wanting the square of the thickness of the plank, to make another, similar to one of the six squares of which we made the box. This will be evident, if the box be cut into two parts, and one of them placed on the other. Therefore, we divide 3235.75 by 7, because there are materials sufficient for 7 squares, and the quotient is 462.25. The square root of this number is the length of one of the squares of which the box is made. $\sqrt{462.25} = 21.5$ inches. To this number we add the thickness of the plank, and we have the width of the required box. $21.5 + 2.5 = 24$ inches, width of the box; $24 \div 2 = 12$ inches, height; and $24 \times 2 = 48$ inches, length, Ans.

We have copied, by permission, from the *Massachusetts Teacher*, the following illustrations, with diagrams, of the 23d and 24th problems.

(23.)

FIG. 1.



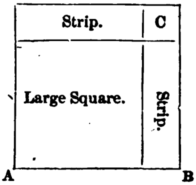
Suppose the 6 pieces of which the box is made to be so arranged (Fig. 1) that the edges of the top, bottom, and sides, will appear at the ends, as in the shaded parts about E; and that the edges of the top and bottom will appear at the sides, as in the shaded parts above and below S.

The external area of each face of the box may be considered as divided into, 1st, a large square, as at E; 2d, 4 strips, one on each side of the large square, having each a width equal to the thickness of the board ($1\frac{1}{2}$in.), and the same length as one side of the large square; and 3d, 4 small squares, each having, for the length of a side, the thickness of the board. In the 6 faces of the box, then, there are 6 large squares + 24 strips + 24 small squares.

But the shaded parts, though forming part of the external area of the sides and ends, do *not* form part of the side and end *pieces* of the box,* and are not to be reckoned as consuming any part of the surface of the board. If, then, from the large squares, strips, and small squares, that make up the whole external area of the box, we deduct 12 strips (2 for each side, and 4 for each end), and 16 small squares (4 for each side, and 4 for each end), we have left 6 large squares + 12 strips + 8 small squares = the area of the board = $49\frac{5}{8}$sq. ft. = 7146sq. in.

From this deduct the area of the 8 small squares ($(1\frac{1}{2}\text{in.})^2 \times 8 = 18$sq. in.) and we have 6 large squares + 12 strips = 7146sq. in. — 18sq. in. = 7128 sq. in. Divide this by 6, and we find one large square + 2 strips = $\frac{7128}{6}$sq. in. = 1188 sq. in. This large square and the 2 strips may be arranged as in Fig. 2, which will be an exact square, if we fill the corner C $(1\frac{1}{2}\text{in.})^2$

FIG. 2.



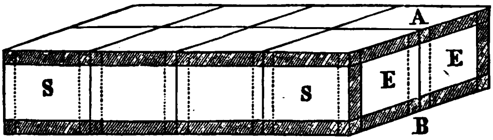
---

* Notice the difference between the side and side *piece*, and the end and end *piece*, of the box.

$= 2\frac{1}{4}$sq. in. Then 1188sq. in. $+ 2\frac{1}{4}$sq. in. = the area of the square (Fig. 2) completed $= \frac{4761}{4}$sq. in. Then, $\sqrt{\frac{4761}{4}}$sq. in. $= \frac{69}{2}$in. = length of one side of the square, or the line A B. This, it is evident (see Fig. 1), is $1\frac{1}{2}$in. less than the length of a side of the box; therefore, $\frac{69}{2}$in. $+ 1\frac{1}{2}$in. = 36in., the length of one side.

(24.)

FIG. 3.



Let, in Fig. 3, the shaded parts about E E represent the edges of the top, bottom, and sides, at the end of the box; and the shaded parts above and below S S, the edges of the top and bottom, at the side. These shaded parts represent the thickness of the plank, and form part of the external area of the side and end of the box, but no part of the side and end *pieces*.

If we draw the line A B vertically across the middle of the end of the box, the whole end is divided into 2 equal squares; but, taking away the shaded portions, it is evident that the end *piece* is not divided by this line into squares, since we have diminished each square of the end in one direction by twice the thickness of the plank, and in the other by only once its thickness.

To get two equal squares in the end *piece*, let there be marked off on each side of A B, by dotted lines, two strips, each as wide as the plank is thick. We may now suppose the *end* of the box (not the end piece) to be divided into, 1st, 2 large squares, E and E; 2d, 8 strips, each equal in length to the side of a *large square*, and in width to the thickness of the plank ($2\frac{1}{2}$in.); and 3d, 8 small squares, each having its side equal in length to the thickness of the plank. Since the box is twice as wide as it is high, and twice as long as wide, the area of an end is doubled in a side, and quadrupled in the top or bottom. Then an end, a side, and the top or bottom, contain 7 times as many large squares, strips, and small squares, as an end; that is, 14 large squares + 56 strips + 56 small squares; and both ends, both sides, and the top and bottom, or the whole external area of the box, = 28 large squares + 112 strips + 112 small squares.

But these exceed the area of the 6 *pieces* of which the box is made, by the strips and small squares in the edges of the top and bottom at the sides, and of the top, bottom, and sides, at the ends. Referring to Fig. 3, and noticing that the strips marked off by dotted lines are part of the pieces in which they are found, and not to be deducted, we see that we must deduct, for each side, 8 strips and 16 small squares; and for an end, 6 strips and 8 small squares; or 28 strips and 48 small squares for both sides and both ends. Subtracting these from the large squares, strips, and small squares of the whole external area of the box, we have 28 large squares + 84 strips + 64 small squares = the area of the 6 pieces = $22\frac{37}{72}$sq. feet. = 3242sq. in. From this deduct the 64 small squares ($(2\frac{1}{2}\text{in.})^2 \times 64 = 400$sq. in.), and we have 28 large squares + 84 strips = 3242sq. in. — 400sq. in. = 2842 sq. in. Divide by 28, and we find 1 large square + 3 strips = $\frac{2842}{28}$sq. in. = $\frac{203}{2}$sq. in.

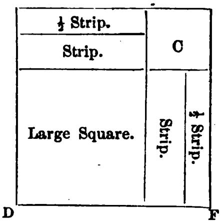


FIG. 4.

This large square and the three strips may be arranged as in Fig. 4; namely, a strip and a half on each of two adjacent sides of the *large square.* This will be an exact square if we fill out the corner C, which contains a square each side of which is one and a half times the width of a *strip* ($2\frac{1}{2}\text{in.} \times 1\frac{1}{2} = \frac{15}{4}\text{in.}$). Adding this small square ($(\frac{15}{4}\text{in.})^2 = \frac{225}{16}$sq. in.), we have $\frac{203}{2}$sq. in. + $\frac{225}{16}$sq. in. = $\frac{1849}{16}$sq. in. = area of the square Fig. 4, completed. Then $\sqrt{\frac{1849}{16}}$sq. in. = $\frac{43}{4}$in. = length of a side of the square, or the line D F.

Comparing this with Fig 3, we see it wants half the thickness of the plank ($2\frac{1}{2}\text{in.} \div 2 = 1\frac{1}{4}\text{in.}$) of being the external height of the box. Then, $\frac{43}{4}\text{in.} + 1\frac{1}{4}\text{in.} = \frac{48}{4}\text{in.} = 12\text{in.}$ = external height of the box; 12in. × 2 = 24in. = external width of the box; and 24in. × 2 = 48in. = external length of the box.

Subtract from each of these dimensions twice the thickness of the plank, and we have 12in. — 5in. = 7in. = internal height of the box; 24in. — 5in. = 19in. = internal width of the box; 48in. — 5in. = 43in. = internal length of the box, and the product of the internal dimensions = 5719 cubic in. = the contents of the box.

25. 3 × 3 × 3 = 27 : 6 × 6 × 6 = 216 : : 4lb. : 32lb., Ans.

26. 1 × 1 × 1 = 1 : 3.5 × 3.5 × 3.5 = 42.875 : : $120 : $5145, Ans.

27. 5ft. 10in. = 70in; 10ft. $4\frac{3}{5}$in. = 124.6in.; 70 × 70 × 70 = 343000; 124.6 × 124.6 × 124.6 = 1934434.936; 343000 : 1934434.936 : : 180lb. : 1015.1+lb., Ans.

28. 2lb. : 2000lb. : : 4 × 4 × 4 = 64in. : 64000;
$\sqrt[3]{64000}$ = 40in. = 3ft. 4in. high;
2lb. : 2000lb. : : 3 × 3 × 3 = 27in. : 27000in.;
$\sqrt[3]{27000}$ = 30in. = 2ft. 6in. wide;
2lb. : 2000lb. : : $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$: $\frac{1000}{64}$;
$\sqrt[3]{\frac{1000}{64}} = \frac{10}{4} = 2\frac{1}{2}$in. thick, Ans.

29. 12096 ÷ 56 = 216; $\sqrt[3]{216}$ = 6; 56 × 6 = 336; 336 × 6 = 2016; 336 and 2016, mean proportions;
Then, 56 : 336 : : 2016 : 12096, Ans

30. 5 × 5 × 5 = 125ft. : 20 × 20 × 20 = 8000ft. : : 1cwt. : 64cwt., Ans.

31. 6 × 6 × 6 = 216ft. : 10 × 10 × 10 = 1000ft. : : 1da. : 4.629+da., Ans.

32. 6 × 6 × 6 = 216ft. : 8 × 8 × 8 = 512ft. : : 600lb. : 1422.2+lb., Ans.

33. 5 × 5 × 5 = 125; 125 ÷ 4 = 31.25; 125 — 31.25 = 93.75; $\sqrt[3]{93.75}$ = 4.542+; 5 — 4.542 = .45+in., the first woman's share. 93.75 — 31.25 = 62.50; $\sqrt[3]{62.50}$ = 3.968; 4.542 — 3.968 = .57+in., share of the second woman. 62.50 — 31.25 = 31.25; $\sqrt[3]{31.25}$ = 3.149; 3.968 — 3.149 = .82+in., third woman's share. 3.149+in., fourth woman's share.

34. If Wells have 100 per cent., Rowe will have 108 per cent., and Northend 110 per cent. But Pierce has 10 per cent. more than Rowe; therefore he will have 108 × 1.10 = 118.8 per cent. 118.8 + 108 + 100 + 110 = 436.8; 5 tons = 100cwt.

Then 436.8 : 118.8 : : 100cwt. : $27\frac{54}{273}$cwt. for Pierce.
And 436.8 : 108 : : 100cwt. : $24\frac{198}{273}$cwt. for Rowe.
And 436.8 : 100 : : 100cwt. : $22\frac{244}{273}$cwt. for Wells.
And 436.8 : 110 : : 100cwt. : $25\frac{50}{273}$cwt. for Northend.

To find the feet in height that each must take, we adopt the following rule, and say, As the relative value of all their shares is to the relative value of each share, so is the cube of the height of the pyramid or stack to the cube of the height of each man's part of the stack. But we are to compute from the top of the stack each time, and then subtract as in the following process:

16 × 16 × 16 = 4096ft., cube of the height of the stack. 436.8 : 118.8 : : 4096 : 1114.02197; $\sqrt[3]{1114.02197}$ = 10.366+ft. in height for Pierce.

436.8 : 118.8 + 108 = 226.8 : : 4096 : 2126.76923076; $\sqrt[3]{2126.76923076}$ = 12.859+ from the top of the stack. From this number we subtract the height of Pierce's stack, 12.859 — 10.366 = 2.493ft. for the height of Rowe's stack.

436.8 : 118.8 + 108 + 100 = 326.8 : : 4096 : 3064.49816849; $\sqrt[3]{3064.49816849}$ = 14.525 from the top of the stack. From this we subtract the height of the other two stacks; 14.525 — 12.859 = 1.666ft. for the height of Wells' stack.

If from the height of the stack, 16ft., we subtract the last root, we have the remaining height of the stack for Northend; thus, 16 — 14.525+ = 1.474ft., height of Northend's stack.

NOTE. — The decimals in the answer to the above question will vary according to the degree of accuracy required.

ARITHMETICAL PROGRESSION.

(Art. **557**, p. **396.**)

2. $\frac{39-3}{19-1}=2$y., Ans.

3. $\frac{45-5}{11-1}=4$m., Ans.

(Art. **558**, p. **397.**)

2. $\frac{51-7}{4}+1=12$da., Ans.

3. $\frac{103-1}{2}+1=52$w., Ans.

(Art. **559**, p. **398.**)

2. $\frac{100+4}{2}\times 17=884$, Ans.

3. $320\times 30=9600=$ rods in 30m.; but there will be one more stone in this distance than there are rods, because there will be a stone at each end of the 30 miles. The man must travel 2 rods to bring the first stone to the basket, and 60 miles and 2 rods to bring the last stone; wherefore the following formula:

$$\frac{\overline{19202+2}\times 9601}{2}=92188802\text{rd.}=288090\text{m. }2\text{rd., Ans.}$$

(Art. **560**, p. **398.**)

2. $\frac{\overline{45+3}\times\overline{45-3+2}}{2\times 2}=528$, sum of the series, Ans.

3. $\frac{\overline{618\times 6}\times\overline{618-6}+12}{2\times 12}=\$162.24$, Ans.

(ART. **561**, p. **399.**)

2. $\frac{100010000 \times 2}{10000} - 20000 = 2$, Ans.

3. $\frac{528 \times 2}{22} - 3 = \$45$; $\frac{45 - 3}{22 - 1} = \$2$ difference, Ans.

(ART. **562**, p. **400.**)

2. $\frac{49 - 4}{6 - 1} = 9$, common difference.

As there are 6 terms, the third and fourth are required; $4 + 9 + 9 = 22$, the third term; $22 + 9 = 31$, the fourth term, Ans.

3. $\frac{30 - 20}{7 - 1} = 1\frac{2}{3}$, the common difference.

$20 + 1\frac{2}{3} = 21\frac{2}{3}$; $21\frac{2}{3} + 1\frac{2}{3} = 23\frac{1}{3}$; $23\frac{1}{3} + 1\frac{2}{3} = 25$; $25 + 1\frac{2}{3} = 26\frac{2}{3}$; $26\frac{2}{3} + 1\frac{2}{3} = 28\frac{1}{3}$. $21\frac{2}{3}$, $23\frac{1}{3}$, 25, $26\frac{2}{3}$, and $28\frac{1}{3}$, Ans.

GEOMETRICAL PROGRESSION.

(ART. **565**, p. **401.**)

2. $5 \times 3^{7-1} = 3645$, seventh term, Ans.

3. $\frac{72}{3^{6-1}} = \frac{8}{27}$, first term, Ans.

4. $\frac{885735}{3^{12-1}} = \frac{885735}{177147} = 5$, twelfth term;

$5 \times 3 = 15$, eleventh term; $15 \times 3 = 45$, tenth term, [Ans.

5. $5 \div (\frac{1}{3})^{7-1} = 5 \div \frac{1}{729} = 3645$, first term, Ans.

6. $50 \times 1.06^{5-1} = 63.123848$, last term, Ans.

7. $2 \times 2^{30-1} = \$10737418.24$, Ans.

8. $\$160 \times 1.06^{7-1} = \$226.96305796096$, Ans.

9. $\$300 \times 1.05^{9-1} = \$443.23,6+$, Ans.

10. $\$100 \times 1.06^{31-1} = \$\,574.3491172913250116264106332310802645846357252196069357387776$, Ans.

(Art. **566**, p. **402.**)

5. $\frac{1.06^4 - 1}{1.06 - 1} \times 50 = 218.7308$, sum of the series, Ans.

6. $\frac{2^{40} - 1}{2 - 1} \times 10 = \$109951162777.50$, Ans.

7. $\frac{2}{3} \div (1 - \frac{1}{2}) = \frac{2}{3} \div \frac{1}{2} = \frac{4}{3} = 1\frac{1}{3}$, Ans.

8. By examining this question, we find there have been 21 deposits. The amount of the last deposit is $10.60, the $10 being on interest only one year. The last but one is $11.236. The last but two is $11.91016. The last but three is $12.6247696, and so on. Thus we have a regular geometrical series, the ratio of which is 1.06, the first term $10.60, the number of terms 21, to find the sum of all the series.

$$\frac{1.06^{21} - 1}{1.06 - 1} \times 10.60 = \$423.922+, \text{Ans.}$$

9. $.008 = \frac{8}{1000}$; $.000497133 = \frac{49733}{1000000000}$, the first term; $\frac{1}{1000000}$, the ratio.
$1 - \frac{1}{1000000} = \frac{999999}{1000000}$; $\frac{437133}{1000000000} \div \frac{999999}{1000000} = \frac{497133000000}{999999000000000} = \frac{497133}{999999000}$; $\frac{497133}{999999000} + \frac{8}{1000} = \frac{8979125}{999999000} = \frac{83}{9768}$, Ans.

10. $10 \div (1 - \frac{9}{10}) = 10 \div \frac{1}{10} = 100$ miles, Ans.

(Art. **567**, p. **403.**)

2. $512 \div 1 = 512$; $\sqrt[9]{512} = 2$ ratio, Ans.

3. $\frac{1328600 - 5}{1328600 - 885735} = 3$ ratio, Ans.

4. $\sqrt[12-1]{\frac{2048}{1}} = 2$ ratio. $\frac{2048 \times 2 - 1}{1} = \$4095$ debt, [Ans.

(Art. **568**, p. **404.**)

2. $128 \div \frac{1}{2} = 256$; $\sqrt[4]{256} = 4$ ratio; $\frac{1}{2} \times 4 = 2$; $2 \times 4 = 8$; $8 \times 4 = 32$. 2, 8, and 32, Ans.

3. $2187 \div 3 = 729$; $\sqrt[6]{729} = 3$; $3 \times 3 = 9$; $9 \times 3 = 27$; $27 \times 3 = 81$; $81 \times 3 = 243$; $243 \times 3 = 729$. 9, 27, 81, 243, and 729, Ans.

(Art. **569**, p. **405.**)

2. $20480 \div 5 = 4096 = 4^6$; $6 + 1 = 7$, Ans.

3. $2048 \div 1 = 2048 = 2^{11}$; $11 + 1 = 12$ months, Ans.

ANNUITIES.

(Art. **574**, p. **408.**)

2. $\frac{1.06^5 - 1}{1.06 - 1} \times \$500 = \$2818.546+$, Ans.

3. $\frac{1.05^9 - 1}{1.05 - 1} \times \$80 = \$882.125$, Ans.

4. $\frac{1.035^{15} - 1}{1.035 - 1} \times \$1000 = \$19295.68$, Ans.

5. $\frac{1.035^6 - 1}{1.035 - 1} \times \$30 = \$196.50$, Ans.

6. $\frac{1.015^{22} - 1}{1.015 - 1} \times \$150 = \$3875.63$, Ans.

(Art. **575**, p. **409.**)

2. $\$6.801692 \times 100 = \$680.169$, Ans.

3. $\$5.786373 \times 200 = \$1157.274$, Ans.

4. $\$13.331709 \times 500 = \$6665.854$, Ans.

5. $\$7.360087 \times 500 = \$3680.043$, Ans.

6. $\$12.561102 \times 80 = \$1004.888$, Ans.

(Art. **576**, p. **409.**)

2. $\$963 \div .06 = \$16050$, Ans.

3. $\$6335 \div .07 = \$90500$, Ans.

4. $\$1200 \div .05 = \$24000$, Ans.

(Art. **577**, p. **410.**)

2. $\$11.469421 - \$6.801692 = \$4.667729$; $\$4.667729 \times 350 = \$1633.705$, Ans.

3. $\$13.590326 - \$6.732745 = \$6.857581$; $\$6.857581 \times 70 = \$480.03$, Ans.

4. $\$240 \div .06 = \$4000$, the present worth of $240 in perpetuity.

   $16.617546, the present value of an annuity of $1, due 100 years hence.

   $\$16.617546 \times 240 = \$3988.211$;

   $\$4000 - \$3988.211 = \$11.789$, Ans.

   or,

   $\$1.06^{100} = \$339.302084+$.

   $\$4000 \div 339.302084 = \$11.789$, Ans.

(Art. **578**, p. **411.**)

2. $\$3680.04 \div 7.360087 = \$500$, Ans.

3. $\$882.12,5 \div 11.026564 = \$80$. Ans.

4. $\$279 \div 6.975319 = \$40$, Ans.

PERMUTATIONS AND COMBINATIONS.

(Art. **581**, p. **412.**)

2. $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ changes, Ans.

3. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 3628800$ days, Ans.

4. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600$ changes. 365d. 5h. 49m. = 525949 min-

utes; 479001600 ÷ 10 = 47900160 minutes; 47900160 ÷ 525949 = 91y. 38801m. = 91y. 26d. 22h. 41m., Ans.

5. $1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12\times13\times14\times15\times16\times17\times18\times19\times20\times21\times22\times23\times24\times25\times26=403291461126605635584000000$ changes, Ans.

(Art. **582**, p. **412**.)

2. $6\times5\times4\times3=360$ changes, Ans.
3. $26\times25\times24\times23\times22\times21=165765600$, Ans.

(Art. **583**, p. **413**.)

2. $\dfrac{10\times9\times8\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}}{1\times2\times3\times\cancel{4}\times\cancel{5}\times\cancel{6}\times\cancel{7}}=\dfrac{720}{6}=120$, Ans.

3. $\dfrac{100\times99\times98\times97\times96\times95\times94\times93\times92\times91}{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}=$ $\dfrac{628156509555294720}{3628800}=\$173103094564.40$, Ans.

ANALYSIS BY POSITION.

(Art. **587**, p. **414**.)

(2.)

Supposed number,	120
$\frac{1}{3}$ of 120 =	40
$\frac{1}{4}$ of 120 =	30
	70
Result found,	50

50 : 60 : : 120 : 144, Ans.

(3.)

Supposed number,	36
$\frac{1}{2}$ of 36 =	18
$\frac{1}{3}$ of 36 =	12
$\frac{1}{4}$ of 36 =	9
Result found,	75

75 : 125 : : 36 : 60, Ans.

(4.)

C's assumed age,	20
B's age =	60
A's age =	120
	200

200 : 140 : : 20 : 14, C's age;
14 × *3* = 42, B's age; 42 × 2 = 84, A's age, Ans.

(5.)

Assumed sum,	$400
	1.60
Amount for 10 yr.,	$640.00

640 : 560 : : 400 : 350, Ans.

(ART. 588, p. 415.)

2. First suppose each laid out $500; then

4)$500	$500
125	225
625 = A's money.	275 = B's money.

Then, by the question, 2 × 275 = $550 should be A's money, but A's money is $625; therefore $625 — $550 = $75 is the first error.

Again, we suppose each laid out $800; then

4)$800	$800
200	225
1000 = A's money.	575 = B's money.

Then, by the question, 2 × $575 = $1150 should be A's money, but A's money is $1000; therefore $1150 — $1000 = $150+, second error; then, by the rule, 75 + 150 : 800 — 500 : : 75 : 100; and 100 + 500 = 600, the sum of dollars each invested, Ans.

3. We first suppose the age of the youngest to be 10. Then, by the question, the age of the next older will be 14, and the next 18, and the oldest will be 22. But, by the supposition, the age of the oldest was twice the age of the youngest, that is, 2 × 10 = 20; but the age of the oldest is 22, therefore the first error is 22 — 20 = 2— too small.

Again, we suppose the age of the youngest to be 16. Then the age of the next older will be 20, and the age of the next 24, and the age of the oldest will be 28. But, by the supposition, the age of the oldest was twice the age of the youngest, that is, 2 × 16 = 32; but the age of the oldest is 28; therefore, the second error is 32 — 28 = 4+ too large; then, by the rule, 2 + 4 : 16 — 10 : : 4 : 4; 16 — 4 = 12, the age of the youngest; 12 × 2 = 24, age of the oldest; 24 — 4 = 20, age of the second; 20 — 4 = 16, age of the third son, Ans.

4. We suppose the value of the first horse to be \$20. Then $20 + 50 = \$70$ will be twice the value of the second horse; that is, the value of the second horse will be \$35. Then, by the question, $35 + 50 = \$85$ will be three times the value of the first horse; that is, the first horse will be worth $\$28\frac{1}{3}$. But, by the supposition, he should be worth but \$20; therefore $28\frac{1}{3} - 20 = \$8\frac{1}{3}$— is the first error. Again, we suppose the value of the first horse to be \$32. Then $32 + 50 = \$82$ will be twice the value of the second horse; that is, the second horse will be worth \$41. Then, by the question, $41 + 50 = \$91$ will be three times the value of the first horse; that is, the first horse will be worth $\$30\frac{1}{3}$. But, by the supposition, he should be worth \$32; therefore, $32 - 30\frac{1}{3} = 1\frac{2}{3}$, the second error; then, $8\frac{1}{3} + 1\frac{2}{3} : 32 - 20 :: 8\frac{1}{3} : 10$; and $10 + 20 = \$30$, value of the first horse; $\frac{30+50}{2} = \$40$, value of the second horse, Ans.

5. If we suppose the time to be 3 o'clock, it will be 9 hours to midnight, and $\frac{2}{3}$ of 3 hours will be equal to $\frac{8}{33}$ of 9 hours; but $\frac{2}{3}$ of 3 hours is $\frac{2}{3} \times \frac{3}{1} = \frac{6}{3} = 2$ hours, and $\frac{8}{33}$ of 9 hours is $\frac{8}{33} \times \frac{9}{1} = \frac{72}{33} = 2\frac{2}{11}$ hours; therefore the first error will be $2\frac{2}{11} - 2 = \frac{2}{11}$—. Again, we suppose the time to be 4 o'clock. Then the time to midnight will be 8 hours; therefore, by the question, $\frac{2}{3}$ of 4 hours will be equal to $\frac{8}{33}$ of 8 hours, but $\frac{2}{3}$ of 4 hours is $\frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3}$ hours; and $\frac{8}{33}$ of 8 hours is $\frac{8}{33} \times \frac{8}{1} = \frac{64}{33} = 1\frac{31}{33}$ hours; therefore, the second error will be $2\frac{2}{3} - 1\frac{31}{33} = \frac{8}{11}+$. Then, by the rule, $\frac{2}{11} + \frac{8}{11} : 4 - 3 :: \frac{20}{110} : \frac{22}{110} = \frac{1}{5}$; $3 + \frac{1}{5} = 3\frac{1}{5}$ hours, the time after noon = 12 minutes past 3, Ans.

6. We first suppose their income to be \$360 each. Then, as A saves $\frac{1}{12}$ of his income, he will spend $\$360 \div 12 = \$30$; $\$360 - \$30 = \$330$ annually. As B spends \$100 per annum more than A, he will spend $\$330 + \$100 = \$430$ each year; and in 10 years he will spend

10 × $430 = $4300. As his income, by the supposition, is only $360 per annum, it would be in ten years 10 × $360 = $3600. His debt would therefore be $4300 — 3600 = $700. But by the question it was only $600; therefore the first error will be $700 — $600 = $100— too small.

Again, we suppose their income to be $300 annually. Then as A saves $\frac{1}{12}$ of his, he will spend $300 ÷ 12 = $25 : $300 — $25 = $275 annually; and B, by the question, will spend $275 + $100 = $375, and in 10 years he will spend 10 × $375 = $3750. B's income by the question will be only 10 × $300 = $3000; therefore his debt will be $3750 — $3000 = $750. But, by the question, it was only $600, therefore the second error will be $750 — $600 = $150— too small; then (the errors being both made by results too small, the difference is taken), 150 — 100 : 360 — 300 : : 100 : 120; 120 + 360 = $480, Ans.

7. It appears, by the tenor of the question, that 90 must be so divided into two parts that, if the larger part be multiplied by 60, and the smaller part by 80, the difference of the products shall be 3300. We therefore suppose the larger part to be 60, and the smaller part 30, and proceed according to the rule, thus: 60 × 60 = 3600; 30 × 80 = 2400. The difference between these products is 3600 — 2400 = 1200, which is 3300 — 1200 = 2100 *less* than it should be. Again, we divide 90 into the two parts 80 and 10, and multiply them by 60 and 80 as before, and find the result as follows: 80 × 60 = 4800; 10 × 80 = 800; 4800 — 800 = 4000, which is 4000 — 3300 = 700 *more* than it should be. Hence we have the statement, 700 + 2100 : 80 — 60 : : 700 : 5; 80 — 5 = 75, the days of labor; and 90 — 75 = 15, days of idleness, Ans.

8. First suppose 48 the weight of the fish; as the body weighs

as much as the head and tail, its weight will be half of the whole fish; half of $48 = 24$; but $24 + 15 + (15 + \frac{1}{6}$ of $24) = 58$; therefore, $58 - 48 = 10$, first error.

Again, suppose the weight of the fish to be 60; but $30 + 15 + (15 + \frac{1}{6}$ of $30) = 65$ for the weight of the fish by the conditions of the question; therefore $65 - 60 = 5$, second error. Then, $10 - 5 : 60 - 48 :: 10 : 25$; therefore, $25 + 48 = 72$, weight of the fish, Ans.

9. Assume 300 pounds to be growing on each acre (any other number would answer as well); then $3\frac{1}{3}$ acres will contain 1000 pounds. We now suppose the weekly increase to be 9 pounds; then $3\frac{1}{3}$ acres will, in four weeks, produce 120 pounds.

$$\begin{array}{rr} & 3\frac{1}{3} \times 300 = 1000 \\ & 9 \times 3\frac{1}{3} \times 4 = \underline{120} \\ \text{Amount of } 3\frac{1}{3} \text{ acres in 4 weeks} = & 1120\text{lbs.} \end{array}$$

Having found that 12 oxen will eat 1120 pounds of grass in 4 weeks, we wish to know how many pounds 21 oxen would eat in 9 weeks, which, by the following process, we find to be 4410 pounds. Thus,

$$\left.\begin{array}{r} 12 : 21 \\ 4 : 9 \end{array}\right\} :: 1220 : 4410\text{lbs.}$$

But we find the amount of the grass of 10 acres in 9 weeks to be 3810 pounds. Thus,

$$\begin{array}{r} 300 \times 10 = 3000 \\ 9 \times 10 \times 9 = \underline{810} \\ 3810\text{lbs.} \end{array}$$

But, by the supposition, 21 oxen in 9 weeks would eat $\underline{4410}$lbs.

The first error will therefore be -600

We next suppose the weekly increase to be 18 pounds per acre; therefore the amount of the grass of $3\frac{1}{3}$ acres in 4 weeks will be 1240 pounds. Thus,

$$\begin{array}{r} 300 \times 3\frac{1}{3} = 1000 \\ \underline{18 \times 3\frac{1}{3} \times 4 = 240} \\ 1240\text{lbs.} \end{array}$$

And the amount of 10 acres in 9 weeks would be 4620lbs. Thus,

$$\begin{array}{r} 10 \times 300 = 3000 \\ 10 \times 18 \times 9 = \underline{1620} \\ 4620\text{lbs.} \end{array}$$

But, by the last supposition, we find that 21 oxen in 9 weeks would eat 4882½ pounds. Thus,

$$\left.\begin{array}{r} 12 : 21 \\ 4 : 9 \end{array}\right\} : : 1240 : 4882\tfrac{1}{2}\text{lbs.}$$

By subtracting 4620 from 4882½ pounds, we find the second error, to be —262½.

$$\begin{array}{r} 4882\frac{1}{2} \\ \underline{4620} \\ -262\frac{1}{2} \end{array}$$

Hence, the statement, $600 - 262\frac{1}{2} : 18 - 9 : : 600 : 16$; and $16 + 9 = 25$ pounds, weekly increase.

Having assumed 300 pounds to the acre, and found the weekly increase to be 25 pounds to the acre, which is 8⅓ per cent., we now proceed to find the amount of the produce of 24 acres for 18 weeks. Thus,

$$\begin{array}{rr} 24 \times 300 = & 7200 \\ 25 \times 18 \times 24 = & \underline{10800} \\ \text{Amount of 24 acres for 18 weeks} = & 18000\text{lbs.} \\ 3\frac{1}{3} \times 300 = & 1000 \\ 3\frac{1}{3} \times 4 \times 25 = & \underline{333\frac{1}{3}} \\ \text{Actual amount of } 3\frac{1}{3} \text{ acres for 4 weeks} = & 1333\frac{1}{3}\text{lbs.} \end{array}$$

The question now is, If 12 oxen eat 1333⅓ pounds of grass in 4 weeks, how many oxen will eat 18000 pounds in 18 weeks? Then,

$$\left.\begin{array}{l} 1333\frac{1}{3}\text{lbs.} : 18000\text{lbs.} \\ 18 \text{ weeks} : 4 \text{ weeks} \end{array}\right\} : : 12 : 36 \text{ oxen, Ans.}$$

10. By trial the required number is found to be between 26 and and 27, which numbers may be assumed successively. Then, by extracting the square root of 26, trebling the root, and taking the result from 26, we get 10.703; 11 —

10.703 = .297, first error; proceeding the same way with 27, the result is 11.412; 11.412 — 11 = .412, second error; then, by the rule, .297 + .412 : 27 — 26 : : .297 : .42; and .42 + 26 = 26.42, the first approximation. Assume now 26.42 for the number; then, 26.42 — 3 times its square root = 10.999883269; 11 — 10.999883269 = .000116731, first error; next assume 26.421; 26.421 — 3 times its square root = 11.000591445; 11.000591445 — 11 = .000591445, second error. Then, .000116731 + .000591445 : 26.421 — 26.42 : : .000116731 : .0001648; and 26.42 + .0001648 = 26.4201648, Ans.

SCALES OF NOTATION.

(Art. **592**, p. **418.**)

(2.)

2)37
2)18 1
2)9 0
2)4 1
2)2 0
1 0

Ans. 100101.

(3.)

3)1000000
3)333333 1
3)111111 0
3)37037 0
3)12345 2
3)4115 0
3)1371 2
3)457 0
3)152 1
3)50 2
3)16 2
3)5 1
1 2

Ans. 1212210202001, in the ternary scale; and

9)1000000
9)111111 1
9)12345 6
9)1371 6
9)152 3
9)16 8
1 7

In the nonary scale, 1783661, Ans.

(4.)

12)476897
12)39741 5
12)3311 9
12)275 *e*
12)22 *e*
1 *t* Ans. 1*tee*95.

(ART. **593**, p. **419**.)

(2.)
234
5
13
5
69, Ans.

(3.)
21122
3
7
3
22
3
68
3
206, Ans.

(4.)
100101
2
2
2
4
2
9
2
18
2
37, Ans.

(5.)
13579
12
15
12
185
12
2227
12
26733

11)26733
11)2430 3
11)220 *t*
11)20 0
1 9
Ans. 190*t*3.

(6.)
123454321
6
8
6
51
6
310
6
1865
6
67167
6
403004
6
2418025

12)2418025
12)201502 1
12)16791 *t*
12)1399 3
12)116 7
9 8
Ans. 9873*t*1.

(ART. **594**, p. **420**.)

(1.)

45324502
25405534
115134440, the sum, } Ans.
15514524, the diff., }

(2.)

2483
589
1*t*985
18502
11184
13122*t*5, Ans.

(3.)

589)1184323(2483, Ans.
*e*56
22*t*3
1*te*0
3*e*32
39*t*0
1523
1523

(4.)

$1\dot{1}1\dot{2}2\dot{4}4\dot{1}$(2405, Ans.
4
44)312
304
5205)42441
42441

DUODECIMALS.

(ART. **598**, p. **421**.)

(1.)

121ft. 3′ 9″
105ft. 11′ 8″
80ft. 0′ 6″
15ft. 10′ 0″ 4‴
Ans. 323ft. 1′ 11″ 4‴

(2.)

462ft. 4′ 9″
307ft. 9′ 1″
Ans. 154ft. 7′ 8″

3. 92ft. 0′ 6″ — 21ft. 9′ 10″ = 70ft. 2′ 8″; and 70 ft. 2′ 8″ + 19ft. 10′ 8″ 6‴ = 90ft. 0′ 11″ 6‴, Ans.

(ART. **600**, p. **422**.)

2. 48ft. 6′ × 24ft. 3′ = 1176sq. ft. 1′ 6″, Ans.

3. 20ft. + 14ft. 6′ = 34ft. 6′, × 2 = 69ft. × 10ft. 4′ = 713ft.; 3ft. 2′ × 6ft. × 2 = 38ft.; 4ft. 4′ × 4ft. = 17ft. 4′, + 38ft. = 55ft. 4′; 713ft. − 55ft. 4′ = 657ft. 8′; 657ft. 8′ ÷ 9 = 73$\frac{2}{27}$ square yards, Ans.

4. 53ft. 6′ × 10ft. 3′ × 2ft. = 1096ft. 9′, Ans.

5. 6ft. 8′ + 5ft. 9′ + 4ft. 6′ + 3ft. 10′ = 20ft. 9′, × 3ft. 5′ × 4 = 283sq. ft. 7′, Ans.

6. 97ft. 9′ × 3ft. 6′ = 342ft. 1′ 6″, ÷ 4 = 85$\frac{51}{256}$ cord ft., ÷ 8 = 10$\frac{177}{256}$ cords, Ans.

7. 100ft. × 6ft. 11′ = 691ft. 8′ ÷ 4 = 172$\frac{11}{12}$ cord ft. ÷ 8 = 21$\frac{59}{96}$ cords, Ans.

(Art. **601**, p. **423**.)

2. 834sq. ft. 3′ ÷ 17ft. 9′ = 47ft., Ans.

3. 18ft. 9′ × 3 = 56ft. 3′; 84ft. 4′ 6″ ÷ 56ft. 3′ = 1ft. 6′, Ans.

Note. — The thickness of the plank, 3′, should not be regarded as 3′ in working the problem, but simply as 3 times the thickness of board measure.

4. 792ft. 6′ 9″ 2‴ ÷ 12ft. 7′ 8″ = 62ft. 8′ 6″, Ans.

MISCELLANEOUS EXAMPLES.

(Page **424**.)

1. As $ 52.50 is the average of salary, $ 52.50 — $ 20 = $ 32.50, average of increase; $ 32.50 × 2 = $ 65, whole increase; $ 65 + 20 = $ 85, salary received last month = last term; then, (Art. **558**) $\frac{85-20}{5}+1=14$ months, Ans.

2. 20ft. + 16ft. 6′ = 36ft. 6′ × 2 = 73ft. × 9ft. 6′ = 693ft. 6′; 20ft. × 16ft. 6′ = 330ft.; 330ft. + 693ft. 6′ = 1023ft. 6′ × 3 = 3070ft. 6′ ÷ 9 = 341yd. 1ft. 6′; 341yd. 1ft. 6′ — 90yd. = 251yd. 1ft. 6′, Ans.

3. (Art. **565**.) $(1\frac{1}{2})^{10}=\frac{59049}{1024}$; $\frac{59049}{1024}\times 1024=$ $ 59049, the share of the eldest, Ans.

4. (Art. **583**.) $\frac{20\times19\times18\times17\times16\times15\times14\times13\times12\times11}{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}$ [= $ 1847.56, Ans.

5. 340ft. ÷ 3ft. 9′ = 90ft. 8′ = 30yd. 0ft. 8′, Ans.

6. By Table (p. 406) the amount of \$1 for 6 years is \$6.975319; therefore, \$700 × 6.975319 = \$4882.72, Ans.

7. (1.) We first suppose the time to be $60\frac{1}{2}$ seconds after 12 o'clock. The hour-hand then will have passed $\frac{60\cdot5}{43200}$ of the distance from 12 to 12 again, and the second-hand will have passed once round and $\frac{1}{120}$ of another time. The difference between these two numbers is $\frac{1}{120} - \frac{60\cdot5}{43200} = \frac{599}{86400}$. The minute-hand will have passed $\frac{60\cdot5}{3600}$ of the distance from 12, and the difference between this number and $\frac{1}{120}$ is $\frac{60\cdot5}{3600} - \frac{1}{120} = \frac{732}{86400}$. We now find the difference between this last number and $\frac{599}{86400}$; $\frac{732}{86400} - \frac{599}{86400} = \frac{133}{86400}$—, first error, too small.

We next suppose the time to be 61 seconds after 12 o'clock. The hour-hand then will have passed $\frac{61}{43200}$ of the distance from 12 to 12, and the second-hand will have passed once round and $\frac{1}{60}$ of another time. The difference of these two numbers is $\frac{1}{60} - \frac{61}{43200} = \frac{659}{43200}$. We now find the difference between the second-hand and minute-hand. The minute-hand has moved in 61 seconds $\frac{61}{3600}$ of the distance from 12 to 12; the difference between $\frac{61}{3600} - \frac{1}{60} = \frac{12}{43200}$. We next find the difference between this last number and $\frac{659}{43200}$; $\frac{659}{43200} - \frac{12}{43200} = \frac{647}{43200} = \frac{1294}{86400}+$, second error. As the denominators of our errors are the same number, we may reject them in the operation; for when fractions have a common denominator their values are as their numerators. $133 + 1294 : 61 - 60.5 :: 133 : \frac{665}{14270}$; and $\frac{665}{1470} + 60.5 = 60\frac{780}{1427}$ seconds, Ans.

(2.) Suppose the time to be $61\frac{1}{2}$ seconds after 12 o'clock. Then the hour-hand will have moved $\frac{61\cdot5}{43200}$ of the distance from 12 o'clock to 12 again, and the minute-hand will have moved $\frac{61\cdot5}{3600}$ of this distance, and the second-hand will have moved once round and $\frac{1\cdot5}{60}$ again. The difference between $\frac{61\cdot5}{43200}$ and $\frac{61\cdot5}{3600}$ is $\frac{676\cdot5}{43200}$. The difference between $\frac{1\cdot5}{60}$ and $\frac{61\cdot5}{3600}$ is $\frac{3420}{43200}$. We then find the difference between $\frac{676.5}{43200}$ and $\frac{3420}{43200} = \frac{334.5}{43200}$—, the first error.

We then suppose the time to be 62 seconds after 12 o'clock. The hour-hand will then have moved from 12 o'clock $\frac{62}{43200}$ of the distance to 12 again, and the minute-hand have moved $\frac{62}{3600}$ of the distance, and the second-hand will have gone once round and $\frac{2}{60}$ of the distance again. The difference between $\frac{62}{3600}$ and $\frac{62}{43200}$ is $\frac{682}{43200}$. The difference between $\frac{2}{60}$ and $\frac{62}{3600}$ is $\frac{696}{43200}$. The difference between $\frac{696}{43200}$ and $\frac{682}{43200}$ is $\frac{14}{43200}+$, second error. $334.5 + 14 : 62 - 61.5 :: 14 : \frac{14}{697}$; and $62 - \frac{14}{697} = 61\frac{683}{697}$ seconds, Ans.

(3.) We will first suppose the time to be 59 seconds after 12 o'clock; the hour-hand will then have advanced $\frac{59}{43200}$ of the distance from 12 o'clock to 12 again, and the second-hand will be within $\frac{1}{60}$ of the distance to 12; therefore, the whole distance between the hour-hand and the second-hand will be $\frac{1}{60} + \frac{59}{43200} = \frac{779}{43200}$; and the distance between the hour-hand and minute-hand will be $\frac{59}{3600} - \frac{59}{43200} = \frac{649}{43200}$, and the difference between $\frac{779}{43200}$ and $\frac{649}{43200} = \frac{130}{43200}$—, the first error.

Again, we will suppose the time to be 58 minutes after 12 o'clock. The distance then between the second-hand and hour-hand will be $\frac{2}{60} + \frac{58}{43200} = \frac{1498}{43200}$. The distance between the hour-hand and minute-hand will be $\frac{58}{3600} - \frac{58}{43200} = \frac{638}{43200}$, and $\frac{1498}{43200} - \frac{638}{43200} = \frac{860}{43200}$— will be the second error. $860 - 130 : 59 - 58 :: 130 : \frac{13}{73}$; and $59 + \frac{13}{73} = 59\frac{13}{73}$ seconds, Ans.

MENSURATION.

(ART. **613**, p. **426.**)

1. $18 \times \frac{12}{2} = 108$, Ans.
2. $15.6 + 9.2 + 10.4 = 35.2$ feet; $35.2 \div 2 = 17.6$ feet; $17.6 - 15.6 = 2.00$; $17.6 - 9.2 = 8.4$; $17.6 - 10.4$

$= 7.2$; $17.6 \times 2 \times 8.4 \times 7.2 = 2128.896$; $\sqrt{2128.896} = 46.139+$ feet, Ans.

3. $40 \times \frac{15}{2} = 300$, Ans.

4. $336 \div 3 = 112$, each side of the field; $112 \div 2 = 56$, half the base; $112^2 - 56^2 = 9408$; $\sqrt{9408} = 97$, the perpendicular height, very nearly; $112 \times \frac{97}{2} = 5432$ rods $= 33$ acres 152 rods, Ans.

(Art. **621**, p. **427.**)

1. $15 \times 2 = 30$, Ans.

2. $128 \times 48 = 6144$, Ans.

3. $12 \times 8 = 96$, Ans.

4. $358 \times 194 = 69452$, Ans.

5. $693 \times 693 = 480249$ft.; $480249 \div 272\frac{1}{4} = 1764$ rods; $1764 \div 160 = 11$ acres 4 rods, Ans.

6. $40 \times 40 = 1600$; $20 \times 20 \times 2 = 800$; $1600 - 800 = 800$, Ans.

7. $\sqrt{3600} = 60$ yards; $3600 \div 2 = 1800$ yards; $\sqrt{1800} = 42.427+$ yards; 60 yards $-$ 42.427 yards $= 17.573$ yards; $17.573 \div 2 = 8.78+$ yards, Ans.

(Art. **622**, p. **428.**)

1. $\frac{75+33}{2} \times 20 = 1080$sq. ft., Ans.

2. $\frac{786+473}{2} \times 986 = 620687$ links; $620687 \div 625$ (the links in a square rod) $= 993$ rods 3 yards $= 6$ acres 33 rods 3 yards, Ans.

(Art. **623**, p. **428.**)

1. Drawing the diagonal divides the garden into two triangles, with sides 328, 598, 298, and 598, 456, 572.

$$\begin{array}{rl}
328 & 612 - 328 = 284, \text{ rem.} \\
598 & 612 - 598 = 14, \text{ rem.} \\
\underline{298} & 612 - 298 = 314, \text{ rem.} \\
2)1224 & \\
\overline{612}, \text{ half sum.} &
\end{array}$$

598	813 — 598 = 215, rem.
456	813 — 456 = 357, rem.
572	813 — 572 = 241, rem.
2)1626	
813, half sum.	

612 × 284 × 14 × 314 = 764059968 ; $\sqrt{764059968}$ = 27641.63 ; 813 × 215 × 357 × 241 = 15038837415 ; $\sqrt{15038837415}$ = 122632.97 ; 27641.63 + 122632.97 = 150274.6sq. ft. = 3 acres 1 rood 31 rods 29 yards 3.85 feet, Ans.

2. $\frac{2.82}{2}$ × 17.56 = 77.4396 square chains, area of one triangle;

$\frac{7.73}{2}$ × 17.56 = 67.8694 square chains, area of the other triangle;

77.4396 + 67.8694 = 145.309 square chains, the area of the whole field; 145.309 ÷ 10 (10 square chains make 1 acre) = 14.5309 acres = 14 acres 2 roods 5 rods, nearly, Ans.

(Art. **626**, p. **429.**)

1. $\frac{172.05}{2}$ × 250 × 5 = 107531.25 square feet, Ans.
2. $\frac{308.305}{2}$ × 356 × 6 = 329269.74yd., Ans.
3. 60^2 × 4.828427 = 17382.3372 yards = 3 acres 2 roods 14 rods 19 yards, nearly, Ans.
4. 243^2 × 7.694209 = 454335.34724 = 10 acres 1 rood 28 rods 24 yards 6.34724 feet, Ans.

(Art. **633**, p. **430.**)

1. 144 × 3.141592 = 452.389248, Ans.
2. 7964 × 3.141592 = 25019.638688, Ans.
3. 512 × 2 × 3.141592 = 3216.9984 feet = 4 furlongs 34 rods 5 yards 1 foot, Ans.

(Art. **634**, p. **430.**)

1. 1043 × .318309 = 331.997, Ans.

2. $25000 \times .318309 = 7957.74$, Ans.
3. $50 \times .318309 = 15.91549$, Ans.

(Art. **635**, p. **430.**)

1. $761^2 \times .785398 = 454840.475158$, Ans.
2. 1 mile $=$ 320 rods; $320 \times 3 = 960$; $960^2 \times .785398 =$ 723822.7968 square rods $=$ 4523.89+ acres, Ans.
3. $1284^2 \times .079577 = 131195.098512$ square yards $=$ 27 acres 17 rods 0.848512 square yards, Ans.
4. $\frac{169}{2} \times \frac{532}{2} = 22477$ inches $=$ 17 yards 3 feet 13 inches, Ans.

(Art. **636**, p. **430.**)

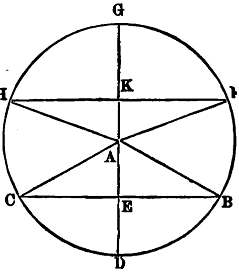
1. $79 \times \frac{47}{2} = 1856.5$, Ans.
2. The area of a circle of which the radius is $25 = 50^2 \times .785398$; then, $360° : 26° : : 50^2 \times .785398 : 141.8$sq. ft., Ans.

(Art. **637**, p. **431.**)

1. $360° : 49.25° : : 24^2 \times .785398 : 61.889$, area of sector A B C E; $\sqrt{12^2 - 5^2} = 10.908 =$ perpendicular D E; $10.908 \times 5 = 54.54$, area of triangle A C E; $61.889 - 54.54 = 7.35$, nearly, area of the segment A B C A, Ans.
2. By the second rule, $\frac{15 \times 24 \times 2}{3} + \frac{15^3}{24 \times 2} = 310.3125$ square rods $=$ 1 acre 3 roods 30 rods 9.4 yards, Ans.

(Art. **638**, p. **431.**)

1. The first step in the working of this problem is to find the height of the segments lying on each side of the zone; the heights are elements necessary in calculating their areas. Let H F B C be the zone; the radii A C, A B, A F, A H, are each 12 feet;



$AH = 12$; $12^2 = 144$;

$HK = \frac{20.8}{2} = 10.4$; $10.4^2 = 108.16$.

$144 - 108.16 = 35.84$; $\sqrt{35.84} = 5.9 = AK$; $12 - 5.9 = 6.1 = GK$, the height of the segment whose cord is 20.8; then, $\frac{20.8 \times 6.1 \times 2}{3} + \frac{6.1^3}{20.8 \times 2} = 90.03$, area of the segment H G F H;

$AC = 12$; $12^2 = 144$;

$CE = \frac{23.25}{2} = 11.625$; $11.625^2 = 135.14$; $144 - 135.14 = 8.86$;

$\sqrt{8.86} = 2.97 = AE$; $12 - 2.97 = 9.03 = DE$, the height of the segment whose cord is 23.25; then $\frac{23.25 \times 9.03 \times 2}{3} + \frac{9.03^3}{23.25 \times 2} = 155.79$, area of the segment C D B C;

$24^2 \times .7854 = 452.39$, area of the whole circle;
$155.79 + 90.03 = 245.82$, area of the two segments;
206.57, area of the zone, Ans.

2. Let 10, the radius, be the hypothenuse of a right-angled triangle, and 8, half the cord, be the base; then, $10^2 - 8^2 = 36$; $\sqrt{36} = 6$, the perpendicular from the centre to the cord; $10 - 6 = 4$, height of each segment; then, by the rule (Art. **637**), $\frac{16 \times 4 \times 2}{3} \times \frac{4^3}{16 \times 2} = 44.66$, area of each segment; $44.66 \times 2 = 89.32$, area of both segments; $20^2 \times .785 =$

314, area of the circle;
89.32, area of the segments.
224.68, or 224.7, nearly,

area of zone, Ans.

(ART. **639**, p. **431.**)

1. By rule, Art. **637**, $\frac{72 \times 30 \times 2}{3} + \frac{30^3}{72 \times 2} = 1627.5$ sq. ft.

$$\frac{72 \times 20 \times 2}{3} + \frac{20^3}{72 \times 2} = \underline{1015.5} \text{ sq. ft.}$$

Ans. 612 sq. ft.

(ART. **640**, p. **431.**)

1. $(20 + 10) \times (20 - 10) \times .7854 = 235.62$ sq. yd., Ans.
2. $(157 + 128) \times (157 - 128) \times .7854 = 6491.331$ sq. yd.
   = 1 acre 1 rood 14 rods 17 yards 7.4 feet, Ans.

(ART. **641**, p. **431.**)

1. $50 \times .886227 = 44.31135$ rods, Ans.
2. $360 \times .282094 = 101.55$ rods, Ans.
3. $10000 \times .282094 = 2820.94$ rods, Ans.

(ART. **642**, p. **432.**)

1. $44.31135 \times 1.12838 = 50.000041113$ rods, Ans.

(ART. **643**, p. **432.**)

1. $30 \times .866025 = 25.98$ inches, Ans.
2. $5000 \times .275664 = 1378.320$ feet, Ans.
3. $80 \times .275664 = 22.05$ inches, Ans.

(ART. **644**, p. **432.**)

1. $30 \times .707106 = 21.21$ inches square, Ans.
2. $80 \times .707106 = 56.56848$ feet, Ans.
3. $5000 \times .225079 = 1125.395$ rods, Ans.
4. $100 \times .225079 = 22.5$ inches square, Ans.
5. $18 \div .225079 = 79.97$ inches, Ans.
6. 20 rods = 330 feet; $330 \div .225079 = 1466.15$ feet, Ans.

(Art. **645**, p. **432.**)

1. $86.2 \div 2.155 = 40$ inches, Ans.

(Art. **647**, p. **433.**)

1. $24 \times 18 \times .785398 = 339.2919$, Ans.
2. $33\frac{5}{12} \times 20\frac{1}{4} \times .785398 = 531.469$sq. feet = 59sq. yd. 67sq. inches, Ans.

SOLIDS.

(Art. **650**, p. **433.**)

1. $23 + 34 + 19 = 76$ inches $= 6\frac{1}{3}$ feet, the perimeter; $6\frac{1}{3} \times 13 = 82.33$ feet, the area of the sides; by Art. **613**, $38 \times 15 \times 4 \times 19 = 43320$; $\sqrt{43320} = 208.1$ inches, the area of one of the ends; $208.1 \times 2 = 416.2$; $416.2 \div 144 = 2.89$, area of both ends in feet;

   82.33, area of sides;
   2.89, area of the ends;
   85.22 feet, surface of the prism, Ans.

2. $33 \times 5 = 165$ inches $= 13\frac{3}{4}$ feet, the perimeter; $13\frac{3}{4} \times 14 = 192.5$ feet, area of the sides; and, $33^2 \times 1.720477 \times 2 = 3747.19$sq. inches $= 26.02$sq. feet, the area of the two ends; then, $192.5 + 26.02 = 218.52$sq. feet, surface of the prism, Ans.
3. 57in. $= 4\frac{3}{4}$ feet; $4\frac{3}{4} \times 13 = 61.75$sq. feet, the convex surface of cylinder; $(4\frac{3}{4})^2 \times .079577 \times 2 = 3.59$sq. ft., area of the two ends; then, $61.75 + 3.59 = 65.34$sq. ft., surface of cylinder, Ans.
4. 1 acre $= 43560$sq. feet; $1\frac{3}{4} \times 3.14159 = 5.49778$ feet, the circumference of the cylinder; $5.49778 \times 5\frac{1}{4} = 28.86334$, convex surface of the cylinder; $43560 \div 28.86334 = 1509.18$ times, Ans.
5. $16 \times 10 \times 4 = 640$sq. feet; $640 \div 9 = 71\frac{1}{9}$sq. yards, Ans.

(Art. **651**, p. **434**.)

1. $2.5^2 - 1.25^2 = 4.6875$; $\sqrt{4.6875} = 2.16506 =$ the perpendicular of the triangle marked by the end of the prism; and 2.16506×1.25 (half the base) $= 2.706 =$ area of the end of the prism; $2.706 \times 12 = 32.47$ cubic feet, Ans.
2. (Art. **613**.) $6 \times 1 \times 2 \times 3 = 36$; $\sqrt{36} = 6 =$ area of the end; then, $6 \times 10 = 60$cu. feet, Ans.
3. $3\frac{1}{6} \times 2\frac{2}{3} \times 2\frac{1}{2} = 21\frac{1}{9}$cu. feet, Ans.
4. $6^2 \times .079577 \times 9 = 25.78$cu. feet, Ans.

(Art. **656**, p. **435**.)

1. $2\frac{2}{3} \times 2\frac{2}{3} = 7\frac{1}{9}$ feet, area of the base; $2\frac{2}{3} \times 4 = 10\frac{2}{3}$ feet, the perimeter of the base; $7\frac{1}{9} + 10\frac{2}{3} \times \frac{3\frac{3}{4}}{2} = 27\frac{1}{9}$sq. feet area, Ans.
2. $9 \times \frac{20}{2} = 90$ feet, Ans.

(Art. **657**, p. **435**.)

1. $(2\frac{1}{2})^2 \times .785398 \times \frac{12.5}{3} = 20.45$, Ans.
2. By Art. **613**, $9 \times 4 \times 3 \times 2 = 216$; $\sqrt{216} = 14.69693$, the area of the base; $14.69693 \times \frac{14.5}{3} = 71.035$cu. feet, Ans.

(Art. **658**, p. **435**.)

(1.)

$$
\begin{array}{llll}
3 \times 5 = 15, & \text{perimeter of smaller end;} & & \\
5 \times 5 = \underline{25}, & \text{perimeter of larger end;} & & \\
\quad \overline{40} \times 5 = 200 & & = & \text{surface of the sides;} \\
3^2 \times 1.720477 & = 15.4842 & = & \text{surface of smaller end;} \\
5^2 \times 1.720477 & = \underline{43.0119} & = & \text{surface of larger end;} \\
 & \ 258.4961 & & \text{inches, surface of the frustum, Ans.}
\end{array}
$$

(2.)

$3\frac{7}{12} \times 3.14159 = 11.2573$, circumference of larger end;
$1\frac{11}{12} \times 3.14159 = 6.02138$, circumference of smaller end;
$\overline{17.27868} \times 4\frac{1}{2} = 77.75406 =$ convex
[surface of the frustum;

77.75406, convex surface;
$(\frac{43}{12})^2 \times .785398 = 10.08472$, surface of larger end;
$(\frac{23}{12})^2 \times .785398 = 2.885246$, surface of smaller end;
$\overline{90.724026}$sq. feet, Ans.

(Art. **659.** p. **435.**)

1. $27 \times 27 = 729$, area of larger end; $16 \times 16 = 256$, area of smaller end;
   $729 \times 256 = 186624$; $\sqrt{186624} = 432$; $432 + 256 + 729 = 1417$; $1417 \times \frac{18\frac{2}{3}}{3} = 8816.888$; $8816.888 \div 144 = 61.228$ cubic feet, Ans.
2. $2^2 \times .7854 = 3.1416$, area of larger end;
   $1^2 \times .7854 = .7854$, area of smaller end;
   $3.1416 \times .7854 = 2.46741264$; $\sqrt{2.46741264} = 1.5708$;
   1.5708,
   3.1416, area of larger end;
   .7854, area of smaller end;
   $\overline{5.4978} \times \frac{40}{3} = 73.304$ cubic feet, Ans.

(Art. **666,** p. **436.**)

1. $24 \times (24 \times 3.14159) = 1809.55$sq. in., Ans.
2. $7957\frac{3}{4} \times 25000 = 198943750$, Ans.

(Art. **667,** p. **436.**)

1. $12^3 \times .523598 = 904.78$, Ans.
2. $25000 \times .31831 = 7957.75$, diameter; $7957.75^3 \times .5236 = 263858149120.06886875$, Ans.

(ART. **668,** p. **437.**)

1. $12\frac{1}{2} \times 3.141592 \times 2 = 78.54$sq. feet, Ans.
2. $7970 \times 3.141592 \times 2143.623553 = 53673093.12$sq. miles, Ans.

(ART. **669,** p. **437.**)

1. $(3^2 + (4\frac{1}{2})^2 \times 3) \times 3 \times .5236 = 109.56$cu. feet, Ans.
2. $(9^2 + 10^2 \times 3) \times 9 \times .5236 = 1795.42$cu. feet. Ans.

(ART. **670,** p. **437.**)

1. $(21 + 4) \times 4 \times 9.8696 = 986.96$sq. in., Ans.

(ART. **671,** p. **437.**)

1. $(25 + 5) \times 5^2 \times 2.4674 = 1850.55$cu. in., Ans.

(ART. **672,** p. **437.**)

1. $20^2 \times 32 \times .523598 = 6702.05$cu. in., Ans.
2. $38^2 \times 48 \times .523598 = 36291.62$cu. feet, Ans.

(ART. **674,** p. **438.**)

1. $24 \times \frac{2}{3} = 16$ feet, Ans.
2. $30 \times 1\frac{1}{3} = 40$ feet, Ans.
3. $(26 + 14) \div 2 = 20$; $30 \times 20 \div 12 = 50$ feet, Ans.

(ART. **675,** p. **438.**)

1. $3 \times 5 \times 15 \times 3 = 675$; $675 \div 12 = 56\frac{1}{4}$ feet, Ans.
2. $2 \times 6 \times 10 \times 20 \div 12 = 200$ feet, Ans.
3. $10 \times 17 \times 20 \div 12 = 283\frac{1}{3}$ feet, Ans.

(ART. **676,** p. **438.**)

1. $30 \times 10^2 \div 144 = 20\frac{5}{6}$ feet, Ans.
2. $50 \times 14^2 \div 144 = 68\frac{1}{18}$ feet, Ans.
3. $90 \times 30^2 \div 144 = 562\frac{1}{2}$ feet, Ans.

(ART. 679, p. 439.)

1. $27 + (8 \times .70) = 32.6$; $32.6^2 \times 45 \times .0034 = 162.602$ wine gallons, Ans.

2. $30 + (8 \times .65) = 35.2$; $35.2^2 \times 42 \times .0034 = 176.9349$ wine gallons, Ans.

3. Consider the tub as the frustum of a cone, and apply the rule in Art. **659**; by this rule get the contents of the tub in inches, and change the inches to their equivalent in gallons;
$30^2 \times .7854 = 706.86$, area of smaller end;
$40^2 \times .7854 = 1256.64$, area of larger end;
$1256.64 \times 706.86 = 888268.5504$; $\sqrt{888268.5504} = 942.48$; $942.48 + 706.86 + 1256.64 = 2905.98$; $2905.98 \times \frac{50}{3} = 48433$ cubic inches; $48433 \div 231 = 209.66\frac{2}{3}$ liquid gallons, Ans.

4. $\frac{10 \times 5 \times 4 \times 1728}{231} = 1496\frac{8}{77}$ wine gallons, Ans.

5. $\frac{12 \times 6 \times 2 \times 1728}{282} = 882\frac{18}{47}$ beer gallons, Ans.

6. $\frac{15 \times 5 \times 7 \times 1728}{2150.42} = 421.8$ bushels, Ans.

TONNAGE.

(ART. 681, p. 440.)

1. $191\frac{6}{12} = \frac{2298}{12}$; $36\frac{5}{12} = \frac{437}{12}$; $\frac{2298}{12} - \frac{3}{5}$ of $\frac{437}{12} = \frac{10179}{60}$; $\frac{10179}{60} \times \frac{437}{12} \times \frac{437}{24} \times \frac{1}{95} = 1184\frac{427}{3200}$ tons, Ans.

2. $184\frac{6}{12} - \frac{3}{5}$ of $38\frac{11}{12} = \frac{9669}{60}$; $\frac{9669}{60} \times 38\frac{11}{12} \times 19\frac{11}{24} \times \frac{1}{95} = 1284\frac{2960047}{5472000}$ tons, Ans.

3. $195\frac{2}{12} - \frac{3}{5}$ of $39\frac{4}{12} = 171\frac{17}{30} \times 39\frac{4}{12} \times 19\frac{8}{12} \times \frac{1}{95} = 1397\frac{182}{12825}$ tons, Ans.

4. $78 - \frac{3}{5}$ of $21 = 65.4$; $65.4 \times 21 \times 9 \times \frac{1}{95} = 130\frac{53}{475}$ tons, Ans.

5. $141 \times 30 \times 15 \times \frac{1}{95} = 667\frac{17}{19}$ tons, Ans.

6. $479 - \frac{3}{5}$ of $80 = 431$; $431 \times 80 \times 40 \times \frac{1}{95} = 14517\frac{17}{19}$ tons, Ans.

MISCELLANEOUS QUESTIONS.

(PAGE 441.)

1. $\frac{2}{7} \times \frac{5}{9} = \frac{10}{63}$; $1 - \frac{10}{63} = \frac{53}{63}$, Ans.

2. As there is a son and a daughter, the son will have $\frac{4}{7}$ of the estate, the wife $\frac{2}{7}$, and the daughter $\frac{1}{7}$. If there had been only a daughter, her share would have been $\frac{2}{3}$; consequently she loses $\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$. Hence

$\frac{8}{21} : \frac{7}{21} : : \$2400 : \$2100$, Ans.

3. From the conditions of the question, it will readily be seen that it was a little more than half-past 5. At $5\frac{1}{2}$ o'clock the minute-hand was at 6, and the hour-hand half-way between 5 and 6, or $2\frac{1}{2}$ spaces from 6. The minute-hand moves twelve times as fast as the hour-hand; hence, while the minute-hand was moving from 6 to the required position, the hour-hand moved $\frac{1}{12}$ as far, and was then as far from the 6 point as the minute-hand was beyond. Therefore, the sum of the spaces passed over by the hour and minute hands $= 2\frac{1}{2}$ minute spaces. Hence $2\frac{1}{2}$m. $= \frac{13}{12}$ of the required time beyond half-past 5, and $\frac{12}{12} =$ 2m. $18\frac{6}{13}$s. 5h. 30m. + 2m. $18\frac{6}{13}$sec. = 5h. 32m. $18\frac{6}{13}$sec., Ans.

(4.)

6)97deg. 55m. 7fur. 35rd. 4ft. 6in.(16deg.

96

1

$69\frac{1}{6}$

6)$124\frac{1}{6}$(20m.

120

$4\frac{1}{6}$

8

6)$40\frac{1}{3}$(6fur.

36

$4\frac{1}{3}$

40

$208\frac{1}{3}$

6)$208\frac{1}{3}$(34rd.

204

$4\frac{1}{3}$

$16\frac{1}{2}$

6)$75\frac{1}{2}$(12ft.

72

$3\frac{1}{2}$

12

6)48(8in.

48

Ans. 16deg. 20m. 6fur. 34rd. 12ft. 8in.

5. $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$; $\frac{1}{3} = \frac{4}{12}$; $\frac{1}{4} = \frac{3}{12}$;
$\frac{7}{12} : \frac{4}{12} :: \$100,000 : \$57,142\frac{6}{7}$, A's part, } Ans.
$\frac{7}{12} : \frac{3}{12} :: \$100,000 : \$42,857\frac{1}{7}$, B's part, }

6. If the first man's share be subtracted from the whole, there will remain $\frac{18}{18} - \frac{7}{18} = \frac{11}{18}$; and $\frac{7}{18}$ of $\frac{11}{18} = \frac{77}{324} =$ the second son's share. And $\frac{7}{18} - \frac{77}{324} = \frac{49}{324} =$ difference of their legacies. $\frac{7}{18} = \frac{126}{324}$; $\frac{126}{324} + \frac{77}{324} = \frac{203}{324} =$ legacy of both sons. Hence $\frac{324}{324} - \frac{203}{324} = \frac{121}{324} =$ wife's legacy. Therefore
$\frac{49}{324} : \frac{121}{324} ::$ 257£. 3s. 4d. : 635£. 0s. 10$\frac{30}{49}$d., Ans.

7. $63 \times 12 \times 12 \times 1000 \times 3 = 27216000$; $27216000 \div 16 = 1701000$lb.; $1701000 \div 2000 = 850\frac{1}{2}$ tons, Ans.

8. 4ft. = 48in.; 6in. × 2 = 12in.; 48 — 12 = 36in.; 36 ÷ 2 = 18in.; 18 + 12 = 30in.; 48in. : 30in. : : 200lb. : 175lb., Ans.
48 — 30 = 18in.; 48in. : 18in. : : 200lb. : 75lb., Ans.

9. 25ft. 4in. = 304in.; 4ft. 5in. = 53in.; 3ft. 5in. = 41in.; $53 \times 41 = 2173$; $53 - 41 = 12$in.; $12 \times 12 = 144$; $144 \div 3 = 48$; $2173 + 48 = 2221$; $2221 \times .785398 = 1744.368958$; $1744.368958 \times 304 \times 8 = 4242305.305856$; $4242305.305856 \div 1728 = 2455.037792 =$ cubic feet in the pillars. $2455.037792 \times 3000 = 7365113.376 =$ weight in ounces; $7365113.376 \div 16 = 460319.586$lbs.; $460319.586 \div 2000 = 230.15+$ tons, Ans.

10. If $\frac{3}{7}$ of a certain sum be taken, and \$410 be left, it is evident that \$410 is $\frac{4}{7}$ of that sum, which is $\overline{410 \times 7} \div 4 =$ \$717$\frac{1}{2}$. Now, if \$717$\frac{1}{2}$ remain of a certain quantity after $\frac{1}{4}$ be subtracted, it is certain that the number from which it is taken is $\frac{4}{3}$ of \$717$\frac{1}{2}$ = \$956.66$\frac{2}{3}$, Ans.

11. $\frac{1}{4}$: \$15.60 : : \$100 : \$6240 = sum remitted; \$96 : \$100 : : \$6240 : \$6500 = value of goods sold; \$6500 — \$6240 = \$260 = commission, Ans.

12. \$107.50 : \$100 : : \$9675 : \$9000; $\frac{9}{40}$ × \$9000 = 2025£. sterling = the bill; \$100 — \$0.25 = \$99.75;

$\$100 : \$99.75 :: \$9675 : \$9650.81\frac{1}{4}$; $\$102 : \$100 :: \$9650.81\frac{1}{4} : \$9461.58\frac{3}{34}$ for investment, Ans.

13. $15 \times 30 = 450$; $15 \times 15 = 225$; $225 \div 3 = 75$; $450 + 75 = 525$; $525 \times 220 = 115500$ = contents of the whole monument, and from this we deduct the contents of the cylinder. $15 \times 11 = 165$; $4 \times 4 = 16$; $16 \div 3 = 5\frac{1}{3}$; $165 + 5\frac{1}{3} = 170\frac{1}{3}$; $170\frac{1}{3} \times .785398 = 133.779459\frac{1}{3}$; $133.779459\frac{1}{3} \times 220 = 29431.481053\frac{1}{3}$ = contents of the cylinder; $115500 - 29431.481953\frac{1}{3} = 86068.51894+$ cubic feet of the monument, Ans.

(14.)

$$\begin{array}{rl}
\frac{4}{12} \times 1 + \frac{4}{12} \times \frac{1}{2} + \frac{4}{12} \times \frac{1}{3} = \frac{22}{36}, & \text{A's product;} \\
\frac{4}{12} \times \frac{1}{2} + \frac{4}{12} \times \frac{1}{3} = \frac{10}{36}, & \text{M's product;} \\
\frac{4}{12} \times \frac{1}{3} = \frac{4}{36}, & \text{P's product;} \\
\hline
\frac{36}{36}, & \text{sum of the products.}
\end{array}$$

$$\left.\begin{array}{l}
\frac{36}{36} : \frac{22}{36} :: \$300 : \$183.33\frac{1}{3} = \text{A pays,} \\
\frac{36}{36} : \frac{10}{36} :: \$300 : \$\ \ 83.33\frac{1}{3} = \text{M pays,} \\
\frac{36}{36} : \frac{4}{36} :: \$300 : \$\ \ 33.33\frac{1}{3} = \text{P pays,}
\end{array}\right\} \text{Ans.}$$

(15.)

$$\begin{array}{lrcl}
\text{A} & 20 \times \$132 & = & \$2640 \\
\text{B} & 25 \times \$120 & = & \$3000 \qquad 85 \times \$6 = \$510. \\
\text{C} & 40 \times \$100 & = & \$4000 \\
\hline
 & 85 & & \$9640
\end{array}$$

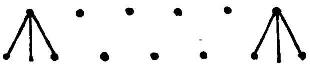
$$\left.\begin{array}{l}
\$9640 : \$2640 :: \$510 : \$139\frac{161}{241}, \text{ A receives,} \\
\$9640 : \$3000 :: \$510 : \$158\frac{172}{241}, \text{ B receives,} \\
\$9640 : \$4000 :: \$510 : \$211\frac{149}{241}, \text{ C receives,}
\end{array}\right\} \text{Ans.}$$

(16.)

$$\begin{array}{l}
\$100 \times \$5.00 = \$500 \text{ given for the flour;} \\
\$500 \times \$0.20 = \underline{\$100} \text{ gained on the flour;} \\
\qquad\qquad\qquad\quad \$600
\end{array}$$

$\$100 \times \$0.03,0\frac{1}{2} = \$3.05$ bank interest of $\$100$ for 6 months; $\$100 - \$3.05 = \$96.95 : \$100 :: \$600 : \$618\frac{1698}{1939}$; $\$618\frac{1698}{1939} \div 100 = \$6.18\frac{1698}{1939}$, Ans.

17. We first find the number of square feet in an acre. $160 \times 272\frac{1}{4} = 43560$ feet. If we extract the square root of this number, we obtain the side of a square field that will contain an acre; thus, $\sqrt{43560} = 208.712+$ feet. We now divide this number by 3.5 feet, and obtain $59.632+$, the number of divisions in the first row. We perceive, therefore, that there will be 60 hills, there being one more hill than divisions. Now, if we divide $208.712+$ by 59, the quotient will be 3.537 feet; that is, the hills in the first row may be 3.537 feet apart, instead of 3.5 feet. Thus, our first row will contain 60 hills, which will be 3.537 feet apart. Our next row will contain but 59 hills, the hills being planted in the quincunx order, thus:



To find the distance between the rows, we square $3.5 = 12.25$; we then take half of $3.537 = 1.768$, which we square $= 3.125824$; we subtract this last number from 12.25, and obtain 9.124176. The square root of this number is 3.0206 feet, equal the distance between the rows. Now, if we divide $208.712+$ by 3.0206, we obtain $69+$; therefore, the number of rows will be 70. To obtain the number of hills in the field, we multiply 70 by $60 = 4200$. But, as there are 70 rows, and as half of the rows contain only 59 hills, we subtract 35 from 4200. Thus, $4200 - 35 = 4165$ hills, Ans.

18. \$300 : \$700 : : 20 months : $46\frac{2}{3}$ months, Ans.

19. $\$1500 \div 150 = 10$; $10 + 2 = 12$ children. $\$1500 \times 2 = \$3000$; $\$3000 \div 3 = \$1000$; $\$3000 + \$1000 = \$4000$; $\$4000 \div 4 = \$1000$; $\$4000 + \$1000 = \$5000$; $\$5000 \times 2 = \$10,000$, Ans.

20. $7 - 5 = 2$ miles which B gains each day, and he will have to gain 80 miles before he overtakes A. 2 miles : 80 miles : : 1 day : 40 days, the time which it takes B to

overtake A. And as B travels 7 miles each day, he will have to go, before he overtakes A, $40 \times 7 = 280$ miles, Ans.

21. 16lb. : 80lb. : : 24.4in. : 122 cubic inches of lead. $\frac{1}{4} \times 2 = \frac{1}{2}$in.; 1in. + $\frac{1}{2}$in. = 1.5 inches; $1.5 \times 1.5 = 2.25$; $2.25 \times .785398 = 1.7671455$ = area of a section of the pipe. From this we subtract the area of a section of the calibre of the pipe. $1 \times .785398 = .785398$; $1.7671455 - .785398 = .9817475$; $122 \div .9817475 = 124.26+$ inches, $= 10.35+$ feet, Ans.

22. $.785398 \times 2 \times 2 = 3.141592$; $\frac{3}{4} \times \frac{3}{4} \times .785398 = .441786$; $3.141592 - .441786 = 2.699806$; $2.699806 \times 8 = 21.598448$; $\frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$; $\frac{3}{4} + \frac{3}{4} = 1.5$; $1.5 \times 1.5 = 2.25$; $.785398 \times 2.25 = 1.7671455$; $1.7671455 - .441786 = 1.3253595$; $21.598448 \div 1.3253595 = 16.29+$in., Ans.

23. Let $100 represent the *relative* value of what D pays; then $\$100 \times 1.08 = \$108$ will represent what C pays; $\$108 \times 1.10 = 118.80$, what B pays; and $\$118 \times 1.08 = \$133.056$, what A pays. $\$100 + \$108 + \$118.80 + \$133.056 = \$459.856$.

$459.856 : $100 : : $100: 21.74\frac{17066}{28741}$, D pays.
$459.856 : $108 : : $100: 23.48\frac{16132}{28741}$, C pays.
$459.856 : $118.80 : : $100: 25.83\frac{11997}{28741}$, B pays.
$459.856 : $133.056 : : $100: 28.93\frac{12287}{28741}$, A pays.

$20 \times 20 \times 20 = 8000$; $459.856 : $133.056 : : 8000 : 2314.742006; $\sqrt[3]{2314.742006} = 13.22+$ft., which A takes; $459.856 : $251.856 : : 8000ft. : 4381.47594ft.; $\sqrt[3]{4381.47594} = 16.36+$ft.; $16.36 - 13.22 = 3.14+$ ft. which B takes. $459.856 : $359.856 : : 8000ft. : 6260.324971ft. $\sqrt[3]{6260.324971} = 18.42$ft.; $18.42 - 16.36 = 2.06+$ft. which C takes. $20 - 18.42 = 1.58+$ft. which remains for D.

24. As B paid 20 per cent. more than A, and 10 per cent. less than C, we obtain their shares paid as follows:

For each dollar A pays, B pays $\$1.20$, and C pays $\$1.33\frac{1}{3}$; $\$1 + \$1.20 + \$1.33\frac{1}{3} = \$3.53\frac{1}{3}$. Therefore,

$\$3.53\frac{1}{3} : \$1.00 :: \$10.60 : \$3.00$, A paid.
$\$3.53\frac{1}{3} : \$1.20 :: \$10.60 : \$3.60$, B paid.
$\$3.53\frac{1}{3} : \$1.33\frac{1}{3} :: \$10.60 : \$4.00$, C paid.

As A paid $\$3.00$, his share of the stone will be

$$\frac{300}{1060} = \frac{15}{53};\ \text{B's},\ \frac{360}{1060} = \frac{18}{53};\ \text{C's},\ \frac{400}{1060} = \frac{20}{53}.$$

$65^2 = 4225$, square of the diameter of the stone.
$3^2 = 9$in., square of the place for the axle.
$4225 - 9 = 4216$, to be divided among A, B, and C.
$4216 \times \frac{15}{53} = 1193.20$, A's part.
$4216 \times \frac{18}{53} = 1431.84$, B's part.
$4216 \times \frac{20}{53} = 1591$, C's part.
$4225 - 1193.20 = 3031.80$; $\sqrt{3031.80} = 55$ inches.
$65 - 55 = 10$in.; $10 \div 2 = 5$ inches, A grinds off.
$3031.80 - 1431.8 = 1600$; $\sqrt{1600} = 40$ inches.
$55 - 40 = 15$; $15 \div 2 = 7\frac{1}{2}$ inches, B grinds off.
$1600 - 1591 = 9$; $\sqrt{9} = 3$ inches.
$40 - 3 = 37$; $37 \div 2 = 18\frac{1}{2}$ inches, C grinds off.
A grinds off 5 inches, B $7\frac{1}{2}$in., and C $18\frac{1}{2}$in., Ans.

NOTE. — In the solution of this problem we have omitted small fractions.

25. It is evident that in every case the drawing off of one gallon from the cask full leaves in it $\frac{9}{10}$ of its previous contents. Hence the quantity of wine left the first day is $\frac{9}{10}$ of 10 gallons; the second day, $\frac{9}{10}$ of that; and so on, till at the 20th day it is only 10 gallons multiplied by the twentieth power of $\frac{9}{10}$; and if this quantity be taken from 10 gallons, the remainder will be the quantity of water. By similar reasoning it would be shown that the quantity of water contained in the cask, at the end of the second period of twenty days, would be equal to the quantity last mentioned, multiplied also by the twentieth power of $\frac{9}{10}$. Now, the twentieth power of $\frac{9}{10}$, or .9, is .12157665459. ($.9^4 = .6561$; $.9^8 = .43046721$; $.9^8 \times .9^8 \times$

$.9^4 = .9^{20} = .12157665459.$) The product of this by 10 taken from 10, the remainder is 8.7842334541, the quantity of water in the cask at the end of 20 days; and the product of this by $.9^{20}$ is 1.0679577+ gallons, or more than a gallon and half-pint, Ans.

26. $18.5 \times 18.5 \times 18.5 \times 8 = 50653$; $\sqrt[3]{50653} = 37$in. wide; $8 \times 8 \times 8 \times 8 = 4096$; $\sqrt[3]{4096} = 16$in. deep, Ans.

27. As the metal is 1in. thick, the diameter of the inner sphere is 3in.; $5 \times 5 \times 5 \times .5236 \times \frac{129}{500} = 16.8861$lb., weight of the shell, if it were solid iron; $3 \times 3 \times 3 \times .5236 \times \frac{129}{500} = 3.6473976$lb., weight of the inner sphere, if it were iron; $16.8861 - 3.6473976 = 13.2387+$lb., Ans.

28. As the two hands had precisely changed positions, they together had passed round through all the spaces of the dial-face; but, as the minute-hand always goes through 60 spaces while the hour-hand goes through 5, both going through 65, therefore 65 : 5 : : 60 (the distance passed over by both hands) : $4\frac{8}{13}$min. or spaces, passed over by the hour-hand, and which is also the distance the minute-hand was in advance of the hour-hand. But at 2 o'clock the hour-hand was 10 minutes in advance of the minute-hand; consequently the minute-hand had gained 10min. $+ 4\frac{8}{13} = 14\frac{8}{13}$min. on the hour-hand; then, since the minute-hand always gains 55min. in 60min., how long was it in gaining $14\frac{8}{13}$min.?

55 : 60′ : : $14\frac{8}{13}$: $15\frac{135}{143}$min. = 15 min. $56\frac{92}{143}$sec. after 2, Ans.

29. $20 \times 20 = 400$; $400 \div 3 = 133.\dot{3}$; $\sqrt{133.\dot{3}} = 11.5469$; $\overline{11.5469}^3 = 1539.58+$cu. in., Ans.

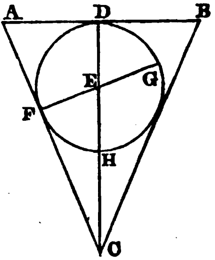
30. $90 \times 40 = 3600$; $\sqrt{3600} = 60$lb., true weight, Ans. $90 - 60 = 30$lb.; $60 - 40 = 20$lb.; 30lb. : 20lb. : : 3ft. : 2ft.; that is, the arms of the scales are to each other as 2ft. to 3ft., Ans.

31. Both wheels being of the same height, and the outer

wheel making two turns while the inner one makes only one turn, it will follow that the outer ring will be twice the circumference of the inner ring. The distance between the rings being 5 feet, and the circumferences of circles being as their diameters, it will also follow that the diameter of the inner ring will be 10 feet, and the diameter of the outer ring 20 feet. And if the diameter be 20 feet, the circumference will be 62.83+ feet, Ans.

32. $72 \times 72 \times 3.141598 = 57001$d. $=$ 237£. 10s. 1d., Ans.

33. The annexed diagram may represent the conical glass, A B C being the cone, and F D G H a globe or sphere immersed in it. If A B be 5 inches, A D will be 2.5 inches, because A D is half of A B. A D C is a right-angled triangle; therefore the side A C may be found; thus, $\sqrt{AD^2 + DC^2} = AC$; $\sqrt{2.5 \times 2.5 + 6 \times 6} = 6.5 = AC$. Because A D E F is a regular figure, and the angles A D E and A F E being equal, each being a right angle, and the sides D E and F E being also equal, because they are radii of the circle D F H G, the sides A D and A F are also equal. A D is 2.5 inches; A F is also 2.5 inches. If A C be 6.5 inches, and A F 2.5 inches, F C will be 4 inches; 6.5 — 2.5 = 4 inches. Then, by similarity of triangles (see page 389), C D : D A : : C F : F E; 6in. : 2.5in. : : 4in. : $1\frac{2}{3}$in. = F E. If F E be $1\frac{2}{3}$in., F G will be $3\frac{1}{3}$in. = $\frac{10}{3}$in, because F G is the diameter of the sphere, and F E the radius, or semidiameter. By mensuration of solids (see pages 435 and 436), we find the contents of the cone and sphere in the following manner:



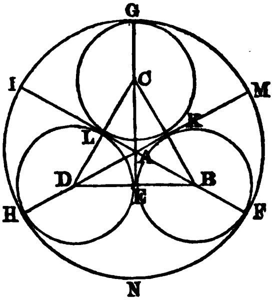
$5 \times 5 \times .785398 \times 2 = 39.2699$in. = contents of the cone;
$\frac{10}{3} \times \frac{10}{3} \times \frac{10}{3} \times .5236 = \underline{19.3925}$in. = contents of the sphere;
19.8774in. = the cubic inches of water

that will remain in the cone after the sphere is immersed. Having taken it for "granted" that cones, spheres, and all similar bodies, are to each other as the cubes of their homologous sides, we say, As the quantity of water it requires to immerse the sphere in the *given* cone is to the cube of the diameter of the sphere, so is any other quantity of water in the conical glass to the cube of the diameter of a sphere that may be immersed in it. Now, the quantity of water given to immerse the *required* sphere is $\frac{1}{5}$ of the contents of the conical glass $= \frac{39.2699}{5} =$ 7.85398 cubic inches. The cube of the diameter of the *given* sphere is $\frac{10}{3} \times \frac{10}{3} \times \frac{10}{3} = \frac{1000}{27}$in. Therefore

$$19.8774\text{in.} : \tfrac{1000}{27}\text{in.} : : 7.85398\text{in.} : 14.634114529+\text{in.};$$

$$\sqrt[3]{14.634114529} = 2.445+\text{in., Ans.}$$

34. Let the larger circle of the annexed figure represent the farm of the lady, and the three smaller circles the farms of her daughters. To construct this figure, make the equilateral triangle C B D, each of whose sides is 10 rods, or inches. Bisect each of the sides D C, C B, B D, in the points L, E, K; and draw the lines I F, E G, H M, at pleasure. Upon the points C, D, B, as centres, and with the distance C L as a radius, describe the circles G L K, L H E, K E F. Upon the point A, as a centre, where the lines I F and H M intersect each other, and with the radius A G, describe the circle G I H N F M, and it will touch the peripheries of the smaller circles without cutting them. The sides of the triangle being 10, the diameter of each of the smaller circles will be 10. Because C D E is a right-angled triangle, C E $= \sqrt{CD^2 - DE^2}$; $\sqrt{10 \times 10 - 5 \times 5} = 8.660254+$;



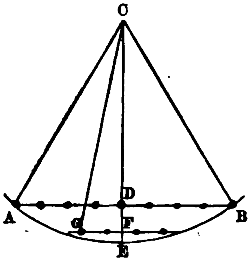
and as C D E and A D E are similar triangles, C E : C D : : D E : D A; that is, 8.660254 : 10 : : 5 : 5.7735027 = A D. If we add H D = 5 to D A, we have the semi-diameter of the larger circle, 5.7735027 + 5 = 10.7735027. By multiplying this last number by 2, we have the diameter of the larger circle, 10.7735027 × 2 = 21.5470054. As the area of a circle may be found by multiplying the square of the diameter by .785398, therefore, by dividing the area by .785398, the quotient will be the square of the diameter. The area of the lady's field is 500 acres = 80000 square rods; 80000 ÷ .785398 = 101859.18, square of the diameter; $\sqrt{101859.18}$ = 319.154006+ rods = diameter of the lady's farm. To find the diameter of each of the daughters' farms, we say, As the diameter of the larger circle in the diagram is to, the diameter of one of the smaller circles in the diagram, so is the diameter of the lady's farm to the diameter of either of her daughters' farms. 21.5470054rd. : 10rd. : : 319.154006rd. : 148.119889+rd. = diameter of the daughters' farms; and the distance of their houses from each other, 148.119889 × 148.119889 × .785398 = 17231.2406+ square rods in each of the daughters' farms; 17231.24+rd. ÷ 160 = 107A. 2R. 31.24+ rods = acres, &c., in each of their farms; 107A. 2R. 31.24rd. × 3 = 323A. 0R. 13.72rd., amount of the 3 daughters' farms; 500A. — 323A. 0R. 13.72rd. = 176A. 3R. 26.28rd. the lady retained. To find the distance of the lady's dwelling-house from those of her daughters, we subtract the semi-diameter of either of the daughters' farms from that of the lady's; thus, 319.154+ rd. ÷ 2 = 159.577+rd.; 148.119+rd. ÷ 2 = 74.059+ rd.; 159.577+rd. — 74.059+rd. = 85.518+rd.

We therefore find that each daughter's farm contained 107A. 2R. 31.22p. The mother retained 176A. 3R. 26.34p. The distance from one daughter's house to the other was 148.119817+ rods. The mother's dwelling-house was distant from her daughters, 85.51+ rods, Ans.

This question can also be solved by using the principle laid down in Art. **645.**

35. The pupil, to understand this problem, will first obtain the number of feet in the diameter of the garden; 10 × 16.5 = 165 feet; 165 — 5 = 160. The trees are, therefore, to be set on a piece of ground 160 feet in diameter. Let the pupil place 1 tree in the centre of the garden; around this let him place 6 other trees, at the distance of 10 feet from each other; he will then perceive that they stand in a hexagonal form. Let him enlarge this hexagon by placing another row of trees around it at the distance of 10 feet each; and this will require 12 additional trees. If we examine this hexagon, we shall find that each side of it contains 3 trees. Let us enlarge this hexagon, by placing another row of trees around it, and we shall find it will require 18 trees, and that each side of the hexagon contains 4 trees. We continue thus to enlarge the hexagon, until we have set 8 rows round the centre tree. Each side of the hexagon will then contain 9 trees. To compute the number of trees in the hexagon, we find the number of trees that compose the periphery of the first hexagon to be 6 trees, and the number that compose the periphery of the larger hexagon to be 48. We therefore add 6 to 48, and multiply the sum by the half of 8 = 4; thus, 6 + 48 = 54; 54 × 4 = 216. To this we add the tree in the centre, 216 + 1 = 217. If we now examine our figure, we find we can set 4 more trees at the base of each side of the hexagon, within the limits of the prescribed field. Therefore, 4 times 6 = 24, to be added to 217; thus 217 + 24 = 241 trees, Ans.

NOTE. — As the radius of the prescribed limits is 80 ft., $80 \div 10 = 8 =$ number of hexagons.



To prove that there can be 4 trees, and *only* 4, placed beyond each side, let A B represent one side of the outer hexagon, C being the centre of the garden, and the arc A E B the prescribed limit, $2\frac{1}{2}$ feet from the outside of the garden. Draw C E perpendicular to A B, and it will also bisect it. $\overline{CD}^2 = \overline{CA}^2 - \overline{AD}^2 = 6400 - 1600 = 4800$; $\sqrt{4800} = 69.28 + = CD$. $CE - CD = 80 - 69.28 = 10.72 = DE$; hence it is evident that there can be another row placed below A B. To find the distance of this row from A B, we have to find the altitude of an equilateral triangle, each side of which is 10 ft. $10^2 - 5^2 = 75$; $\sqrt{75} = 8.66 + = DF$. $69.28 + 8.66 = 77.94 + = CF$. Having placed 4 trees on this row, we wish to ascertain whether the one at G is within the limit. $\overline{CG}^2 = \overline{CF}^2 + \overline{FG}^2 = 6075 + 225 = 6300$; $\sqrt{6300} = CG$. As this is less than 80, G is *within* the limit. If we add one more tree on a line with F G, its distance from C $= \sqrt{6075 + 625} = \sqrt{6700}$, which is greater than 80, and consequently the tree would be *without* the limit. Hence there can be *only* 4 trees added on each side.

36. 90s. = 1080d.; 3s. 9d. = 45d. As A would reap the field in 9 days, he would in 5 days reap $\frac{5}{9}$ of the field; therefore, $1 - \frac{5}{9} = \frac{4}{9}$ = the part of the field which B and C reap; $\frac{45}{1080} = \frac{9}{216}$; $\frac{4}{9} \div \frac{9}{216} = \frac{864}{81}$; $\frac{864}{81} - \frac{81}{81} = \frac{783}{81}$; $\sqrt{81} = 9$; $783 \div 9 = 87$; $87 \div 2 = 43.5$; $(43.5)^2 = 1892.25$; $1892.25 - 1080 = 812.25$; $\sqrt{812.25} = 28.5$; $43.5 - 28.5 = 15$ days = the time B would reap the field. We therefore perceive that A would do $\frac{5}{9}$ of the work, and B $\frac{5}{15}$ of it, in 5 days; $\frac{5}{9} + \frac{5}{15} = \frac{8}{9}$ of the work would be performed by A and B in 5 days. Therefore, $1 - \frac{8}{9} = \frac{1}{9}$ would be performed by C in 2 days, or $\frac{1}{18}$ in 1 day. And if $\frac{1}{18}$ of it be reaped in 1 day, it is evident that it would require 18 days for C to perform the whole labor. Therefore we find that B would reap the field in 15 days, and C in 18 days, Ans. [See *solution, p. 252.*]

OPERATION BY ALGEBRA.

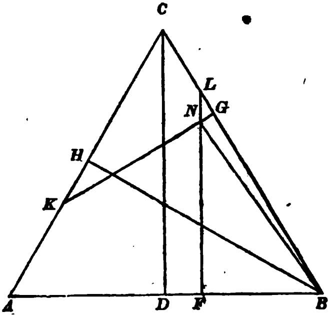
90s. = 1080d.; 3s. 9d. = 45d. Let x = the time in which B can do the work. Then $9 : x :: 45 : \frac{45x}{9}$ = the sum which C must receive from A in part payment for his labor. Then $45 + \frac{45}{x}$ = the money received by C for his 2 days' labor. Now, it is evident that the sum received by C must bear the same proportion to the sum received for the whole work, as the part of the work which he performs bears to the whole work.

Hence $\frac{45 + \frac{45x}{9}}{1080} = \frac{x+9}{216}$ denote the part of the work performed by C. Therefore $\frac{x+9}{216}$ work : 1 work : : 2 days : $\frac{\frac{2}{x+9}}{216} = \frac{432}{x+9}$ = the time in which C would do the whole work. Now, since from the question and the operation we see that A performs $\frac{5}{9}$ of the work, B $\frac{x}{9}$ of it, and C $\frac{x+9}{216}$ of it, it is evident that $\frac{5}{9} + \frac{x}{9} + \frac{x+9}{216} = 1$ work. Or, $\frac{x}{9} + \frac{x+9}{216} = 1 - \frac{5}{9} = \frac{4}{9}$, which, being reduced and transposed, gives $x = 15$ days = the time in which B would reap the field. $\frac{432}{x+9} = \frac{432}{24} = 18$ days = the time in which C would reap the field, Ans.

37. $4 - \frac{1}{8} = 3\frac{7}{8}$; $4 : 3\frac{7}{8} :: 40 : \frac{155}{4}$; $1\frac{1}{2}$yd. = 24na.; 24na. — $1\frac{1}{2}$na. = $22\frac{1}{2}$na.; 24na. : $22\frac{1}{2}$na. : : $2\frac{1}{4}$yd. : $2\frac{7}{64}$yd. = $\frac{135}{64}$yd.; $\frac{155}{4} \times \frac{135}{64} = \frac{20925}{256} = 81\frac{189}{256}$yd.; 5qr. = $1\frac{1}{4}$yd.; $20 - 1\frac{1}{4} = 18\frac{3}{4}$yd.; $\frac{18\frac{3}{4}}{20} = \frac{15}{16}$; 5qr. = 20na.; $20 - \frac{1}{2} = 19\frac{1}{2}$; $19\frac{1}{2} \times \frac{15}{16} = 18\frac{9}{32}$na.; $81\frac{189}{256}$yd. = $1307\frac{13}{16}$na.; $1307\frac{13}{16} \div 18\frac{9}{32} = 71\frac{7}{13}$yd., Ans.

38. Let the tower at A be 30 feet high; that at B, 40; at C, 50.

First. What point F in the side A B is equally distant from the top of the tower at A and the top of that at B?



The square of the distance from F to the top of the tower A is the square of its distance from the foot of A + 900; the square of the distance from F to the top of B is the square of the distance from the foot B + 1600; if the distances of F from the tops of A and B are equal, the square of its distance from the foot A is greater by 700 than the square of its distance from the foot B. $AF^2 - BF^2 = 700$. But $AF + BF = 200$; now, $AF^2 - BF^2 = (AF + BF)(AF - BF)$, $AF - BF = 3\frac{1}{2}$; and therefore $AF = 101\frac{3}{4}$, $BF = 98\frac{1}{4}$.

Finding a similar point G on B C, we get $BG = 102\frac{1}{4}$, $CG = 97\frac{3}{4}$.

At what point L does the line F L perpendicular to A B meet B C? Draw C D perpendicular to A B.

Then, as A and B are similarly situated with regard to C, $AD = DB$, and each $= 100$.

Then $DF = 1\frac{3}{4}$; therefore $CL = 3\frac{1}{2}$, as D F must be $\frac{1}{2}$ of C L, if D B is $\frac{1}{2}$ of C B.

If $CL = 3\frac{1}{2}$, L G must be $CG - CL = 97\frac{3}{4} - 3\frac{1}{2} = 94\frac{1}{4}$. Draw G K perpendicular to B C.

Now, the foot of the ladder is on the line F L, for every point in F L is equally distant from the top of the two towers A and B. Again, it is on the line G K, for every point in G K is equally distant from the top of the towers B and C; therefore it is on the intersection N of F L and G K.

Now, the triangle L N G is similar to C D B; then L N is 2 times N G, and $\overline{LN}^2 = 4$ times $\overline{NG}^2$; then $\overline{LG}^2 = 3$ *times* $\overline{NG}^2$; $\overline{LG}^2 = 8883.0625$, then $\overline{NG}^2 = 2961.0208\dot{3}$.

The square of the length of the ladder = height of tower $C^2 + C\,G^2 + N\,G^2$.

Square of height of C, .	2500
$C\,G^2$, ...	9555.0625
$N\,G^2$, ...	$2961.0208\dot{3}$
Square of length of laddder,	15016.08333
Length of ladder, . .	122.54+

To find the distance of the foot of the ladder from the foot of each tower, we subtract the square of the height of each tower from the square of the length of the ladder. This gives

$\overline{N\,A^2} = 14116.08\dot{3}$ $N\,A = 118.811$

$\overline{N\,B^2} = 13416.08\dot{3}$ $N\,B = 115.827$

$\overline{N\,C^2} = 12516.08\dot{3}$ $N\,C = 111.875$

SECOND SOLUTION.

A line drawn from either angle to the middle of the opposite side divides the garden into two equal right-angled triangles; and the length of this line, found in the usual way, is 172.2+ft. Draw, in the same manner, lines from the other two angles, and the three lines will intersect each other at the centre; and the garden will be divided into six equal triangles, *similar* to the first two. Then 172.2+ : 200 : : 100 : 115.6ft. = distance of the centre from the foot of either tower. A ladder placed on this centre, reaching to the top of one of the towers, will be the hypothenuse of a vertical triangle, of which the tower is the perpendicular, and 115.6ft. the base. Now, since the three vertical triangles have equal bases, and, since the height of the tower B is an arithmetical mean between the heights of the towers A and C, it follows that the square of one base, plus one third of the squares of the three perpendiculars, will equal the squares of an *average* hypothenuse, or the length of a ladder, which, placed at some point, will reach to the top of each of the towers. Finally, find the distance of this point from the foot of each tower, as in the *last* paragraph of the first solution.

ADDITIONAL SOLUTIONS.

Ex. 16. Arith. p. 295 (Key, p. 156). It is evident that the interest on the note, for the required time, at 6 per cent., is equal to the interest of the *nominal* present worth for the same time, at $6\frac{1}{2}$ per cent. Therefore, the note must exceed the nominal present worth in the ratio of $6\frac{1}{2}$ to 6, or of 13 to 12; hence the *nominal* discount, or interest of the note, must have been $\frac{1}{13}$ of the note. Then, since the interest of any sum for one year at 6 per cent. is $\frac{6}{100}$ of the same sum, we have $\frac{6}{100} : \frac{1}{13} :: 1$ year : 1 year, 3 months, $11\frac{7}{13}$ days, Ans.

Ex. 36. Arith. p. 444 (Key, p. 248). As A could reap the field in 9 days, he would in 5 days reap $\frac{5}{9}$ of the field, and B and C would, in 5 days, reap $1 - \frac{5}{9} = \frac{4}{9}$ of the field. Now, the whole sum received for reaping the field was 90 shillings; consequently A would receive $\frac{5}{9}$ of 90s. = 50s. for his labor, and B and C $\frac{4}{9}$ of 90s. = 40s. for their labor.

But, by the question, B receives 3s. 9d. = 3.75s. less by employing C. Hence, as the sum which B receives is to the sum A receives, so is the sum B receives less by employing C to the sum A receives less by employing C. Of this proportion there are given only the second term, 50s., and the third term, 3.75s.

Since the product of the extremes is equal to the product of the means (Art. 336), it is evident that the product of the first and fourth terms will be $50 \times 3.75 = 187.5$.

Also, since the sum of the first, third, and fourth terms is 40s., and the difference between 40s. and the third term, 3.75s., is 36.25s., it is evident that the sum of the first and fourth terms will be 36.25.

Then, having the sum of the first and fourth terms, 36.25, and their product, 187.5, these terms may be found by Art. 552.

Thus, $36.25 \div 2 = 18.125$, half the sum of the two terms; $(18.125)^2 = 328.515625$, the square of half the sum; $328.515625 - 187.5 = 141.015625$, the square of half the difference; $\sqrt{141.015625} = 11.875$, half the difference of the first and fourth terms; $18.125 + 11.875 = 30$s., the first term, or the sum B receives; and $18.125 - 11.875 = 6.25$s., the fourth term, or the sum A receives less by employing C.

As B received 30 shillings, C received $40 - 30 = 10$s.

30s. : 90s. :: 5 days : 15 days, the time B would reap the field.

10s. : 90s. :: 2 days : 18 days, the time C would reap the field.

THE END.

"STANDARD AND ENDURING."

GREENLEAF'S

New Mathematical Series.

MERITORIOUS AND ECONOMICAL,

Tested and proved, these books have an unprecedented popularity.

Over 1,600,000 Copies in use!

DISTINGUISHING FEATURES.

1. A complete course for all classes of learners, upon the same plan and by one author.
2. The Primary Books are strictly elementary, and not mere abstracts or abridgments of larger works.
3. Each book is complete in itself, but so constructed as to prepare the learner for systematic progress.
4. The methods and processes are entirely modern, and most happily combine science and practice.
5. The course is arranged with a view of saving the time of the learner, and of expense, by the use of the smallest number of text-books.
6. The several books are beautifully illustrated, printed upon sized paper, and handsomely and durably bound.

SCHOOL OFFICERS

Contemplating a change of Mathematical Text-Books, are invited to examine GREENLEAF'S NEW SERIES. Address,

ROBERT S. DAVIS & CO.,

36 Bromfield St., Boston.

GREENLEAF'S

NEW SERIES OF ARITHMETICS.

Each Book is complete in itself, and sold separately.

Superior to all others in use.

1. In clearness of statement.
2. In progressive development.
3. In logical arrangement.
4. In conciseness and accuracy.
5. In simplicity of analysis.
6. In improved processes.
7. In conformity to business usages.
8. In variety of practical problems.
9. In conformity to new legal standards.
10. In adaptation to all classes of pupils.

NEW PRIMARY ARITHMETIC.

This book is on the plan of OBJECT TEACHING, and is beautifully illustrated.

It combines WRITTEN AND ORAL EXERCISES, in lessons, all of an inviting and attractive nature.

PRIMARY CLASSES are fully prepared by its use for the next book of the series.

NEW ELEMENTARY ARITHMETIC.

A COMPREHENSIVE MANUAL, at once inductive, analytic and useful, expressly prepared to provide for new educational demands.

It is especially adapted to INTERMEDIATE CLASSES, and to the wants of pupils whose limited opportunities require that they should learn much in the shortest possible time.

It brings together in a brief form the essentials of arithmetic, mental and written, the fundamental rules, fractions, percentage and interest, and with the *New Primary Arithmetic* forms a COMPLETE COURSE, in only TWO BOOKS.

GREENLEAF'S NEW MATHEMATICAL SERIES.

NEW PRACTICAL ARITHMETIC.

A COMPLETE ARITHMETIC, entirely modern in its methods, and combining throughout practical utility with scientific accuracy.

It was the first to make written exercises intellectual, to thoroughly enforce educational results by means of systematic review questions and exercises, and to introduce certain TECHNICAL WORK of great value to Business Men, Farmers, Masons and Carpenters.

The extent to which this book has caused like books to be revised or re-written, is a sufficient evidence of its standard character.

Its important features, such as the early introduction of the decimal point, the avoiding of many special rules, and of obsolete or extraneous matter, and the introduction of valuable new applications, have aided to make this work the most popular of its kind now before the public.

In it may be found, treated in a practical manner, the Metric System, Stocks and Government Securities, new par of English Exchange, and the new legal values of Foreign Coins.

It will be noted that this work, with Greenleaf's New Primary and New Elementary Arithmetics, forms a clear, practical, comprehensive system, a time-saving and labor-saving course,

COMPLETE IN ONLY THREE BOOKS,

and is believed to be the cheapest, most convenient and productive, that has ever appeared.

By their use, in comparison with some *extended* courses of five or more books, it is safe to say that the learner may not only make better progress, but at least

Save Two Years of Valuable Time!

GREENLEAF'S ARITHMETICS

have been approved for, and are now used, in the most important Schools of

BOSTON, NEW YORK CITY, PHILADELPHIA,

and many other cities; and have been recently introduced into upwards of **6,800 Schools** in the EASTERN AND MIDDLE STATES, including *entire uniformity* in numerous counties.

GREENLEAF'S is the standard in upwards of **1,000** CITIES and TOWNS in the NEW ENGLAND STATES alone.

TESTIMONIALS.

From REV. LEVERETT GRIGGS, *School Visitor, Bristol, Conn.*, April 3, 1875.—"Some years since, Greenleaf's Arithmetics were used in our schools. They were displaced by ————. Last Fall the Board unanimously adopted Greenleaf's New Series, to the great satisfaction of our most accomplished teachers, as well as that of our school officers."

From REV. A. E. DENNISON, *Acting School Visitor, Plainville, Conn.*, June, 1875.—"Greenleaf's New Series of Arithmetics give satisfaction. We are well satisfied with the change from ————.

From S. P. WILLIAMS, A. M., *Principal of Union Graded School, Plainville, Conn.*, June, 1875.—"Greenleaf's New Series has many merits. I am much pleased with the change."

From NATHANIEL WILCOXEN, *Chairman of School Board, Oxford, Conn.*, March 11, 1875.—"I have given Greenleaf's New Series of Mathematics a careful examination. I find it, I may well say, the best adapted to our schools of any with which I am acquainted."

From LINUS T. GILBERT, *School Visitor, Litchfield, Conn.*, Jan. 2, 1875.—"Greenleaf's New Series of Arithmetics are just what we have wanted for some time. I think them *far superior* to ————, formerly in use. I am pleased that they have been introduced in our schools. *There* are many new features in the books that must commend them to towns now using books *like* ————, *as* we formerly had."

From S. T. Dutton, *Principal South Norwalk School, Ct.*, April, 1875.—"I have used Greenleaf's Mathematical Works, and take pleasure in testifying to the rare excellencies which they possess. So well are they adapted to the popular wants that I never heard a teacher speak of them as deficient in any respect."

From E. T. Hayward, *Principal W. District Grammar School, Rockville, Ct.*, April, 1875. "Having tested Greenleaf's New Arithmetics in my school for a term of fourteen weeks, I cheerfully recommend them to the favorable consideration of all teachers in need of good text-books. The prominence given to the principles embodied in the fundamental rules, and the constant attention to practical applications throughout the works are prominent excellencies."

From Heman R. Timlow, *Acting School Visitor, Southington, Ct.*, Dec., 1874.—"I have examined with care Greenleaf's New Arithmetics, and for three months they have been in use in the eleven school districts of this town. The period of use is too brief to justify the commendation a longer experience might contribute. But after repeated personal examinations of the series, and quietly watching the fruits as they appear, I am prepared to defend the change the School Board made last spring; and not only this, but I feel warranted in stating the conviction that the Greenleaf's New Series is surpassed by none in the country"

From W. E. Hawkins, *East Hartford, Conn.*, April, 1875.—"I have used Greenleaf's Arithmetics for some years, and can heartily recommend the New Series as meeting the wants of the school room in every particular."

From L. N. Cummings, *Principal of High School, Portland, Conn.*, April 29, 1874.—"I have recently introduced into my school Greenleaf's New Practical Arithmetic. Having previously examined many other editions, I can truly say it is the most instructive and complete of any Arithmetic I have ever seen, and I would heartily recommend it to the consideration of teachers and school officers everywhere."

From R. B. Clarke, *Fitchburg*, Jan. 23, 1875.—"We have used Greenleaf's Series, as a whole, or in part, for several years, with entire satisfaction. In my opinion they have not been surpassed in excellence as text-books."

From Frank M. Wilkins, *Principal of High School, Canton, Mass.*—"We use in our school Greenleaf's National Arithmetic, Algebra, Geometry and Trigonometry. I know of no series of text-books on any subject which I use with so much pleasure and satisfaction in school. The Algebra especially is above praise."

GREENLEAF'S NEW MATHEMATICAL SERIES.

From Percy S. Bryant, *Principal of High School, Thompsonville, Conn.*—"I have carefully examined Greenleaf's New Mathematical Series, and consider it superior to any with which I am familiar. I gladly recommend it to all who desire first-class text-books."

From H. P. Topliff, *Principal, Graded School, Brooklyn, Conn.*—"I consider Greenleaf's New Elementary Algebra the *best* of the best, having used it as a text-book in the school-room for some years. It is the text-book on Algebra adopted for this town."

From A. L. Gerrish, ***President of Maine Central Institute.***—**"Many years acquaintance with Greenleaf's Arithmetics, with a good opportunity to compare them with others, has led me to the conclusion that they are *the best extant*, and that their great popularity has been won by real merit."**

From E. D. Sanborn, *Professor in Dartmouth College.*—"Greenleaf's works have all a deserved popularity, and their general use attests their excellence. *The most experienced teachers adopt them.*"

From W. L. P. Boardman, *now Principal of Lewis Grammar School, Boston.*—"I have always liked Greenleaf's Series of Arithmetics. The improvements contained in the new practical Arithmetic, make it in my estimation, one of the *very best* books before the public."

From I. Van Houten, *President of Board of Education, Paterson, N. J.*—"At the last meeting of the Board of Education of this city, Greenleaf's New Series was adopted for exclusive use in our schools."

From Alanson Palmer, *Superintendent of Schools, Long Island City, N. Y.*—"A trial of three years has satisfied all our principals of the general superiority of Greenleaf's Arithmetics, and all are favorable to the books. The New Practical we consider *perfection*, both as to arrangement and on account of new and important matter contained in it."

From George C. Fenn, *Principal of Grammar School, Ware, Mass.*—"For accuracy and conciseness, Greenleaf's New Series of Arithmetics, now used in all our schools, is unsurpassed."

From A. M. Beecher, *Principal of Seminary, Hartford, Ct.*—"We have recently adopted *Greenleaf's Series.* Our professor gives the books cordial commendation."

GREENLEAF'S NEW MATHEMATICAL SERIES.

From Asa Morehouse, *School Commissioner, Second District, Orange County, N. Y.*—"Greenleaf's New Series of Arithmetics are quite extensively used in this county, and wherever used they are giving good satisfaction."

From A. C. Brackett, *Principal of Storer Normal School, W. Va.*—"We shall hereafter use Greenleaf's Arithmetics, and no others, in this institution."

From D. C. Stone, *Professor in Oakland College, Cal.*—"I have used Greenleaf's Arithmetics for several years, and am not disposed to change."

From J. S. Cilley, *Principal of Graded School, Brandon, Vt.*—"We have used Greenleaf's Arithmetics in the Graded School for five years, and we shall continue to use them in years to come."

From Isaiah N. Leigh, *County Examiner, Hunterdon Co., N. J.*—"I have tested in the class-room, during a period of thirteen years, the different series of Arithmetics claimed to be the best, and found none to suit the wants of pupils better than Greenleaf's Series. Greenleaf's Arithmetics are used with great uniformity in this county.

From J. H. Walker, *County Examiner, Bergen County, Englewood, N. J.*—"Having used Greenleaf's Mathematical Text-books in the school-room for several years, it gives me pleasure to state that I *know of no books so well adapted* to the wants of our public schools."

From Dr. John M. Brewster, *Superintendent of Public Schools, Pittsfield, Mass.*—"Greenleaf's Arithmetics were introduced last Fall, by the unanimous vote of the School Committee, into all the public schools of Pittsfield; and all concerned, teachers and pupils, are glad of the change. We feel confident that *now* we are in possession of the best Arithmetics our pupils have ever used."

From Charles T. Whitman, *Town Supervisor of Schools, Norway, Me.*—"Greenleaf's New Practical Arithmetic *is the best Arithmetic we have ever used.* These and similar words of commendation are the universal sentiments of our teachers."

GREENLEAF'S
HIGHER SCHOOL MATHEMATICS.

NEW ELEMENTARY ALGEBRA.

This is a well-known popular favorite, and admitted on all hands to be a work of rare excellence. Its extraordinary success has led to the publication of a number of imitation treatises, but all falling far short of the merit of the original.

NEW SHORTER COURSE IN GEOMETRY.

The increased attention given of late to the application of Geometry in Elementary Education, has made a necessity a brief course like this. It contains all the more important theories and problems of the science, with exercises for original thought and practical applications. The success of this work shows that it is meeting a general want.

RECOMMENDATIONS.

From REV. GEORGE GANNETT, *Principal of the Gannett Institute for Young Ladies, Chester Square, Boston.*—"We have for some years, and exclusively, used in our Institute, Greenleaf's Mathematical Series. With these works, improved and perfected as they have been from time to time, I have been perfectly satisfied, and have no desire to exchange them for any others with which I am acquainted. They combine all the best qualities which I can conceive of as desirable in such books. With the New Elementary Algebra I am especially pleased."

From EDWARD G. WARD, *Principal of Public School No.* 11, *Jersey City, N. J.*—"My experience with Greenleaf's Mathematics justifies me in speaking of them in terms of the highest commendation. The Geometry, the Algebra, the Arithmetic,—are all good."

From J. D. SMITH, *Principal of Worcester Academy, Mass.*—"My experience has convinced me that Greenleaf's Elementary Algebra stands among *the very best* works on the subject. We use the work in this institution."

From GEORGE A. WENTWORTH, *Professor of Mathematics, Phillips Academy, Exeter, N.H.*—"I have used Greenleaf's New Elementary Algebra for more than six years. It has proved so satisfactory that I have no desire or inclination to change for any other."

From ISAAC N. CARLETON, *Principal of Normal School, Conn.*—"Although sorely pressed to adopt some other Algebra, we still continue to use Greenleaf's in the Normal School, and consider it *the best.*

GREENLEAF'S
UNIVERSITY MATHEMATICS.

New Higher Algebra.

This elegant treatise, containing many important improvements, meets very fully the requirements of the best standard of instruction in Agricultural Colleges, Institutes of Technology and Universities.

Elements of Geometry.

Its arrangement is the simple order adopted by LEGENDRE. The acknowledged improvements of M. DECUNHA have been carefully incorporated into the work. Numerous miscellaneous Geometrical Exercises are a distinctive feature, and make the course both complete and practical.

Elements of Trigonometry.

In this comparatively short course, Plane and Spherical Trigonometry is *analytically* treated and *practically* applied. The Trigonometric Functions, in accordance with the modern method, have been regarded as *ratios*. The work is *full* and *comprehensive*, containing the most useful applications, with full *tables*.

IMPORTANT TESTIMONY.

From C. O. THOMPSON, *now Principal of Institute of Technical Science, Worcester, Mass.*—"Greenleaf's Algebras work well in my school. The New Elementary is logical, lucid, progressive, and dignified, and the NEW HIGHER is, if possible, better than the Elementary."

From CHARLES H. SMITH, *Professor of Mathematics, Bowdoin College, Me.*—"I have carefully examined Greenleaf's Algebras, Geometry, and Trigonometry of the New Comprehensive Series, and find they are admirably adapted to the purpose for which they are designed. Clear, sufficiently concise, and printed in excellent taste, they can hardly be surpassed as introductory text-books. Greenleaf's New Higher Algebra has long been a popular text-book in this college."

From J. D. RUNKLE, *now President of Massachusetts Institute of Technology, Boston.*—"The greatest care has evidently been taken with the style of the Geometry. After a careful search we have not been able to find one expression which we felt sure could be improved."

From SELDEN J. COFFIN, *Adjunct Professor of Mathematics, Lafayette College, Pa.*—"Greenleaf's New Higher Algebra, by its judicious arrangement and the practical nature of its examples, commends itself to the favor of teachers. The work excels in the mode of presenting the general theory of Equations; and the sections on Variation, Useful Formulas, and the chapter on Radical Quantities are of great practical value."

From E. J. THOMPSON, *Professor of Mathematics, University of Minnesota.*—"I have examined Greenleaf's Trigonometry, and like it well. I am pleased with its systematic and logical treatment of the subject, and also with the practical work it introduces to the student. I shall use it in my future classes."

PARKER'S EXERCISES IN COMPOSITION:

REVISED AND ENLARGED,

BY JAMES H. HAMILTON, M. D.

◆

A COMPLETE COURSE IN ONE BOOK.

The attention of Teachers is specially invited to the improvements contained in the present edition of this useful MANUAL, which has been a great favorite in the School Room for so many years. It now appears in a new and more attractive dress, and in a form to render it more useful.

It is believed that "PARKER'S EXERCISES," in its revised and enlarged form, will prove a useful auxiliary in teaching Composition and Rhetoric, for the following reasons:—

1. The work is *progressive*, commencing with very easy and simple exercises, and concluding with the general principles of Rhetoric,—thus comprising a complete course in one book.

2. Capitals and Punctuation are clearly and fully treated where they should be, in the body of the work, and not in a brief appendix.

3. The whole work is divided into four distinct parts, presenting in order the different departments of Composition; as words, sentences, discourse, etc.

4. A Chapter of Miscellaneous Exercises is given as a general Review of all the Lessons.

5. The exercises in this volume have stood the test of the School Room. Most of the original exercises have been retained, and a number of new ones added to render the work more complete.

This work has been approved for, and is now used in some of the most important schools of New York City, Boston, Worcester, Lowell, Providence, New Haven, etc., and in the public or private schools of many other important places in different States.

WHAT IS SAID OF IT.

From Rev. Edward Cooke, D. D., *late Principal of Wesleyan Academy, Wilbraham, Mass.*—"We have used Parker's Progressive Exercises in English Composition, in the institution the past year and are well pleased with the work. It possesses the advantage of presenting just what the pupil needs to learn, and in the briefest possible manner. I know of no similar work which answers our wants so well; "*multum in parvo*," is practically realized in it."

From Prof. F. A. Allen, *late Principal of State Normal School, Pa.*—"I am glad to see this new little gem of a book, Parker's Progressive Exercises in Composition. Many thanks to Mr. Hamilton for revising this best of old standard works. We like the book and its division into four parts. Success to it.

From Prof. Alexander Winchell, LL.D., *of University of Michigan.*—"I always liked Parker's Exercises in English Composition, and I like still more the revised and enlarged edition. My tastes, no less than occasional occupations, have led me to become acquainted with works of this class, and I do not express a recently formed opinion, when I say that Parker's Exercises has always had my highest approval. The writing of compositions becomes here an amusement. It is generally the dreaded drudgery of the school.

"*This work begins at the level of the lowest capacity, and floats him along without his consciousness, certainly without a wearisome effort, to the mastery of a finished style.* I presume it was intended for preparatory schools; but I am sure a Freshman in college could have no better rhetorical drill than to begin with the first lesson and faithfully follow on to the last. Small as the book is, unpretending as it is, it teaches exactly those things in which my experience informs me that even college students are very apt to be sadly deficient."

From Joseph Hall, A. M., *Principal of Public High School, Hartford, Ct.*—"We continue to use the revised edition of Parker's Exercises in English Composition, with constant and uniform success. I consider it to be by far the best work of the kind that I have ever seen."

From J. P. Sherman, A. M., *President Pennsylvania Female College.*—"I have used Parker's Exercises in English Composition, and have found it to be an excellent work, and one in which the young take great interest.

"The new enlarged edition is much improved, and will doubtless have a large circulation."

From H. Dame, *Principal of High School, Bangor, Me.*—"It gives me pleasure to state that Parker's Exercises in Composition has been in use in our High School during the past three years, to our entire satisfaction. I know of no work so admirably adapted to give the younger classes in a High School that training in the Art of English Composition, so essential to a good education, and yet, unfortunately, so often neglected, or crowded out by other studies. The principles are stated with clearness and conciseness, the exercises for practice are numerous and varied, and cover sufficient ground to occupy pupils, from two to three years; indeed, with proper teaching, it may be made to take the place of a regular text-book in Rhetoric. I can fully endorse it as a school-book leaving nothing to be desired."

PARKER'S EXERCISES IN COMPOSITION.

From E. O. Hovey, A. M., *Principal of High School, Newark, N. J.*—"Many thanks for Parker's Exercises in English Composition, revised by Hamilton. Parker's Exercises was always a good book, but it is now *the* English Composition *par excellence.*"

From Edwin Davenport, A. M., *St. John's School, Sing Sing, N. Y.*—"I have had occasion to admire in Parker's Exercises in English Composition, as revised by Dr. Hamilton, the completeness and the system of the work. Full as it certainly is, in every matter pertaining to the important Art of Composition, it is as far as possible from being an undigested mass of grammatical, rhetorical, and miscellaneous information. By a careful method the student is led to a practical acquaintance with all the requirements of correct expression even down to those last minutiæ of advice, which many a practical writer would find of no small service. *The system of the book, in its arrangement, divisions, judicious fullness of illustration, and examples for practice, leaves nothing to be desired.*

"The topic of Punctuation is treated in a particularly sensible way.

"The work is excellently adapted to pupils passing through the last three or four years of our schools. No other is so comprehensive and thorough."

THE QUESTION OF EXPENSE.

How great is the cost of time to the learner by the use of badly constructed text-books, or such as are behind the time. Since time is money, it may often be positive economy to adopt improved text-books. Labor-saving tools are as much a necessity in the school-room as elsewhere. A cost of a change at introduction rates, is hardly more than nominal.

The old book in use, when worn out would have to be replaced at full retail price. The introduced book can be had after the first supply at only two-thirds of the retail price, and if the one in use is surrendered, even at half-price.

The question, then, of expense is readily answered. Books are the tools of the school-room, and you cannot afford to be without the best.

School Boards and Teachers,

are invited to correspond freely with us.

All of our educational publications are of a standard character, and will be supplied for *first introduction* at two-thirds retail prices. Or, when a corresponding old book in use is exchanged, at one-half of the retail price.

Address

ROBERT S. DAVIS & CO., Publishers, BOSTON,

OR THEIR INTRODUCING AGENTS:

ORLANDO LEACH,
142 and 144 Grand Street, New York.

BENJAMIN H. SANBORN,
Office at Brown & Gross' Book-store, Hartford,

Who will supply Committees and Teachers on Favorable Terms.